

MATHEMATICAL MODELING OF MICROWAVE HEATING OF PRODUCTS WITH CENTRAL SYMMETRY

V.A. Karelin^{1,2}, V.I.V. Salomatov^{1,2},

¹*Novosibirsk State University, Pirogova 2, Novosibirsk 630090, Russia*

²*Kutateladze Institute of Thermophysics SB RAS, Kutateladze 2, Novosibirsk 630090, Russia*
vad2hen@mail.ru

Keywords: microwave, radio-frequency heating, power applications, modeling, central symmetry

Microwave heating has found widespread use in the energy, construction, forestry, chemical and food industries, etc. There are a number of publications that discuss the main mechanisms that occur during microwave heating and microwave drying [1,2]. For a better understanding of these processes and the development of highly efficient microwave installations, mathematical modeling is needed. As a rule, nonlinear models that most adequately describe these phenomena use a numerical algorithm for calculations. The authors of this work are engaged in approximate analytical approaches for microwave heating and microwave drying of bodies, which, with controlled error, allow displaying the main processes and estimating such heating and drying parameters as: temperature and moisture distribution, heating time, drying rate, reaching maximum values and etc.

In this paper, a model of microwave heating of a body in the form of a ball with uniform irradiation with microwave energy in the conditions of radiation-convective interaction of the product with the environment is considered. The absorption of the microwave inside the material is given by the law of the Bouguer. In this case, a number of simplifications were made: the electrophysical and thermophysical properties of the material are constant, the material is homogeneous in composition and properties. The equations defining this problem are as follows:

$$\frac{\partial T(r,t)}{\partial t} = \alpha \left[\frac{\partial^2 T(r,t)}{\partial r^2} + \frac{2}{r} \frac{\partial T(r,t)}{\partial r} \right] + \frac{q_0}{c\rho} e^{-k(r_0-r)} \quad (1)$$

$$0 < t < t_d; 0 < r < r_0; T_0 < T < T_d \quad (2)$$

$$\lambda \frac{\partial T(r_0,t)}{\partial r} = \varepsilon \sigma_0 [T^4(r_0,t) - T_c^4] + \alpha [T(r_0,t) - T_c] \quad (3)$$

$$\lambda \frac{\partial T(0,t)}{\partial r} = 0 \quad (4)$$

$$T(r,0) = T_0 \quad (5)$$

Using the conditions for the uniqueness of the problem, we represent this system in the following dimensionless variables.

$$\xi = \frac{r}{r_0}; Fo = \frac{\alpha t}{r_0^2}; \theta(\xi, Fo) = \frac{T(r,t) - T_c}{T_c}; Po = \frac{q_0 r_0^2}{\lambda T_c}; Bi = \frac{\alpha r_0}{\lambda}; Sk = \frac{\varepsilon \sigma_0 T_c^3 r_0}{\lambda}; Bu = k r_0$$

As a result, the system (1-5) will go to the following form:

$$\frac{\partial \theta}{\partial Fo} = \frac{\partial^2 \theta}{\partial \xi^2} + \frac{2}{\xi} \frac{\partial \theta}{\partial \xi} + Po e^{-Bu(1-\xi)} \quad (6)$$

$$-\frac{\partial \theta(1,Fo)}{\partial \xi} = Sk [\theta(1,Fo)^4 - 1] + Bi [\theta(1,Fo) - 1] = Ki(Fo) \quad (7)$$

$$\frac{\partial \theta(0,Fo)}{\partial \xi} = 0 \quad (8)$$

$$\theta(\xi,0) = \theta_0 \quad (9)$$

Apply the Laplace transform to this system:

$$s\theta_L(\xi, s) - \theta_0 = \theta_L''(\xi, s) + \frac{2\theta_L'(\xi, s)}{\xi} + \frac{Po}{s}e^{-Bu(1-\xi)} \quad (10)$$

$$-\theta_L'(1, s) = Ki_L(s) \quad (11)$$

$$\theta_L'(0, s) = 0 \quad (12)$$

We will find the solution of this system as the sum of the solution of a homogeneous problem and the solution with the inhomogeneous term taken into account:

$$\theta_L(\xi, s) = Y_1(\xi, s) + Y_2(\xi, s)$$

The solution of a homogeneous problem in general: $Y_1(\xi, s) = \frac{Ae^{\sqrt{s}\xi} + Be^{-\sqrt{s}\xi}}{\xi}$

The non-uniform part should be: $Y_2(\xi, s) = C(\xi)e^{-Bu(1-\xi)} + \frac{\theta_0}{s}$

