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Interpreting and validating models of loans
Interpretación y validación de modelos de préstamos

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Abstract

Due to developments in the financial marketplace, digitalization of finance and easy access to financial products, financial literacy is becoming increasingly important. The paper at hand introduces parts of a learning environment for mathematics classes that focuses on mathematical modelling and financial literacy. After theoretical considerations, two worksheets are presented, which deal with influences of the interest rate on the repayment of loans. Based on the description of a test run, the usability of the worksheets in class is concluded.

Debido a la evolución del mercado financiero, la digitalización de las finanzas y el fácil acceso a los productos financieros, la alfabetización financiera es cada vez más importante. El artículo que nos ocupa presenta partes de un entorno de aprendizaje para clases de matemáticas que se centra en la modelización matemática y la alfabetización financiera. Tras las consideraciones teóricas, se presentan dos propuestas de trabajo que tratan sobre la influencia del tipo de interés en la devolución de los préstamos. A partir de la descripción de una prueba, se concluye la utilidad de las hojas de trabajo en clase.

Keywords: Mathematical modelling, financial literacy, loans, worksheets
Palabras clave: Modelización matemática, educación financiera, préstamos, hojas de cálculo

1. Introduction

The paper at hand pursues the goal of preparing students for the life situation of planning to raise a loan by modelling amortization schedules with random interest rates. This is not far-fetched according to the EOS debt study (2017), 75 % of the German population raise at least one loan in their lifetime and in Austria, every second person has debts resulting from his or her consumer behaviors (c.f. ING-DiBa, 2016). One might also recall the real estate bubble burst in Spain. Furthermore, multiple studies find that people's financial literacy skills are in a bad shape. For example, according to OECD (2020), around 25 % of the 15-year-olds who took part in the financial literacy survey in 2018 are unable to make even simple decisions about everyday spending. Adults similarly do not do well on financial literacy tests, which has been known for some time now. Lusardi and Mitchell (2014), for instance, report poor performances. In their study, they asked the subjects three questions. Any person who could not answer all three questions was said to be financial illiterate. In the USA, about 70 % were not able to answer all three questions correctly, while in Germany, the percentage amounted to approximately 50 %. The OECD (2016) confirms the above presented result, they report on relatively low levels of financial literacy among adults.

This paper examines parts of a learning environment published by Dorner (2017) that focuses on mathematical modelling and financial literacy. To do so, the article begins with theoretical considerations of mathematical modelling and financial literacy. Then, two worksheets from the above-mentioned learning environment are presented. They deal with influences of the interest rate on the repayment of loans. After the description of a test run of using the teaching materials in a classroom, a conclusion about the usability of the worksheets in class with focus on the theoretical foundation is drawn.

2. Theoretical background

2.1. Mathematical Modelling in mathematics classes

Pollak (1977) distinguished four definitions of applied mathematics, whereby he understood modelling, as something as repeating the modelling cycle several times. Since then, modelling has been considered as a cyclic process between reality and mathematics. Over the years, in German speaking countries, multiple detailed descriptions of modelling cycles have been provided, especially in the context of education (c.f. Schupp, 1989; Maaß, 2006; Blum and Leiß, 2007). Typically, this process is partitioned into subprocesses such as simplifying, mathematizing, working with mathematics, interpreting and validating. According to Borromeo Ferri (2011), these phases are observable during students' modelling processes, although not always in the order listed above. Rather, students jump between certain phases - in other words, they pursue individual modelling routes. Similarly, modelling competence as a whole can be divided into sub-competencies according to the stages of modelling cycles. Hankeln and Greefrath (2020) empirically showed the existence of different modelling sub-competencies.

Students' difficulties while modelling have been known for quite a long time. According to Wess (2020), struggles of students can occur in every modelling phase. According to the focus of the paper, a few selected study results on students' troubles during the specific phases interpreting and validating will now follow. According to Blum (2011) and Maaß (2004), students can forget the meaning of the calculated results or omit an interpretation entirely. The process of validation causes problems as well. Students are not used to validating their results, furthermore, some students fear a devaluation of their calculated results if they try to validate them. There are also students' conceptions to take into account, which include the idea of the teacher being the only authority who is allowed to validate their results. Hence, they may think such a process is part of the teachers' activities.

