# OBTENCIÓN DE PERSPECTIVAS CABALLERAS Y MILITARES A PARTIR DE MODELOS TRIDIMENSIONALES

*Pedro M. Cabezos Bernal, Juan J. Cisneros Vivó*

Los grandes avances en el campo de la informática que se han producido en los últimos años, han posibilitado el desarrollo de aplicaciones de diseño, cada vez más potentes, que se han convertido en una herramienta fundamental para el proyectista y han generado, además, una revolución en la docencia de la Geometría Descriptiva. Sin embargo, los programas informáticos no son perfectos, ya que tienen algunas limitaciones que hemos tratar de superar, por lo que no debemos rendir pleitesía a un determinado programa, ni resignarnos a lo que se nos ofrece, sino que tenemos que dar solución a los inconvenientes que se plantean.
Este artículo se centra en resolver el problema de obtener perspectivas militares o caballeras a partir de un modelo tridimensional, ya que es un defecto común en la mayoría de los programas de CAD del mercado, que se limitan a obtener proyecciones cilíndricas ortogonales o proyecciones cónicas del modelo, lo que ha provocado que el uso de este tipo de representaciones se haya reducido drásticamente.

## Axonometrías oblicuas como sistema de representación

La perspectiva militar y  la caballera son sistemas de representación de gran interés y expresividad, que han sido empleadas con gran acierto a lo largo de la historia, por su carácter divulgativo. Ejemplo de ello lo podemos encontrar en algunos dibujos de Leonardo da Vinci para describir sus máquinas, así como algunos detalles arquitectónicos de Miguel Ángel, autores más recientes como Auguste Choisy en su obra "Historia de la arquitectura" analiza gráficamente los edificios utilizando para ello axonometrías militares, con lo que se obtiene información precisa de la planta, además de percibir el espacio arquitectónico.

Los arquitectos de la Bauhaus utilizaron  a menudo las axonometrías oblicuas, junto con el color, con lo que consiguieron representaciones de gran plasticidad, que competían con obras pictóricas de la época, incluso Piet Mondrián las empleó en alguno de sus cuadros.

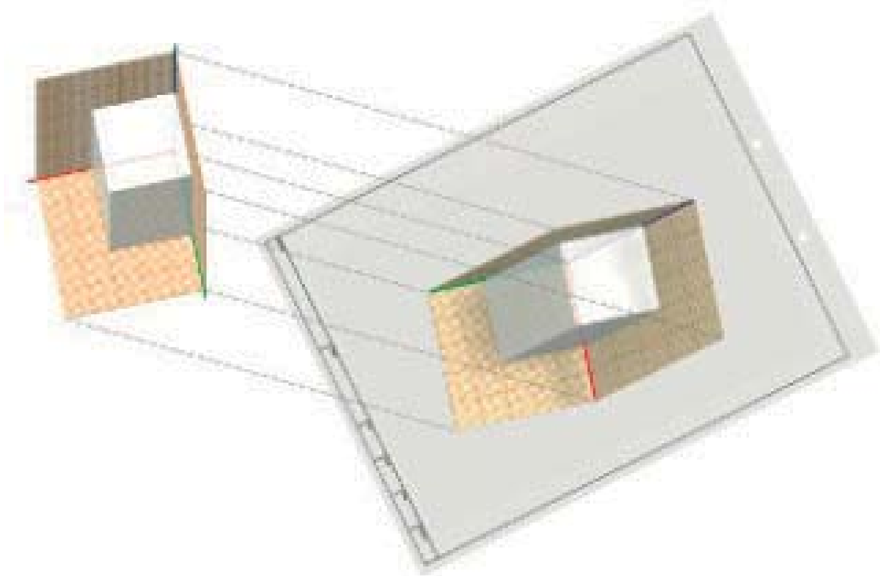
Autores contemporáneos como Tadao Ando, Arata Isozaki, John Hejduk, Peter Eisenman, entre otros, han elegido las axonometrías oblicuas como recurso gráfico para explicar sus proyectos.

Una axonometría oblicua, es fruto de la proyección cilíndrica oblicua de un modelo sobre un plano, que puede estar en cualquier posición con respecto al objeto, podemos hablar de axonometrías militares, si el plano de proyección es paralelo a planos horizontales del objeto, o bien axonometrías caballeras si lo es a planos verticales del objeto representado.

La distorsión del objeto dependerá de la dirección de proyección, que determina el coeficiente de reducción que se aplicará sobre el eje perpendicular al plano coordenado que vemos en verdadera magnitud.

Si no queremos tener una percepción distorsionada del modelo representado, deberemos proyectarlo según una dirección que forme con el plano de proyección, un ángulo de entre 45º y 63º, lo que supondrá obtener coeficientes de reducción de entre 0.5 y 1.

La elección del coeficiente de reducción idóneo depende de la percepción de cada persona, la figura 2 muestra distintas axonometrías militares y caballeras de un cubo con coeficientes de reducción entre 0.5 y 1. Deberemos elegir el que nos produzca una menor distorsión de la figura representada. La percepción puede ocasionar que el coeficiente que creamos óptimo para una militar, no tiene por que ser el idóneo en una caballera.



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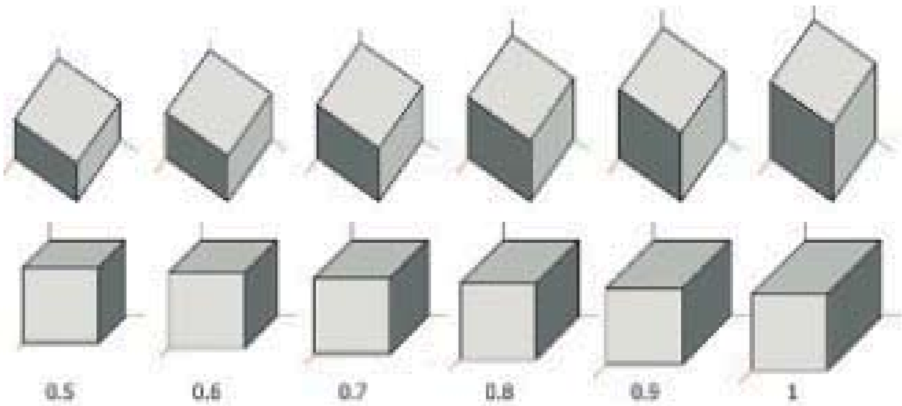