Next, we substitute a general solution $\theta_L(\xi, s) = \frac{Ae^{\sqrt{s}\xi} + Be^{-\sqrt{s}\xi}}{\xi} + C(\xi)e^{-Bu(1-\xi)} + \frac{\theta_0}{s}$ of inhomogeneous problem in the system (10-12), we find the constants:

$$C = \frac{Po(2Bu - Bu^2\xi + s\xi)}{(Bu^2 - s)^2s\xi} + \frac{e^{-(Bu+\sqrt{s})\xi}C_1}{\xi} + \frac{e^{-(Bu-\sqrt{s})\xi}C_2}{2\sqrt{s}\xi}$$

As a result, after substituting the constant C:

$$\theta_L(\xi, s) = \frac{Ae^{\sqrt{s}\xi} + Be^{-\sqrt{s}\xi} + \frac{2PoBue^{-Bu}}{(Bu^2 - s)^2s}e^{Bu\xi} + C_1e^{-Bu}e^{-\sqrt{s}\xi} + C_2\frac{e^{-Bu}}{2\sqrt{s}}e^{\sqrt{s}\xi}}{\xi} - \frac{Po}{(Bu^2 - s)s}e^{-Bu(1-\xi)} + \frac{\theta_0}{s}$$

Constants with corresponding exponent degrees can be combined:

$$\theta_L(\xi, s) = \frac{A'e^{\sqrt{s}\xi} + B'e^{-\sqrt{s}\xi} + \frac{2PoBue^{-Bu}}{(Bu^2 - s)^2s}e^{Bu\xi}}{\xi} - \frac{Po}{(Bu^2 - s)s}e^{-Bu(1-\xi)} + \frac{\theta_0}{s}$$

From conditions (11, 12) we get:

$$A' = M(Po, Bu, s) - \frac{2PoBue^{-Bu}}{(Bu^2 - s)^2s}$$

$$B' = -M(Po, Bu, s)$$

$$\text{Where } M(Po, Bu, s) = \frac{Ki_L + \frac{2PoBue^{-Bu}e^{\sqrt{s}(1-\sqrt{s})}}{(Bu^2-s)^2s} + \frac{2PoBu^2 - 2PoBu}{(Bu^2-s)^2s} - \frac{PoBu}{(Bu^2-s)s}}{-\sqrt{s}(e^{\sqrt{s}} + e^{-\sqrt{s}}) + e^{\sqrt{s}} - e^{-\sqrt{s}}}$$

As a result:

$$\theta_L(\xi, s) = \frac{(M(Po, Bu, s) - \frac{2PoBue^{-Bu}}{(Bu^2 - s)^2s})e^{\sqrt{s}\xi} - M(Po, Bu, s)e^{-\sqrt{s}\xi} + \frac{2PoBue^{-Bu}}{(Bu^2 - s)^2s}e^{Bu\xi}}{\xi} - \frac{Po}{(Bu^2 - s)s}e^{-Bu(1-\xi)} + \frac{\theta_0}{s}$$

This solution has a rather complicated dependence on the parameter s and it is not possible to take the inverse Laplace transform. However, from the terms not containing $e^{\sqrt{s}\xi}, e^{-\sqrt{s}\xi}$ the inverse transformation is possible to obtain:

$$\theta_L(\xi, Fo) = \frac{2Poe^{-Bu+Bu\xi}(1 - e^{Bu^2Fo} + Bu^2e^{Bu^2Fo}Fo)}{Bu^3\xi} + \frac{Po(e^{Bu^2Fo} - 1)}{Bu^2}e^{-Bu(1-\xi)} + \theta_0$$

In order to obtain the inverse Laplace transform from the rest of the solution, we find the asymptotic expansions for large and small values of the parameter s. This will make it possible to find approximate solutions for large and small values of the time parameter, which allows determining the temperature by layer with high accuracy and depending on time.

Large values of s (small Fo)

For large values of s:

$$M(Po, Bu, s) = \frac{Ki_L + \frac{2PoBue^{-Bu}e^{\sqrt{s}(1-\sqrt{s})}}{(Bu^2-s)^2s}}{e^{\sqrt{s}(1-\sqrt{s})}} = \frac{Ki_L}{e^{\sqrt{s}(1-\sqrt{s})}} + \frac{2PoBue^{-Bu}}{(Bu^2-s)^2s}$$

Consider the terms containing $e^{\sqrt{s}\xi}$, $e^{-\sqrt{s}\xi}$:

$$\frac{(M(Po, Bu, s) - \frac{2PoBue^{-Bu}}{(Bu^2-s)^2s})e^{\sqrt{s}\xi} - M(Po, Bu, s)e^{-\sqrt{s}\xi}}{\xi} = \frac{\frac{Ki_L}{e^{\sqrt{s}(1-\sqrt{s})}}e^{\sqrt{s}\xi} - (\frac{Ki_L}{e^{\sqrt{s}(1-\sqrt{s})}} + \frac{2PoBue^{-Bu}}{(Bu^2-s)^2s})e^{-\sqrt{s}\xi}}{\xi}$$

