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"Tug of war" Maths & Sport Project

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Abstract

Tug of war is a sport which originated from rituals practiced by ancient populations and now is an organized sport. In this paper we present the project that we proposed to a group of high school students, in order that they analysed the physics of tug of war and produced a model. The project was carried out with the support of the math and physics teachers of the students involved. After some introductory lessons, the students worked independently producing results that were discussed with us. Virtual simulations were carried out in the school laboratory using first Excel and then the open source LÖVE graphics engine. The experiments were done in the school gym. The students asked the collaboration of players of some teams of tug of war to validate the expected tactics of the mathematical model.

Keywords: *Mathematics, Physics, Sport*

1. Introduction

The authors of the present paper are engaged in interdisciplinary research initiatives of the FDS Laboratory of the Politecnico di Milano (**F**ormation, **D**idactics, **S**cience Communication) and, in particular, they are interested to "contamination" projects between scientific thoughts, artistic insights and sport. It is well known that Mathematics plays a very important role from a cultural point of view in the modern world and the students realize that Mathematics is also a powerful tool, rather than being a closed discipline. Mathematical concepts connect new ideas to other ideas learned previously or in other educational experiences, helping to learn concepts used in other disciplines.

In 2002, the FDS laboratory conceived a program called "Progettiamo con la Matematica" (Planning with Mathematics) to let students and teachers of upper secondary schools know how Mathematics can enter in many aspects of human life, through the modeling of phenomena, situations, forms or allowing predictive studies. The experience aims to develop in the working group the ability to analyze a problem, its adequate formalization, identification of a suitable resolution strategy, and a subsequent verification of the results obtained. The project also allows an unusual collaboration between students, teachers and university tutors; it favors the ability to communicate both by means of information technology and orally. Students can take advantage of the research developed both in the school career and in national / international competitions.

In this article, we present a project that combines Mathematics, Physics and Sport.

We propose the "Tug of War" project to a group of pupils of the last year high school. They already knew the fundamental laws of dynamics and in particular the theoretical point of view of the second law of Newton.

2. Historical hints

The *Oxford English Dictionary* says that the phrase "tug of war" originally meant "the decisive contest; the real struggle or tussle; a severe contest for supremacy". Only in the 19th century it was used as a term for an athletic contest between two teams who haul at the opposite ends of a rope.

The origins of tug of war are uncertain, but this sport was practiced in ancient Egypt, Greece and China, where it was held in legend that the Sun and Moon played Tug of War over the light and darkness. In ancient Greece the sport was called *helkustinda* (Greek: ἑλκυστίνδα), *efelkustinda* (ἐφελκυστίνδα) and *dielkustinda* (διελκυστίνδα), which derives from *dielkō* (διέλκω), meaning amongst others "I pull through", all deriving from the verb *helkō* (ἕλκω), "I draw, I pull".*Helkustinda* and *efelkustinda* seem to have been ordinary versions of tug of war, while *dielkustinda* had no rope, according to Julius Pollux. It is possible that the teams held hands when pulling, which would have increased difficulty, since handgrips are more difficult to sustain than a grip of a rope. Tug of war games in ancient Greece were among the most popular games used for strength and would help build strength needed for battle in full armor. Tug of war stories about heroic champions from Scandinavia and Germany circulate Western Europe where Viking warriors pull on animal skins over open pits of fire in tests of strength and endurance, in preparation for battle and plunder.

In the XVI and XVII century, tug of war is popularized during tournaments in French châteaux gardens and later in Great Britain.

In XIX century, tug of war begins a new tradition among seafaring men who were required to tug on lines to adjust sails while ships were under way and even in battle.

3. Tug of war games

Tug of war was part of the Olympic Games from 1900 until 1920, but has not been included since.