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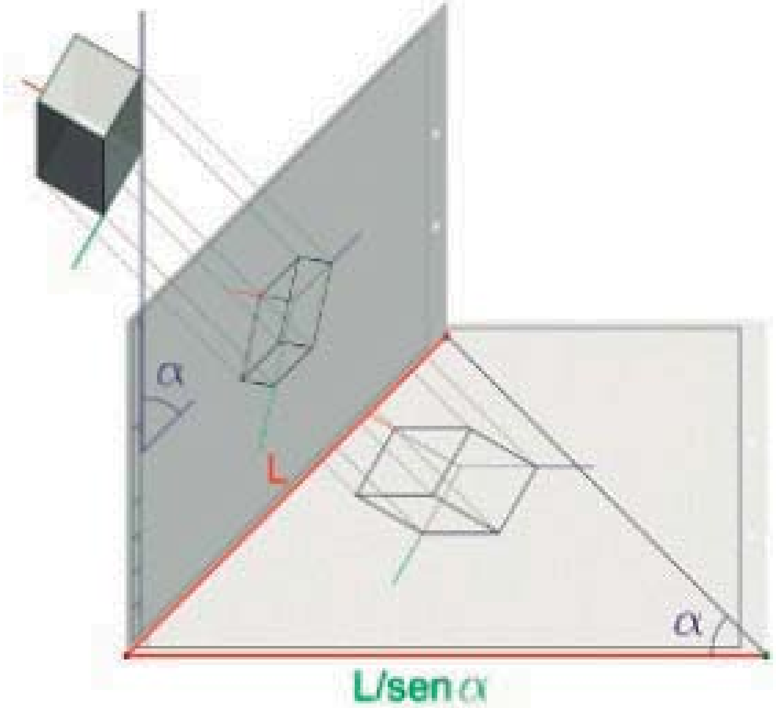


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1. Generación de una proyección oblicua.
2. Axonometrías militares y caballeras de un cubo con distintos coeficientes de reducción.
3. Relación entre proyección ortogonal y oblicua.

## Transformar proyecciones ortogonales en oblicuas

La primera parte del estudio se centra en establecer la relación existente entre una axonometría ortogonal y una oblicua.

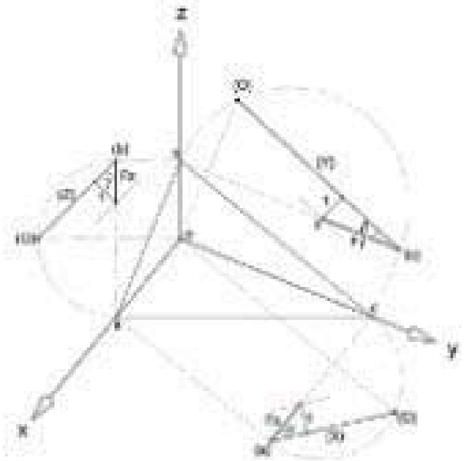
En la figura 3 se analiza la relación entre una axonometría ortogonal y una oblicua, mediante la representación de dos planos secantes sobre los que se proyecta un cubo en el espacio, siendo uno de los planos ortogonal a la dirección de proyección, obtenemos por tanto sobre él una proyección ortogonal del cubo, mientras que el otro forma un ángulo α con la dirección de proyección, obteniendo, en este caso, una proyección oblicua, en concreto una militar.

Podemos observar que el ángulo α es el mismo que forma el eje Z en el espacio con el plano ortogonal a la dirección de proyección.

Para establecer la relación entre ambas proyecciones, consideremos las proyecciones como una hoja de papel, y pensemos que ocurre si abatimos la proyección ortogonal sobre la oblicua, utilizando como charnela la arista común. Es obvio que no concuerdan, pero bastaría aplicar a la proyección ortogonal un cambio de escala en la dirección del eje z, para que ésta coincida con la perspectiva militar.

Esta deformación o cambio de escala en la dirección del eje z tendrá una magnitud concreta, es decir tendremos que aplicar un factor de escala adecuado, que hemos de determinar.

Consideremos la longitud L, equivalente a un lado del papel en la proyección ortogonal que, después del cambio de escala, deberá medir L/sen α, para concordar con el lado del pa-

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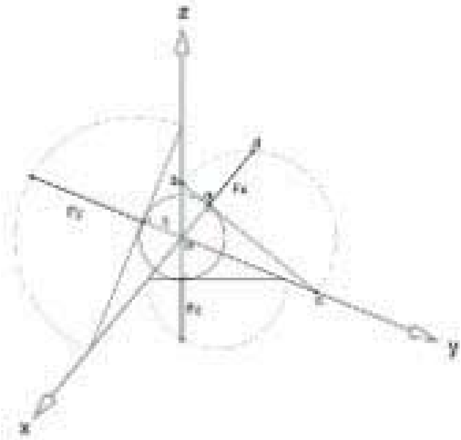


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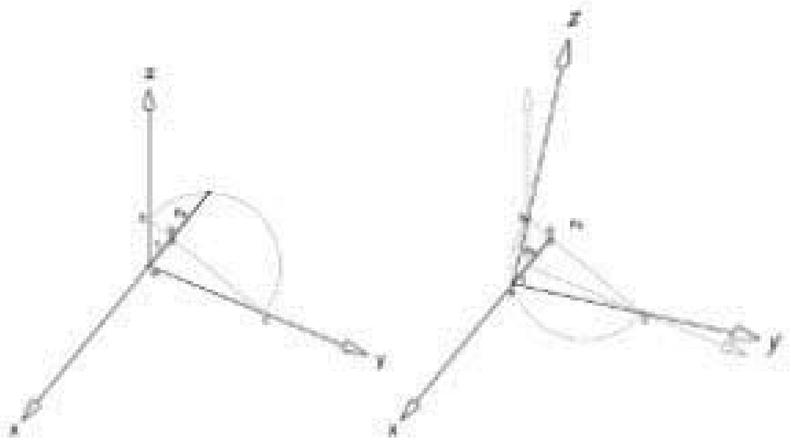


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4. Obtención gráfica de los factores de escala para cada eje.
5. Obtención gráfica de los factores de escala para cada eje.
6. Razonamiento del procedimiento gráfico.

pel en la proyección oblicua, siendo el factor de escala que hemos de aplicar para transformar la ortogonal en oblicua igual a L/sen $\alpha$/L = 1/sen $\alpha$.