According to Fischer (2001) and his concept of *"general higher education"*, graduates of upper secondary schools should be able to communicate with experts, which means understanding expert opinions in order to make effective decisions. In the context of mathematics, this mainly includes the interpretation of mathematical results.

According to multiple – mostly qualitative – studies, the use of DGS supports students at various stages of the modelling process (Hankeln and Greefrath, 2020). Following Greefrath and Vorhölter (2016), this is particularly true for the processes interpreting and validating. Furthermore, DGS are likely to increase the diversity of solutions found by students. According to Siller and Greefrath (2010), when using digital tools while modelling, the modelling cycle has to be expanded because mathematical expressions have to be translated into the language of the used digital tool and then the results of the digital tool must again translated into mathematical results. Dynamic geometry software (DGS), in this case GeoGebra, plays a significant role in the modelling processes intended to be considered when using the below presented worksheets. GeoGebra supports various modelling processes, especially, because of diverse representation options (Pead et al., 2007; Moreno-Armella et al., 2008; Mousoulides, 2011; Greefrath et al., 2018). Hence, GeoGebra seems to be an appropriate choice. Due to possible computer calculations, e.g. many different loan repayments could be considered in a short period of time.

The modelling process described in the two worksheets below involves stochastic simulations. In accordance with Greefrath and Vorhölter (2016), this paper regards such simulations as experiments that use models, hence, such simulations provide insights into the real system. Thus, students gather experiences in probabilistic situations and can improve their stochastic thinking skills. Furthermore, through the use of random numbers, risks of loans become visible. A few studies state positive effects of simulations in mathematics classes. For example, simulations promote students' motivations according to Mills (2002), with weaker students, in particular, benefitting from them.

2.2. Financial literacy

According to the OECD (2019b), shrinking public and private support systems (risk shift), aging of the population, developments in the financial marketplace, digitalization of finance (digital financial products and services), as well as easy access to money and financial products and services from a young age (bank accounts) are reasons why being financially literate becomes more and more important. Since 2002, the OECD has committed itself to financial education - at first by initiating far-reaching education projects and later on by assessing the financial literacy of students in its PISA study. The first large scale study took place in 2012, in the course of which a financial literacy framework was created. Within this framework, the term financial literacy is defined as follows:

"Financial literacy is knowledge and understanding of financial concepts and risks, as well as the skills and attitudes to apply such knowledge and understanding in order to make effective decisions across a range of financial contexts, to improve the financial well-being of individuals and society and to enable participation in economic life." (OECD, 2019b, p. 18)

Considering the interplay with other literacies, the OECD (2019b) states that students who want to perform well need to have certain levels of reading literacy and mathematical literacy. There is a strong positive correlation between financial literacy and reading literacy as well as between financial literacy and mathematics (Sole, 2014). Considering the focus of the paper, the latter one is of special interest. The OECD framework of mathematical literacy encompasses four main areas: "*change and relationships*", "*space and shape*", "*quantity*"and "*uncertainty*". According to the OECD (2019a), only quantity, in more detail, numeracy is necessary for financial literacy. Basic calculations, like percentages, are required for some financial decisions.

It should be noted that the financial literacy framework is aimed at students of up to 15 years, in other words, lower secondary level. The paper at hand focuses on a learning environment for students who attend upper secondary level. Not only for that reason, the intersection of mathematical literacy and financial literacy will be understood in a broader sense but also other studies state a greater intersection of these two fields. Ozkale and Erdogan (2020), for example, investigated mathematical literacy tasks and financial literacy tasks of PISA and emphasized that mathematics serves not only as a calculation tool for financial literacy. They point out the necessity of using mathematical contents and processes when solving financial problems. Furthermore, Sole (2017) describes how one can incorporate financial topics into mathematics lessons. She demonstrates that function analysis and algebra are needed for optimizing one's outcome. Following this argumentation, multiple questions may arise in the context of financial risk considerations: Considering buying financial products, when does one take a risk? How much gain or loss can be expected? What is the meaning of a low probability of a high interest rate? How can I reduce risk? In order to answer these questions, one must exhibit stochastic competencies.