For large values of the s parameter:

$$\frac{Ki_L}{e^{\sqrt{s}(1-\sqrt{s})}\xi}e^{\sqrt{s}\xi}$$

Inverse transform through convolution from a given addend:

$$\theta(\xi, Fo) = -\frac{1}{\xi} \int \frac{Ki(y)}{\sqrt{\pi(Fo-y)}} e^{\frac{-(\xi-1)^2}{4(Fo-y)}} dy$$

To obtain an explicit calculated expression, decompose $Ki(y)$ near $y = Fo$ into a Taylor series:

$$Ki(y) = Ki(Fo) + (y - Fo)Ki'(Fo) + \dots$$

Restricting ourselves to the first member of the expansion, we have:

$$\theta(\xi, Fo) = -Ki(Fo) \frac{1}{\xi} \int \frac{e^{\frac{-(\xi-1)^2}{4(Fo-y)}}}{\sqrt{\pi(Fo-y)}} dy$$

After taking the integral, we finally get:

$$\theta_L(\xi, Fo) = Ki(Fo)D(\xi, Fo) + \frac{2Poe^{-Bu+Bu\xi}(1 - e^{Bu^2Fo} + Bu^2e^{Bu^2Fo}Fo)}{Bu^3\xi} + \frac{Po(e^{Bu^2Fo} - 1)}{Bu^2}e^{-Bu(1-\xi)} + \theta_0$$

Where D is the function obtained after integration.

Next, we find the boundary temperature $\theta_L(1, Fo)$ from the previous equation.

A typical view of the dependence of temperature on the depth of the layer, as well as the temperature at the boundary, depending on the time is given below:

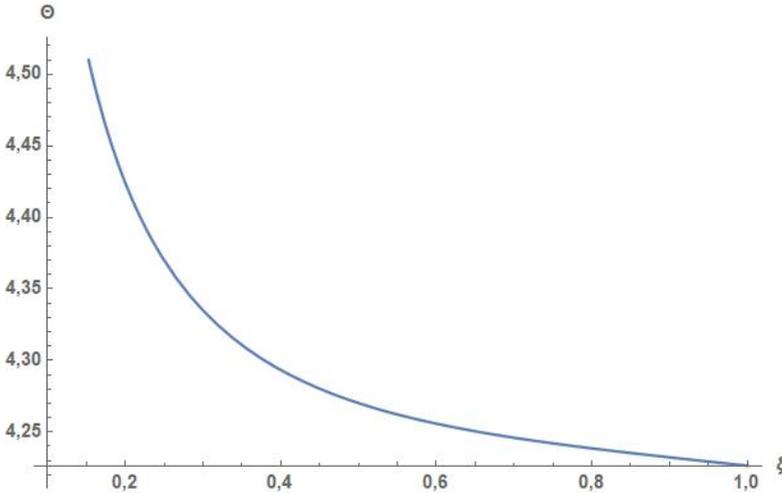


Fig. 1. The dependence of the dimensionless temperature on the surface coordinates ($\xi = 1$ corresponds to the edge of the ball)

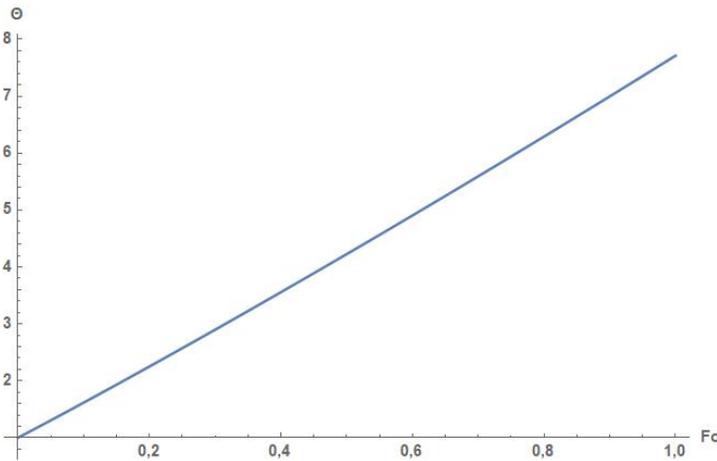


Fig. 2. The dependence of the dimensionless temperature on time at the boundary of the ball.