Podemos realizar el mismo planteamiento para el eje x y para el eje y, obteniendo en estos casos perspectivas caballeras, teniendo en cuenta que el ángulo $\alpha$ será distinto para cada eje, por lo tanto, los factores de escala también lo serán.

Así podemos afirmar que dada una axonometría ortogonal cualquiera, la podemos transformar en una axonometría militar o caballera si aplicamos un cambio de escala en la dirección de los ejes coordenados, de forma que obtendremos una axonometría militar si escalamos en la dirección del eje z, una caballera si escalamos en la dirección del eje x y otra caballera si escalamos en la dirección del eje y, para ello habremos de determinar previamente el factor de escala adecuado para la transformación en cada uno de los ejes coordenados.

Dada una axonometría ortogonal, el conocimiento de los tres factores de escala Fx, Fy y Fz se puede realizar por medios gráficos, para lo que se describen dos procedimientos distintos:

El primero se ejemplifica en la figura 4. Se trata de identificar los ejes coordenados de la axonometría ortogonal original para poder dibujar el triángulo de trazas abc, que es la base de la pirámide cuyo vértice será el origen de coordenadas en el espacio. Si hacemos tres cambios de plano, o tres abatimientos, para poder ver el resto de aristas de esta pirámide en verdadera magnitud, obtendremos el ángulo que forma cada eje coordenado con el plano del cuadro.

Centrándonos en el eje Y, una vez abatido el plano proyectante que contiene al eje Y sobre un plano paralelo al del cuadro, lo vemos en verdadera magnitud (Y), y también el ángulo $\beta$ que forma con el plano del cuadro.

Si en el abatimiento hacemos una paralela al eje (Y) a una unidad de distancia, ésta cortará al segmento b(c) en el punto p, y el segmento p(c) medirá 1/sen $\beta$, puesto que es la hipotenusa de un triángulo rectángulo que tiene por cateto opuesto la unidad y ángulo entre hipotenusa y cateto contiguo igual a $\beta$. Esta longitud será equivalente al factor de escala correspondiente al eje y, que hemos denominado Fy. Haciendo lo mismo con los ejes X y Z, obtendremos respectivamente Fx y Fz.

El otro procedimiento gráfico es más sencillo de realizar, como podemos comprobar en la figura 5. Al igual que en el caso anterior partimos de una disposición de ejes coordenados conocidos, y dibujamos una circunferencia centrada en el origen de coordenadas, de radio una unidad y prolongamos los ejes hasta que la corten. Si nos centramos, por ejemplo, en el eje x vemos que corta a la circunferencia en el punto a, trazamos por ese punto una ortogonal al eje x, o lo que es lo mismo, la tangente a la circunferencia en el punto a , y la prolongamos hasta que corte al resto de los ejes en b y en c, trazamos a continuación el arco capaz que tiene por diámetro el segmento bc, y prolongamos el eje x hasta que corte al arco capaz en el punto d.

La longitud del segmento ad es el factor de escala que buscamos para el

7. Obtención de militar y caballeras a partir de una axonometría ortogonal.

eje x, es decir Fx. Si repetimos el proceso para el resto de los ejes, obtendremos también Fy y Fz.

La figura 6, compara el procedimiento, aplicado sobre el eje x, con el razonamiento gráfico que lo origina, que es el de aplicar un escalado en la dirección del eje x para obtener una caballera. Consideremos el punto a como base de la deformación, por lo tanto permanecerá inmóvil, distando del origen de coordenadas una unidad; para saber cuánto se alargará el segmento oa para que los eje z e y formen un ángulo recto, bastará dibujar el arco capaz que tenga por diámetro el segmento bc.

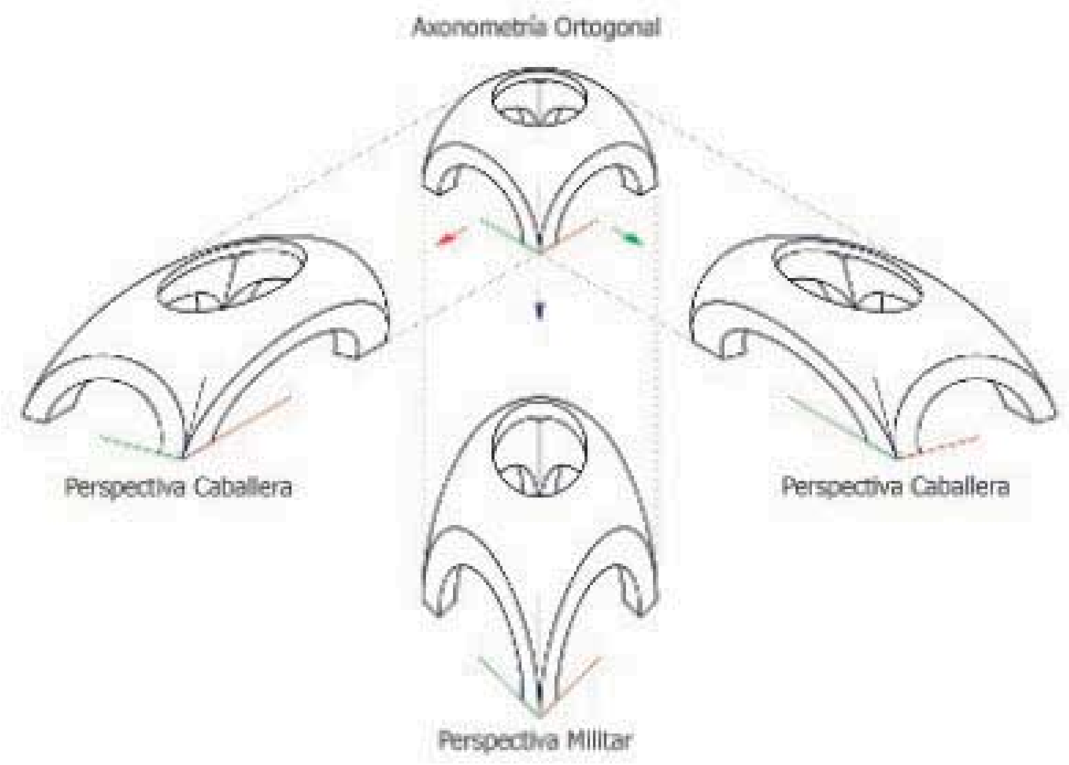
Una vez conocida esta relación entre las dos proyecciones, la podemos aplicar para transformar dibujos o imágenes que representen proyecciones ortogonales, en proyecciones oblicuas, tal como muestra la figura 7, en donde, mediante el escalado de una axonometría ortogonal en la dirección de los ejes coordenados, se obtienen tres axonometrías oblicuas, que son una militar y dos caballeras.