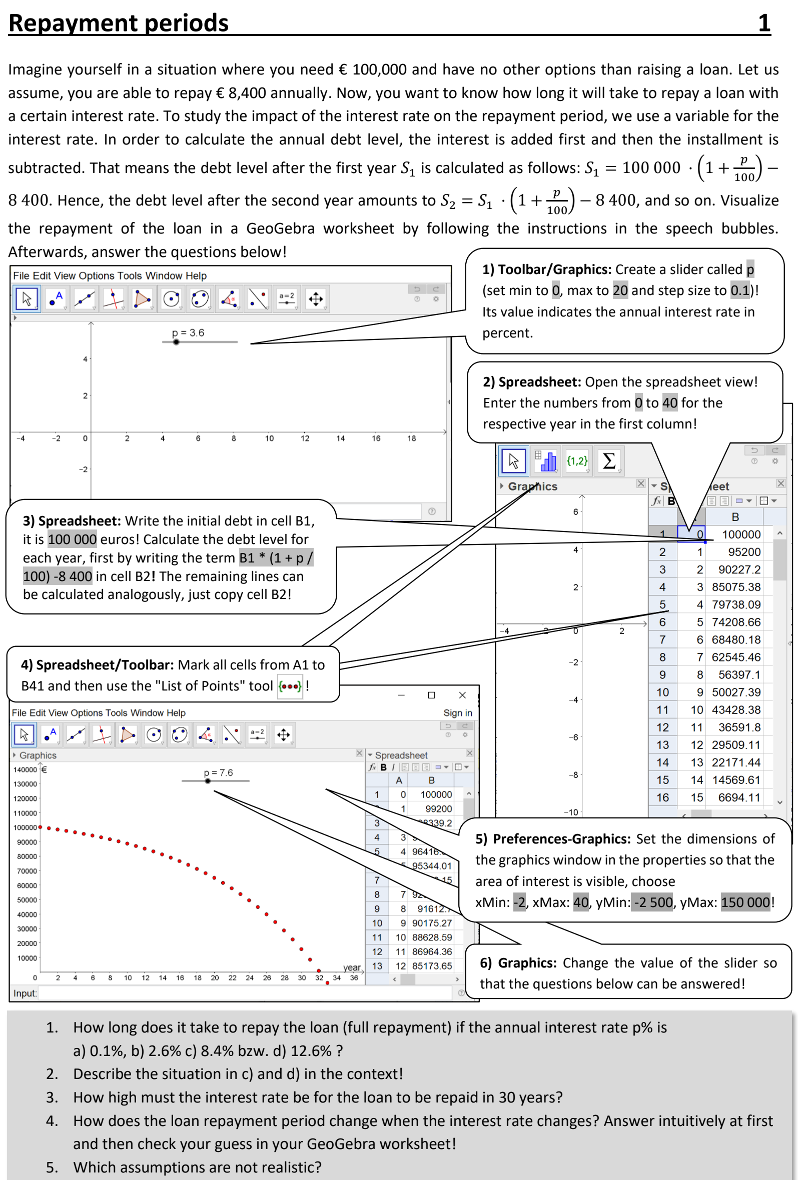
Summing up, being financially literate also means applying mathematical concepts in a financial context in a broader sense. The paper at hand will use the above-mentioned definition of financial literacy by the OECD, but with an understanding of a greater intersection with mathematical literacy, as described above.

3. The lessons: Modelling interest rates of loans

The design of the double lesson for mathematics classes is based on two worksheets that are part of a greater learning environment in Dorner (2017). They cover the effects of interest rate changes on the repayment of a certain loan. The first worksheet guides the students in creating a repayment schedule with the interest rate being modelled via a slider. With the second worksheet, probabilistic considerations come into play when students have to model the interest rate with random numbers. Thus, a large number of repayment scenarios can be considered and analyzed.

The worksheets, see Figure 1 and Figure 2, were created for the specific situation of loans in Austria. Here, a large number of loans are granted with a variable interest rate, even for mortgages. In fact, the situation used to be even worse: Until the bank crisis in the year 2008, private persons could raise foreign currency loans in order to finance their homes. Typically, however, borrowers have to choose between loans with a fixed interest rate or a floating interest rate. As a rule, fixed rates are higher than floating rates at the time of borrowing, which represents a kind of risk premium. Considering floating rates, one must be alert to interest rate changes as years go by. In order for the students to be prepared to make effective decisions when raising loans, the worksheets make interest rates and their impacts a subject of discussion.