Small values of s (large Fo)

For small values of s:

$$M(Po, Bu, s) = \frac{Ki_L + \frac{2Po(e^{-Bu} + Bu - 1 - \frac{Bu^2}{2})}{Bu^3s}}{-\sqrt{s}}$$

Consider the terms containing $e^{\sqrt{s}\xi}, e^{-\sqrt{s}\xi}$ for small values of the parameter s:

$$2M\sqrt{s} - \frac{2Poe^{-Bu}}{Bu^3s\xi} = 2 \frac{Ki_L + \frac{2Po(e^{-Bu} + Bu - 1 - \frac{Bu^2}{2})}{Bu^3s}}{-s} - \frac{2Poe^{-Bu}}{Bu^3s\xi}$$

Find the inverse Laplace transform from this part:

$$-2Ki - \frac{2PoFo(e^{-Bu} + Bu - 1 - \frac{Bu^2}{2})}{Bu^3} - \frac{2Poe^{-Bu}}{Bu^3\xi}$$

As a result, for final solution:

$$\theta(\xi, Fo) = -2Ki - \frac{2PoFo(e^{-Bu} + Bu - 1 - \frac{Bu^2}{2})}{Bu^3} - \frac{2Poe^{-Bu} - 2Poe^{-Bu+Bu\xi}(1 - Bu^2Fo + Bu^2e^{Bu^2Fo}Fo)}{Bu^3\xi} + \frac{Po(e^{Bu^2Fo} - 1)}{Bu^2} e^{-Bu(1-\xi)} + \theta_0$$

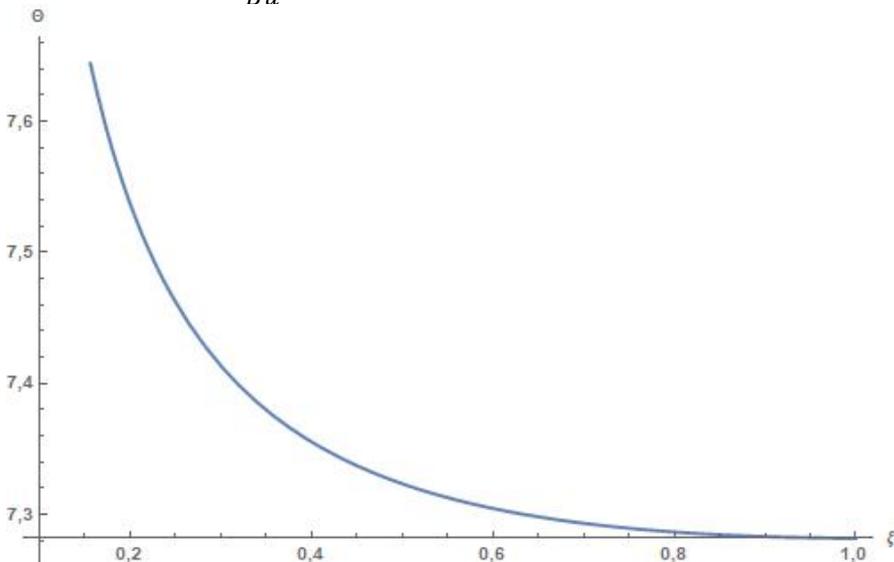


Fig. 3. The dependence of the dimensionless temperature on the surface coordinates ($\xi = 1$ corresponds to the edge of the ball)

Comparing with the numerical solution of a similar problem, it was found that for the corresponding solutions for large and small Fo , they give an error of no more than 7% with $Fo < 0.1$ and $Fo > 0.3$, respectively. These solutions allow you to qualitatively take into account the main processes occurring during the heating of a spherically symmetric particle.

As a result, using asymptotic procedures, approximate solutions were obtained for the temperature field depending on the time and intensity of processing, the electrical and thermal properties of the material, which allow to find the warm-up time to a certain temperature, the start time of drying, the maximum temperature coordinate, etc.

Acknowledgement

The statement of the problem and the solution for the stage of sphere heating were performed within the framework of the state task IT SB RAS, the search for approximate analytical solutions was done with the support of the Russian Foundation for Basic Research, grant No. 17-08-00752

References

1. Feng Gao, Comparison of microwave drying and conventional drying of coal. A thesis submitted to the Department of Mining Engineering in conformity with the requirements for the degree of Master of Applied Science Queen's University Kingston, Ontario, Canada, 2010, 167.
2. Khaidurova A.A., Fedchishin V.V., Konovalov N.P. Mikrovolnovaya sushka buryh uglej i povyshenie ih tekhnologicheskikh harakteristik [Microwave drying of brown coals and increasing their technological characteristics]. Izv. Universities. Problems of energy. - Kazan: Publishing house KGEU. - 2010. - P. 31-35. In Rus.