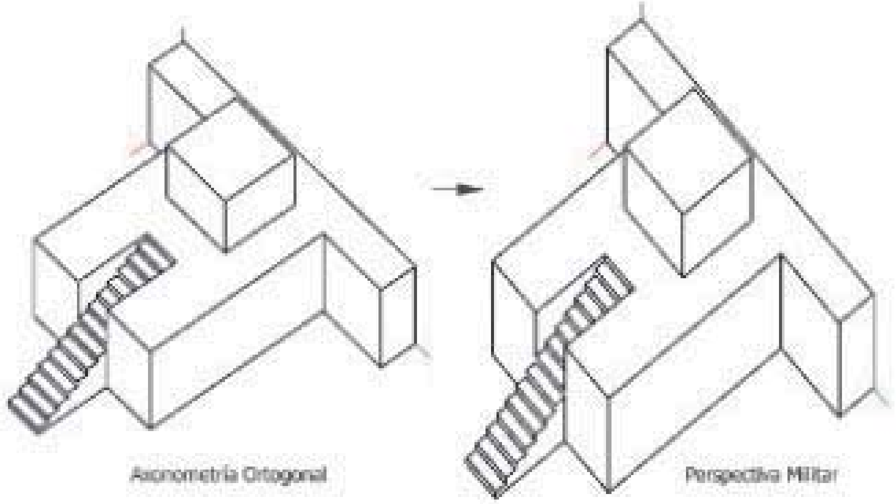
Como podemos apreciar en el ejemplo, las caballeras y la militar obtenidas, presentan unas distorsiones muy grandes que dificultan la lectura del objeto representado, debido a que el ángulo que forman los ejes coordenados con el plano del cuadro en la axonometría inicial no es el adecuado.

Para solventar este inconveniente interesa partir de axonometrías donde el eje coordenado según el que se haga la transformación forme un ángulo de entre 45º y 63º con el plano del cuadro para obtener, en las axonometrías resultantes, factores de reducción de entre 1 y 0.5.

Es evidente que si partimos de axonometrías ortogonales existentes, no es posible controlar la distorsión, pero si tenemos un modelo tridimensional, podremos orientarlo correctamente con respecto al plano de proyección y conseguiremos fácilmente una proyección ortogonal adecuada del modelo, mediante la que podremos obtener una militar o caballera con coeficientes de reducción apropiados.

En la figura 8, vemos una axonometría ortogonal obtenida a partir de un modelo tridimensional cuyo eje Z formaba un ángulo de 55 grados con el plano de proyección, lo que origina que tras la transformación obtengamos



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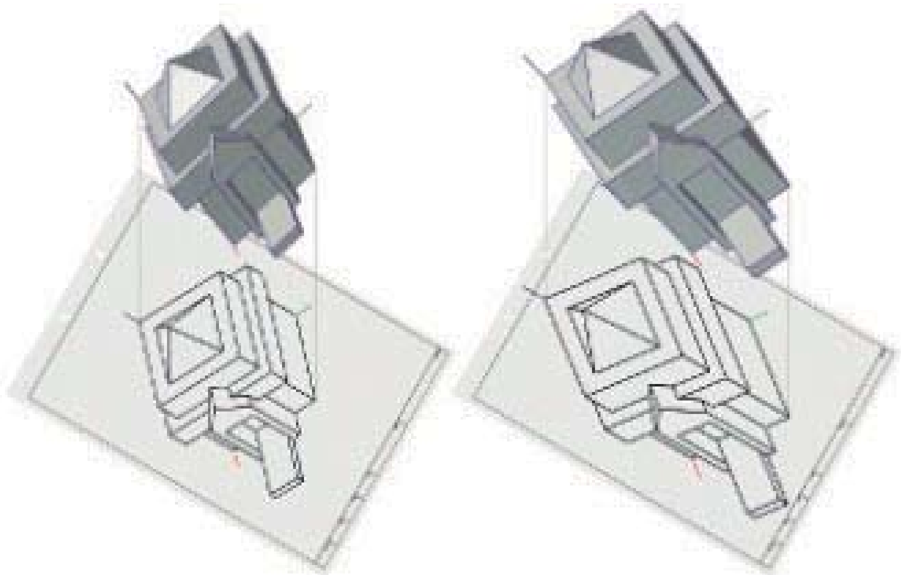

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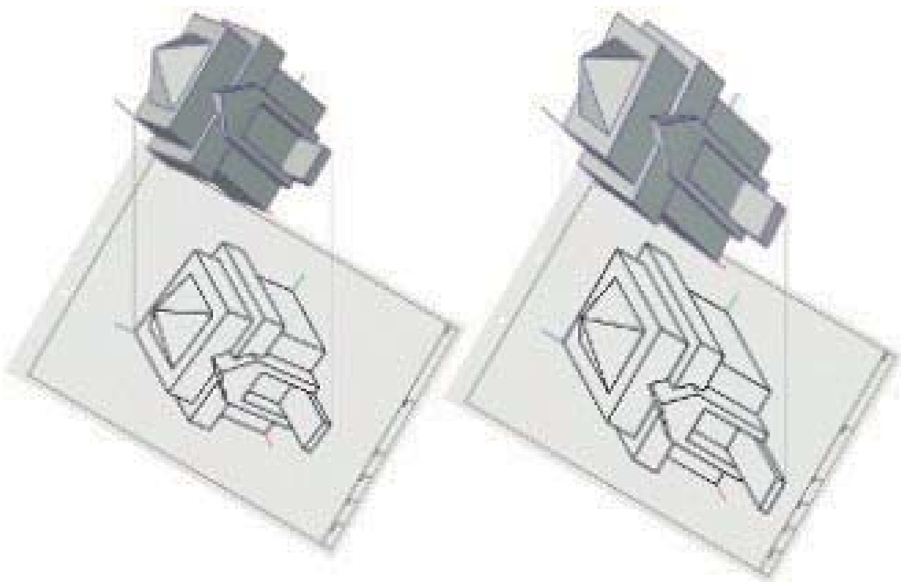

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8. Imagen antes y después de la transformación.
9. Proyecciones ortogonales de la figura original (izquierda) y de la figura deformada mediante un cambio de escala en la dirección del eje z para simular una perspectiva militar.
10. Proyecciones ortogonales de la figura antes (izquierda) y después de la transformación para simular una axonometría caballera.

una axonometría militar con un coeficiente de reducción adecuado Cz= 0.7.