The lesson plan only requires few prerequisites that the participating students should have covered already, namely students should be able to calculate the annual debt level of a loan given the initial debt, the interest rate and the installment by using recursive sequences. Also, students should have gained some experiences with random experiments, events and random numbers. In order to assure smooth working on the tasks, the students should already be used to the software GeoGebra. With the goal of reducing complexity, technical jargon and legal terms are avoided.



Repayment periods **1**

Imagine yourself in a situation where you need € 100,000 and have no other options than raising a loan. Let us assume, you are able to repay € 8,400 annually. Now, you want to know how long it will take to repay a loan with a certain interest rate. To study the impact of the interest rate on the repayment period, we use a variable for the interest rate. In order to calculate the annual debt level, the interest is added first and then the installment is subtracted. That means the debt level after the first year S_1 is calculated as follows: $S_1 = 100\,000 \cdot \left(1+\frac{p}{100}\right) - 8\,400$. Hence, the debt level after the second year amounts to $S_2 = S_1 \cdot \left(1+\frac{p}{100}\right) - 8\,400$, and so on. Visualize the repayment of the loan in a GeoGebra worksheet by following the instructions in the speech bubbles. Afterwards, answer the questions below!

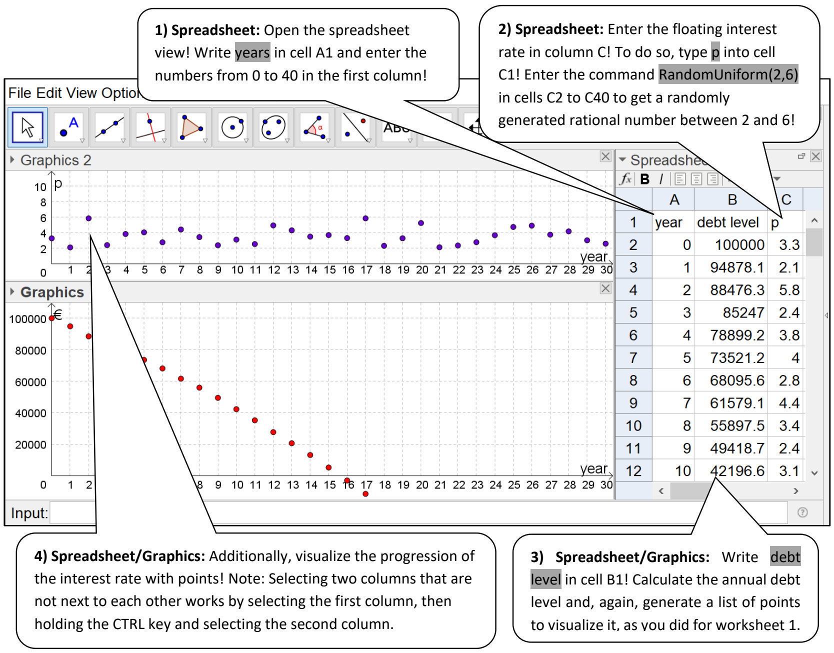
1. How long does it take to repay the loan (full repayment) if the annual interest rate p% is a) 0.1%, b) 2.6% c) 8.4% bzw. d) 12.6% ?
2. Describe the situation in c) and d) in the context!
3. How high must the interest rate be for the loan to be repaid in 30 years?
4. How does the loan repayment period change when the interest rate changes? Answer intuitively at first and then check your guess in your GeoGebra worksheet!
5. Which assumptions are not realistic?

Figure 1 – Worksheet 1 - Repayment periods (see Dorner, 2017, p. 226, translated)

Floating interest rates **2**

The previous model assumes that the annual interest rate remains constant over time, which is not the case for all kinds of loans. Fluctuating interest rates represent a risk in loans with variable interest rates. Many borrowers overlook this risk. When raising a loan, a conversation with a bank employee takes place in which repayment scenarios such as the ones in the previous task are considered. In most cases, the employee only presents one possible repayment schedule that usually depicts a favorable situation for borrowers. Repayment plans that are bad for the borrowers are usually not shown. The bank just wants to sell its products, so this strategy makes sense.