Figure 1. Olympic Games - St.Louis 1904. Source:Public Domain

The sport is part of the World Games. The Tug of War International Federation (TWIF) organizes World Championships for nation teams biannually, for both indoor and outdoor contests, and a similar competition for club teams. In England, the sport was formally governed by the AAA until 1984, now the Tug of War Association (formed in 1958) is the delegate authority. Instead the Scottish Tug of War Association (formed in 1980) organizes the competitions in Scotland. The sport also features in Highland Games there. The sport is played almost in every country in the world. However, a small selection of countries has set up a national body to govern the sport. Most of these national bodies are associated with the

International governing body call TWIF, which stands for The Tug of War International Federation. As of 2008 there are 53 countries associated with TWIF, among which are Scotland, Ireland, England, India, Switzerland, Belgium, Italy, South Africa and United States.

3.1 Fundamental Rules

1- Two teams formed of eight players (only women or only men), whose total mass must not exceed a maximum weight as determined for the class, align themselves at the end of a rope approximately 11centimeters in circumference and 33.5meters long. The rope is marked with a "center line" and two markings 4 meters either side of the centerline. The teams start with the rope's centerline directly above a line marked on the ground, and once the contest (the "pull") has commenced, attempt to pull the other team such that the marking on the rope closest to their opponent crosses the centerline, or the opponents commit a foul.

2- Lowering ones elbow below the knee during a “pull”- known as “Locking” - is a foul, as is touching the ground for extended periods. The rope must go under the arms; actions such as pulling the rope over the shoulders may be considered a foul. These rules apply in highly organized competitions such as the World Championships.
However, in small or informal entertainment competitions, the rules are often arbitrarily interpreted and followed.

4. General analysis

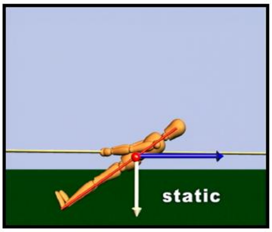
While the sport is practiced both outdoors and indoors, we considered the indoor variant of tug of war only, as the soil and ground conditions would furthermore complicate the study, as for the possibility for the players to sink their heels in the ground.

Figure 2. Indoor and outdoor matches. Source:Public Domain

We can distinguish two player roles:

- The first seven players (pullers)
- The eighth player (anchor man)

Figure 3. Players. Source:Public Domain



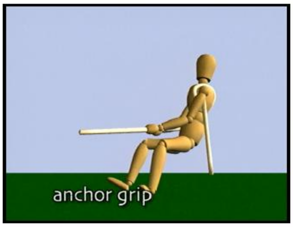


Figure 4. Scheme of Players. Source:Our Creation (2018)

4.1 The first seven players

The elements that characterize these players are:

- the position maintained during the race, characterized by an angle α between the body and the platform
- the force exerted in the shooting action
- the position of the rope always parallel to the platform

4.2 The eighth player

The elements that characterize this player are:

- the position of the rope that is passed diagonally along the body
- this player has an angle β of different inclination respect to the angle of the other players

- the section of rope between him and the seventh player is not parallel to the platform but forms an angle θ with respect to it

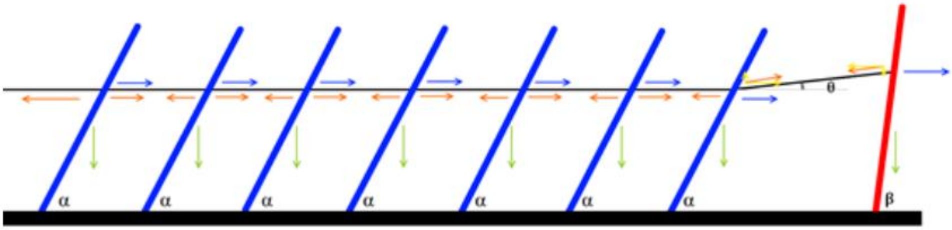


Figure 5. Sport’s Diagram. Source:Our Creation (2018).

5. Physical model

Initially, we examine the conditions for static equilibrium of the first six pullers. We assume that the rope is massless, parallel to the ground except for the last two pullers and we approximate the athletes’ bodies to rigid bars of the same length.

By applying Newton’s second law, we can draw an equation, which ties the tilt angle α of a puller to the force he exerts on the rope. This force is equal to the difference between the tension coming from the front of the athlete and one coming from behind.