## Transformación directa de modelos tridimensionales

Basándonos en las primeras consideraciones, aplicadas sobre proyecciones ortogonales de un modelo, comprobaremos que igualmente podemos realizar la transformación directamente sobre el modelo tridimensional.

Se trata de deducir cómo hemos de deformar la geometría original para que, elegido el punto de vista, la proyección ortogonal de la figura deformada sea equivalente a una proyección oblicua de la figura original.

En esencia, se trata de aplicar los mismos principios vistos anteriormente, al modelo tridimensional, siendo que la deformación que hemos de producir al objeto obedece a un cambio de escala en la dirección de los ejes x, y, z, que son la proyección de los ejes X, Y, Z en el espacio sobre el plano de proyección.

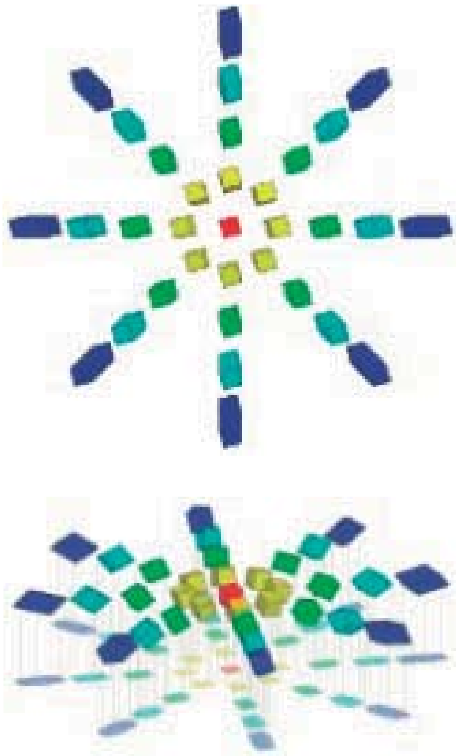
Escalando el modelo en la dirección del eje z, obtendremos una militar, y si lo hacemos en la dirección del eje x o del eje y , una caballera.

El factor de escala a aplicar será el mismo que hemos deducido en las reflexiones anteriores, es decir, 1/sen $\alpha$, recordemos que $\alpha$ es el ángulo que forma el eje coordenado en cada caso, sea X, Y ó Z con el plano de proyección.

La figura 9 muestra cómo una figura tridimensional se proyecta ortogonalmente sobre el papel, antes y después de aplicar un cambio de escala en la dirección del eje z, para obtener así el equivalente a una axonometría militar.

La figura 10 muestra cómo se consigue el equivalente a una axonometría ca-

11. Aparentes proyecciones oblicuas de un cubo.
12. Ejemplo aplicado a la modelización de una bóveda perteneciente al “Tratado de Arquitectura de Alonso de Vandelvira" escrito entre 1575 y 1591, que el autor llama "capilla cuadrada por cruceros".



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ballera haciendo un cambio de escala del objeto original en la dirección del eje x

La figura 11 muestra, a la izquierda, lo que parecen varias proyecciones oblicuas de un cubo, desde ángulos distintos, lo que originaría axonometrías oblicuas con distintos coeficientes de reducción. Sin embargo, a la derecha, se muestra que, en realidad, no se trata de proyecciones oblicuas de un cubo, sino de proyecciones ortogonales de prismas oblicuos obtenidos a partir de la deformación del cubo.

## Conclusiones y ejemplos de aplicación

El hecho de trabajar directamente sobre el modelo 3D presenta una serie de ventajas como la posibilidad de aplicar texturas, iluminación, además de poder generar animaciones.

La aplicación de las deducciones del estudio se puede llevar a la práctica mediante cualquier programa de CAD que permita la escala no uniforme del modelo.

La figura 12 muestra un ejemplo de aplicación del método a la modelización tridimensional de una “capilla cuadrada por cruceros” perteneciente al “Tratado de Arquitectura de Alonso de Vandelvira, con lo que se ha obtenido una animación del despiece de la bóveda en perspectiva caballera, de la que se muestran únicamente tres fotogramas.

Otros ejemplos lo constituyen la mayoría de las figuras ilustrativas del presente artículo que, como se puede comprobar, son perspectivas militares.

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25 / PORTALÉS, C. and GINER, F. Back to the 70,s, AC SIGCHI, International conference on advanced in Computer entertainment Technology, Valencia 2005.



understand that it is necessary to illuminate zenithal the alley, since it is very long and dark, and that in addition it would be suitable to create a landing after the door of entry, to facilitate to overcome seven steps that exist between the street and the interior of the step. (Fig 13 Left.).

The following challenge is to see how is possible to return this sketch to hand lifted to the model 3D, since the authors do experimentally before mentioned. For the present time our test-prototype passes for the use of the *plugin* Photomatch® in the patternmaker NPR SketchUp who invests the transformation - model of camera to the model 3D, previous definition of the axes of reference of the system of coordinates. (Fig. 13 cen.) For it we invoke the apprehension of screen while the virtual model was observed and in that we realize some sketches of adjustment and on this object 2D, there are defined the orthogonal planes of reference and axes which automatically a model allows to create 3D vectorially.

In parallel we have tested another option of AR's utilization in which the observer just receives local information that has before if and that identifies with a code QR. With a mobile telephone type iPhone® or similar, connected, it does a photo of this code and Kaywa®'s reading application to Internet, sends him to an Internet address, URL, Uniform Resource Announcer, which invokes a web page. In this case we have tried with that of the Museum of History of the City. (Fig. 14). Once there can disburden, for example, an Object VR QuickTime® that shows a spherical panorama of the model of the Alley 2 or a conventional video of the court of the Jewish Museum that finds at the end of the alley.