Create a new GeoGebra worksheet in which the annual interest rate changes from year to year! Model the changing interest rate using random numbers! Open a new GeoGebra worksheet and follow the instructions in the speech bubbles!



1. Pressing the F9 key causes a new calculation of the random numbers. Simulate multiple repayment scenarios by pressing the button! Describe what has happened!
2. Which interest rate curves are favourable for the borrower and which are not?
3. Increase the range in which the random interest rate may be to the interval]0.1; 12 [! Describe what has happened!
4. Improve the model: The interest rate for the next year should stay within a small interval (How small?) around the interest rate of the current year. Modify column C accordingly and simulate several scenarios! What do you notice?
5. Does it even make sense to simulate the loan interest rate with random numbers?

Figure 2 – Worksheet 2 - Floating interest rates (see Dorner, 2017, p. 227, translated)

The two worksheets above can be regarded as authentic tasks, which provide the students with an appropriate mathematical model, where interpretation and validation are at least partly left open to the students. Considering the expanded modelling cycle by Siller and Greefrath (2010), the translation process from computer results to mathematical results is left open to the students as well. Due to the complexity of the actual process of raising a loan and based upon the above-mentioned concept of general higher education and the financial literacy framework of the OECD (2019b), these worksheets mainly focus on these sub-processes.

There are three major learning objectives: 1) Students should understand the dynamics of loans, which explicitly includes the effects of rising and falling interest rates assuming constant installments. 2) Students should be able to simulate a loan repayment by modelling the interest rate using random numbers. 3) Students can mention good and bad progresses of the interest rate of certain loans from the point of view of a borrower.

4. Teaching experiences

Dorner (2017) reports on a trial of the two worksheets presented above, which took place during three regular mathematics lessons with 16/17-year-old students at a secondary school in Austria. Ten out of thirteen students were present during the two lessons. All of these students finished the two worksheets within at most 120 minutes.

In the following, the focus is on written student responses with five tasks being examined more closely. As a quick reminder, task 2 of worksheet 1 reads as follows: "Describe the situations in c) and d) within the context". In case c), the interest rate is 8.4 %, hence, we maintain a constant debt level, even though installments of 8, 400€ are paid every year, because $S_1 = 100000 \cdot (1 + 8.4/100) - 8400 = 100000$. In case d), the interest rate is greater than 8.4 %, therefore $100000 < S_1 < S_2$ and so on. Considering the respective student answers, one could observe a great variety. For example, there are students who give more mathematically bounded responses (see Figure 3), while others describe the situation without using mathematical formulas (see Figure 4).