Table 1. Variables and Forces

Variable	
T_n	forward tension
T_{n+1}	back tension
F_n	force
F_p	weight
F_a	friction

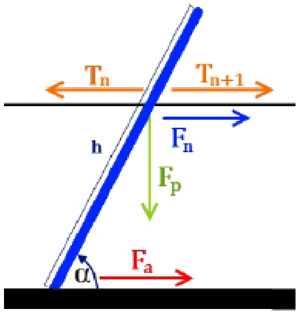


Figure 6. Scheme. Source: Our Creation(2018)

Therefore, we find a minimum tilt angle α_{min} tiled to the coefficient of friction: if the angle α of any puller falls below this value, he falls backwards, due to insufficient friction with the floor. To obtain a measure for the coefficient of friction μ between the athletes' shoes and the floor, we suggest carrying out an experiment in the school gym with dynamometers, some weights and a pair of sports shoes.

The students calculated the static friction force corresponding to an empty shoe and the same shoe with different weights, obtaining the average value of the friction coefficient between the sole of the shoe and the platform.

This mean value is $\mu = 0.71 \pm 0.02$, with R^2 coefficient very near to 1.

As regards the last two athletes, we can draw a similar equation to describe the tension of the rope in front of the seventh puller, which depends on three variables: the tilt angles α and β of the two pullers and the bending angle θ of this section of rope between them.

Thanks to information from Mr. Zoccoli, a professional player, we are able to tie θ to α, and then define β from both, making the equation dependent only on the first angle.

Making a system of the last equation and the equations describing the first six pullers, we are able to write a new equation describing the total force exerted on the team in function of the tilt angle α of the first seven pullers.

The last equation, however, contains a large number of transcendent functions and thus it is impossible to find the value of α by a known tension T. In order to do that, we use a numerical approach that is the bisection method.

We obtain , using the μ value 0.71, $\alpha_{min} \approx 55°$.

The physical analysis and the simulation developed are limited to modeling pulls into a static equilibrium situation. In a next project, we can carry out modeling pulls in dynamic situation and evaluate the strategic elements that characterize the tug of war.

The program chosen by the students calculates in real time the angle of inclination of each player, using the tension coming from the left and the bisection method.

The tensions of each string sections and the forces applied by each player are calculated. The masses are randomly generated so that their sum is equal to 720kg.

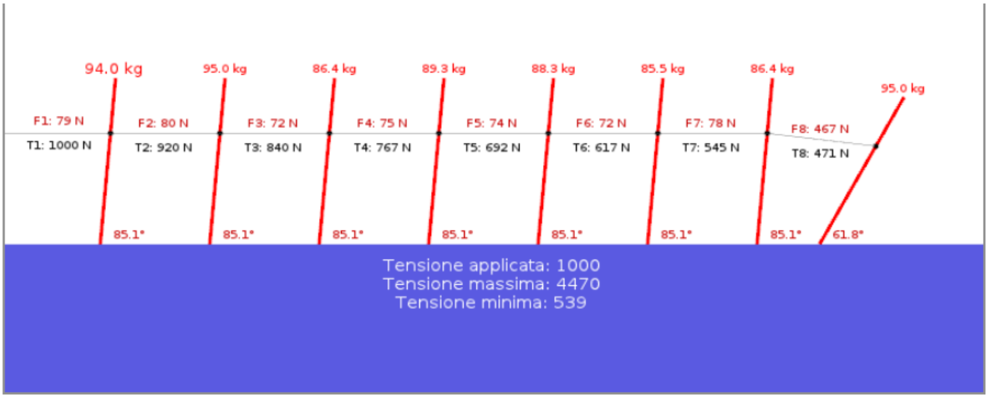


Figure 7. Simulation. Source: Our Creation(2018)

6. Conclusion

The conventional belief has always been that students interested in scientific thought should develop strong math skills. However, it might actually be the other way around. Teachers think that activities in art or in history of science can help students build math skills and make math learning more fun. FDS give both projects to enhance the mathematical knowledge and projects to build a solid math foundation. Here we have presented an example in which students can combine mathematics, physics and sport and the aim is to help students visualize the mathematical abstract concepts and its contributions to the cultural heritage.

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