## 5. Conclusions and future work

### *5.1 Conclusions to the First Part*

From the standpoint of urban historical research, the main contribution is the demonstration that both streets are great historical interest and that is supports the interest of the UMAT by incorporating there into their base cartographic and historical archive of town hall to proceed with its documentation. Because of it is necessary to continue being employed at his consolidation, and our offer goes in this direction though it is necessary to improve the local recognition on the part of the archaeologists and with topographers' help, to realize an exhaustive drawing of the same ones and his immediate environment. In this sense already we have demonstrated to the beginning of this one work that we define in general terms as the first phase of a more extensive investigation. For the present time we contribute with a first drawing (Fig 16) and the recognition and local documentation.

### *5.2 Conclusions to the Second Part*

The architectural offers arisen from the integration of the methodologies SBIM and 13 digital sketches which different databases join *on line*, joined those of the Augmented Reality at the moment of solving a project in an environment urban, we understand that it are a good conclusion and an original and unpublished experience that we want to make come to the university community from the intermediate position that occupies always an applied investigation of the area of knowledge of the graphical architectural presentation.

We have finally demonstrated the viability and interest of the applied investigation exemplified in this case of study. Our offer of use of not immersive augmented reality, in an urban environment for the evaluation of a project with portable devices, is original and allows advancing in the development of an investigation of major packing in the one that will be indispensable to integrate the environment in the virtual model. The tests of Portalés, C. and Giner, F[25] demonstrate that it is feasible if channels apply alpha to the model of the real environment, but it is without solving the lighting of the interior. Because of it I motivate we must continue working.

## FIGURES

1. Gerona's Historic Centre and the "Call". Source: Gerona City Hall Web Site.
2. Location of number1 and number 2 roads. Source: Gerona City Hall Web Site.
3. Current situation of the roads to recover. Source: ER, Ernest Redondo.
4. Plan of the Gerunda, 300 A.C. Of the closing of the Call, s the XIV th, Emplacement of last synagogue 1434 and reconstruction of la Estima of 1535. Source Urban History of Gerona. Cartographic reconstruction of the city. Vol. 1, 6, 7 and 8. 1995-2008. Town hall Gerona.
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9. Different phases of production of the model 3D of the Call. Source ER, Ernest Redondo.
10. Different sketches digital in the capture of information and preliminary studies. Source ER, Ernest Redondo.
11. Scheme of the development of the Augmented Reality and his basic equipment. Sources: Becker, To. MxR Project. 2008. Bimber, O + Raskar, R. Spatial Augmented Reality. 2005 And Ar-media.
12. Preliminary tests and process of visualization AR of the model of the alley 2. Source ER, Ernest Redondo.
13. Process of adjustment in situ and restitution of the model AR to the model 3D. Source ER, Ernest Redondo.
14. Test of application of the codes QR of the address URL of the LMVC from a mobile and of the VRObject and a video linked. Source ER, Ernest Redondo.
15. Final planimetry of the alleys and potentials places where to develop this application. Source ER, Ernest Redondo.

# OBTAINING OBLIQUE PERSPECTIVES FROM THREE-DIMENSIONAL MODELS

**by Pedro M. Cabezos Bernal, Juan J. Cisneros Vivó**

## Introduction

The great advances in the field of computers, have led the development of more powerful design applications, which have become an essential tool for the designer and have generated a revolution in descriptive geometry teaching.

However, design software isn't perfect, as it has some limitations that we try to overcome, so that, we shouldn't comply with a particular application, or resign us to obtain only what it offers, so we have to find solutions to that limitations.

This article focuses on solving the problem to obtain oblique perspectives from a three-dimensional model, because it's a common lack in most CAD programs, since they only can obtain orthogonal projections and linear perspectives from the model. This inconvenient drives to the fact that employment of this type of representations it's been drastically reduced.

## Oblique perspective as representational system

Oblique perspective is an interesting representational system, due to his expressiveness, and has been employed, with great success, in the course of history as for example in some Leonardo da Vinci's drawings, that describes their machines, as well as some architectural details of Michael Angelo. More recent authors as Auguste Choisy, in his work "History of architecture", analyzes architecture graphically with the employment of oblique perspective that gives accurate plan information, and also let us to perceive architectural space.

Bauhaus architects often used the oblique perspective, and colour employment to achieve great plasticity representations, which competed with pictorial works of the moment, even Piet Mondrián used it in some of his paintings.

Contemporary Authors as Tadao Ando, Arata Isozaki, John Hejduk, Peter Eisenman, among others, have chosen oblique perspective too, as a resource to explain their projects.

Oblique perspective is the result of projecting obliquely a model on a plane, which can be in any position with regard to the subject. We say military perspective, when the projection plane remains parallel to horizontal planes of the model, or we say cavalier perspectives, when projection plane is parallel to any vertical plane of the represented object.

The model distortion depends on the projection direction, which determines the reduction coefficient to be applied to the axis that isn't projected in true magnitude. To avoid distorted perceptions of the represented model, it has to be projected by a direction that forms an angle between 45 and 63º with the projection plane, which will produce reduction coefficients between 0.5 and 1.

Choosing suitable reduction coefficient depends on each person perception; figure 2 shows different oblique perspectives from a cube, with reduction coefficients between 0.5 and 1. We must choose which produces less distortion of the represented figure. Our perception can cause that optimal coefficient for a military perspective, doesn't have to match with the suitable coefficient for a cavalier perspective.

### Transforming orthogonal projections in oblique one

First part of the study analyzes the relationship between orthogonal and oblique projections, figure 3 explain this relationship through the representation of two planes on which a cube is projected, a plane remains orthogonal to the projection direction, so we get an orthogonal projection from the cube on it, while the other describes an angle $\alpha$ with the projection direction so an oblique projection is obtained over it. We can observe that the angle $\alpha$ is the same that form the Z-axis with the orthogonal plane to projection direction.

To establish the relationship between both projections, consider the projections as a sheet of paper, and imagine what happens if we rotate the orthogonal projection around the common edge with the oblique projection, and make them be coplanar. It is obvious that they won't match, but scaling properly the orthogonal projection, only by the z-axis direction, will cause them fit.

This deformation or scale change by the direction of z-axis needs to be properly applied, so we have to determine the adequate scale factor.

Consider the length L, equivalent to an edge of the plane which is orthogonal to projection direction. After the scale change, L must measure L/sen $\alpha$, to match with the length of the edge belonging to the other plane, being the scale factor that we must apply equal to L/sen $\alpha$/L = 1/sen $\alpha$.

We can do the same approach considering, by one hand the x-axis, and by the other hand, the y-axis. We will get in these cases cavalier perspectives, taking into account that $\alpha$ angle will be different for each axis, therefore, the scale factors will be so.

As seen, we can transform any orthogonal projection into an oblique one, applying a scale change by the direction of the coordinate axes, so that, we will get a military perspective when scaling by z-axis direction, or a cavalier perspective when scaling by the x-axis direction, or by the y-axis direction. We shall determine previously the suitable scale factor to apply in each transformation.

Given an orthogonal projection, knowledge of the three scale factors called Fx, Fy and Fz, can be performed graphically by two different procedures described bellow.

The first procedure is exemplified in figure 4. The edges of abc triangle, are orthogonal to the coordinate axes. This triangle is the basis of the pyramid whose apex is the spatial origin, so their edges are the spatial coordinate axes that have been rotated to make them lay over the projection plane, so we can see them in true magnitude, also we can obtain the angle between each coordinate axis and projection plan.

Focusing on the Y-axis, the rotation makes it lay over the projection plan and let us see it in true magnitude (Y) and also the $\beta$ angle with the projection plane. Doing a parallel line to the axis (Y) separated one unit from it, will cut the segment b(c) at the point p. The segment p(c) measures 1/sen $\beta$, since it's the hypotenuse of a right angled triangle, which opposite side measures an unit, and the angle between hypotenuse and his adjacent side is equal to $\beta$. This length will be equivalent to the scale factor Fy corresponding to the y-axis. Doing the same with the X-axis and Z-axis, we will obtain respectively Fx and Fz.

The other proceeding figure is easier to carry out, as we can see in figure 5. As in previous case, we start with a distributed coordinate axes and a unit radius circle, centred on the origin of coordinate, it's drawn. The axes are extended until they cut the circle. Focusing in x-axis, we see that cuts the circumference in point a, drawing by that point an orthogonal line to the x-axis, and extend until it cut the rest of the axes in b and c. Then we draw a capable arch, which diameter is the segment bc, and x-axis is extended until it cut the capable arch in point d. Length of segment ad is equivalent to wanted scale factor Fx. Proceeding with y-axis and z-axis, we will obtain respectively Fy and Fz.

Figure 6, compares the procedure, applied to the x-axis, with the figure that originates it, since scaling by x-axis direction a cavalier perspective is obtained. Consider segment ao, it measures an unit, and fix point a as the scaling base point, therefore it will remain immobile, we will draw the capable arch which diameter is segment bc to know how much the segment oa has to be lengthen to achieve a right angle between z-axis and y-axis.

Once known the relationship between both projections types, we can apply the method explained to transform drawings or images that represent orthogonal projections in oblique projections, as shown in figure 7, where an orthogonal perspective has been scaled by the coordinate axes direction to obtain three oblique perspectives.

As we can see in the example, the perspectives obtained, presented heavy distortions, which difficult comprehension of the represented object, due to the inadequate angle between coordinate axes and projection plane.

To solve this problem, we better can use orthogonal projection, where angle between coordinate axes and projection plane, come about 45 and 63º so it may produce reduction coefficient factors between 1 and 0.5. Obviously if the orthogonal projection is already made, is not possible to control the distortion, but if we have a three-dimensional model, we can place it correctly with regard to the projection plane, and easily get an appropriate orthogonal projection of the model, to obtain a military or cavalier perspective with appropriate reduction coefficients.

We see in Figure 8 an orthogonal projection obtained from a three-dimensional model whose Z-axis formed an angle with the projection plane equal to 55º, so after the transformation, we got a military perspective with an appropriate reduction coefficient Cz=0.7.

### Processing directly three-dimensional models

Based on the first considerations, applied to orthogonal projections, we can apply the transformation directly on the three-dimensional model.

Once the point of view is chosen, we have to distort original geometry, to achieve orthogonal projection of the deformed figure to be equivalent to an oblique projection from the model.

Applying same previous principles to the three-dimensional model, will produce an object transformation, so it's made a change of scale in the direction of the x, y, z axes which are the spatial X, Y, Z axis projected.

Scaling the model by z-axis direction, we will get a military perspective, and doing the same by x-axis or y-axis direction we will obtain both cavalier perspectives. The scale factor to be applied will be the same that we have deducted in the earlier reflections, i.e. 1/sen $\alpha$, remember that $\alpha$ is the angle that form coordinate axes in each case, X, Y or Z with the projection plane.

Figure 9 shows how a three-dimensional model is orthogonally projected, before and after applying the scale change by z-axis direction, in order to obtain the equivalent projection to a military perspective.

Figure 10 shows how to get the equivalent projection to a cavalier perspective scaling the original object by the direction of the x-axis.

The figure 11 exhibits, on the left, which seem to be several oblique projections from a cube, by different angles. However, on the right, it's seen that isn't oblique projections, but orthogonal projections of oblique prisms obtained from cube deformation.

### Results and examples

Working directly on the model 3D has some advantages such as the possibility of applying textures, lighting, in addition to generate animations.

The study deductions implementation may be carried out through any CAD program that allows non uniform scale.

Figure 12 shows an example of method implementation to the three-dimensional model from a vault belonging to the Architecture Treaty of Alonso de Vandelvira, where only three frames of the entire animation obtained are shown. Other examples are the illustrative figures of this article since they are oblique perspectives.

1 / Oscar Niemeyer, *Meu Sosia e eu.* 1992, Page. 33.
2 / Idem, p.35.
3 / Idem, p.33.

4 / Oscar Niemeyer, *100 years of Architecture* (Oh, how magical it is to see what appears on a sheet of white paper, a palace, a museum, the beautiful figure of a woman! Like desire and the enjoyment of drawing, like the feeling from the curves of my architecture!" ).
5 / arquia/documental 1/disc 1. Oscar Niemeyer. *Un arquitecto comprometido.* Belgium, 2000; Brazil, 2003.
6 / Idem previous.



## ABOUT THE DRAWINGS OF OSCAR NIEMEYER

**by Manuel Franco Taboada**

This paper is a continuation and development of the work presented to the International Architectural Graphic Expression Congress in Madrid in 2008, under the title: *Simple drawing versus simple architecture.*
In that paper we intended to answer the conundrum in the title. In other words, is there a direct link between an architect's drawings and his or her architecture? Or, on the contrary, is it possible to have simple architecture with project drawings that are baroque and highly detailed, and therefore contradictory?
The text went on to say:
"In the same way, one can ask if from architecture with a taste for elaborate details or decoration one can infer that the drawing is also complex, arabesque, over-drawn.
The initial aim of that paper was therefore very ambitious: the intention was to explore the concept as a whole. It soon became clear that the field is immense, that there is more than enough material for a doctoral thesis. We therefore delimited the objective of the study/field to a few masters of architecture, chosen subjectively, leaving for later the architects initially included in the summary of the paper.
It was therefore an introduction, the exploration of an idea, a taste of the work to come".
Work therefore began to test the relationship between drawing and architecture for the following architects: Mies van der Rohe, Le Corbusier, Oscar Niemeyer, Alejandro de la Sota and Frank Lloyd Wright.
In this paper we are going to explore the work of the Brazilian master Oscar Niemeyer in more depth, through his drawing. We are starting from the view that, as we will explore below, few architects have been as militant in terms of valuing the importance of drawing in architecture, something that we can see not only from the huge number of graphic testimonies, but also from his reflections about drawing as a creative medium.
I consider it a privilege to have been witness to a plastic-communicative expression, unprecedented in the history of architecture, during the extraordinary opportunity I had to meet Oscar Niemeyer, experiencing in person a detailed explanation of his architectural concept, his works, and his legacy, through drawing.
We arrived at the studio in his house in Copacabana, Rio de Janeiro, early in the morning. Today he is 101 years old, but at that time he was 92 years old and it was surprising to see how enormously energetic he was, something that he communicated clearly. Smoking a cigarette and with his hands on his braces, he showed us into his office, the same one that you can see in the photograph taken last year with Alvaro Siza, in which the beautiful photograph at the back was omnipresent, but more of that later.
To see Niemeyer "drawing architecture" in person and at 92 years old is without doubt a privilege, a unique opportunity that chance, or luck, allowed me to enjoy. I am saying "drawing architecture" not "talking about architecture" because Niemeyer does not know how to communicate, to express the depth of his architectural thinking, without the aid of drawing, without using his hand to put black pen on white paper.
Black on white.
The purest contrast, the truth without beating about the bush, without shades of grey, without resorting to artifice.
White and black.
The pure form expressed in lines that reflect the main idea, the idea that defines his architecture so well, that the line is the shape: it is architecture.

### The relationship between DRAWING and ARCHITECTURE

"A Arquitectura se exibe desde o primeiro traço".
"Architecture which seeks out beauty through the imagination, without worrying about small details, working on the structure, which is created and exhibited with the first line"[1]
When Niemeyer says that his architecture is inserted and exhibited in the structure, "*nas quais se insere e se exibe desde o primeiro traço"*, he is talking not only about the first stages of construction, but also the first *lines* of the drawing. From the first moment of conception, his direct and minimalist drawing defines the structure which will be, in the end, the essence of the architecture itself. "Se examinarem o Congreso de Brasilia ou os palácios nela realizados verão que, terminadas sus estruturas, a arquitetura ja estava presente". [2]
For Niemeyer, the beauty of his architecture, in contrast to functionalist architecture, resides in the freedom of the forms, in the lightness, variety and creativity that he gives them through the use of concrete, and which is perfectly expressed by his drawings. He quotes Baudelaire when he says: "L´inattendu, l´irrégularité , la surprise et l´etonnement sont una partie essentielle et une caractéristique de la beauté".[3]
If, for example, we examine the drawing of the Mondadori headquarters at the same time as we look at the building, or at its canonic photograph, we can see an almost complete definition of the building; the drawing represents, through fine lines and without shading, the purity of the form of the building, a drawing in which details scarcely exist and, where they do exist, are unimportant. The lines of the drawing create the outline of the future structural form.
Architecture which is also simple, without details, structural, monumental.

### The importance of Drawing

"Drawing, including figurative drawing, is essential for an architect, but in schools it was replaced by technical drawing and computers a long time ago. A talented child does some fantastic drawings at eight or ten years old, which bad teaching and knowledge of the classics is going to permanently vulgarize".
"Drawing is fantastic. It serves for almost everything. Seeing a child's drawing is always nice because there is freedom. The child doesn't have anything in its head, it is something spontaneous. So freedom and drawing are essential parts of the life of man".[4]
When talking about "desenho" drawing, Niemeyer defends the need for figurative drawing, of the human figure, arguing that it provides the manual ability to draw freehand:
"Ah, como é mágico ver surgir na folha branca de papel um palácio, um museu, uma bela figura de mulher! Como as desejo e gosto de desenhá-las! Como as sinto nas curvas da minha arquitectura!"[5]
In addition, all through his books he presents us with freehand drawings next to the photographs.
We could say that freehand drawing is so important to him that he rejects technical drawing, to such a point that he considers it expendable. To try and find exhaustive documentation on his projects, plans that define the construction, is an arduous task. It is almost as if there was no intermediate step between the figurative drawing and the constructed work.
Niemeyer says that he has often come up with the solution for a project by just thinking about it, without drawing (as Sota says he did), imagining how it is, how it should be, but at other times he arrives at the solution through drawing: "The basis for architecture is drawing and the mind. When a solution is found, and once the drawings are defined, I start to write a text explaining the architecture, and if on reading the text I do not find arguments I go back to the drawing board because something is missing".[6]

### The influence of the landscape and the feminine form on his work

In *Minha Arquitectura*, Niemeyer writes: *"*Não é o ângulo recto que me atrai, nem a linha recta, dura e inflexível, criada pelo homem. O que me atrai é a curva livre e sensual, a curva que encontro nas montanhas do meu país, no curso sinuoso dos seus rios, nas ondas do mar, no corpo da mulher preferida. De curvas é feito todo o universo, o universo curvo de Einstein".
Many of Niemeyer´s drawings are of women, or, perhaps more accurately, of women's curves; and many of the drawings of buildings are equally curvaceous and immediately bring to mind the feminine form.
On one occasion, Le Corbusier told Niemeyer that he had the Rio mountains within him.
Niemeyer replied that this was partly true, that the environment in which he was living was to blame, but