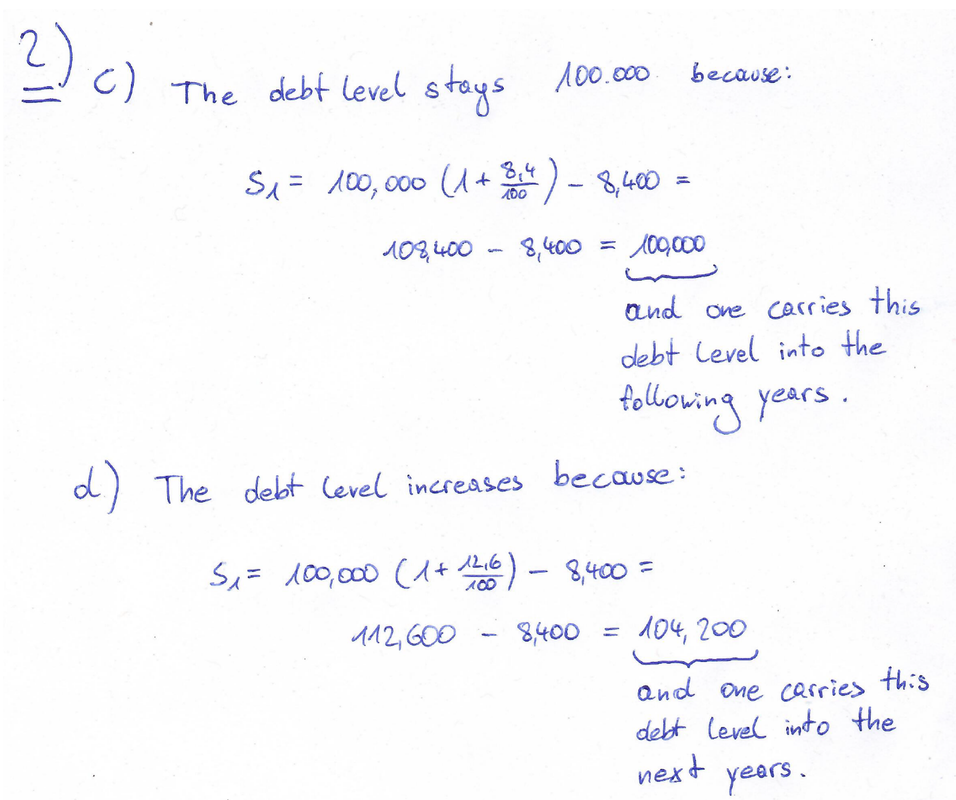
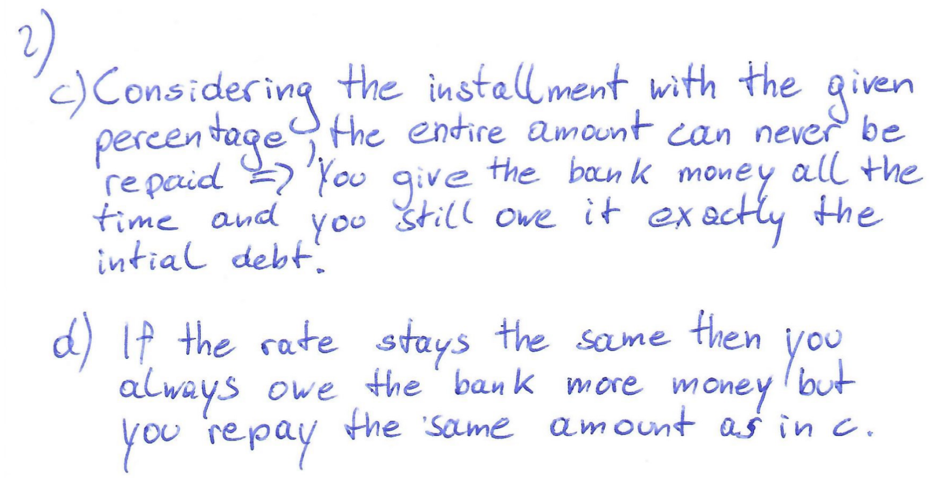


Figure 3 – Student response 1 - worksheet 1, task 2 (see Dorner, 2017, p. 273, translated)


2)
c) Considering the installment with the given percentage, the entire amount can never be repaid => You give the bank money all the time and you still owe it exactly the intial debt.

d) If the rate stays the same then you always owe the bank more money but you repay the same amount as in c.

Figure 4 – Student response 2 - worksheet 1, task 2 (translated)

There are less formal answers as well, see Figure 5. The statement is correct in principle, however, one cannot deduce whether the student has understood the mathematics behind it.

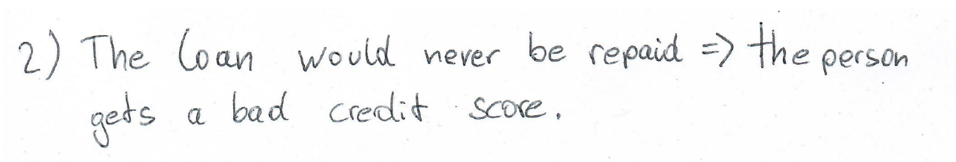

2) The loan would never be repaid => the person gets a bad credit score.

Figure 5 – Student response 3 - worksheet 1, task 2 (translated)

When checking for possible inappropriateness of the assumptions, see task 5 of worksheet 1, the student answers resemble each other. All students wrote that high interest rates were unrealistic, specifically, they rejected the possibility that the interest to be paid could be higher than the installments.

In the context of random interest rates, task 2 from worksheet 2 asks about favourable interest rates from the point of view of the borrower. All students recognized that lower interest rates are better for the borrower, see Figure 6.

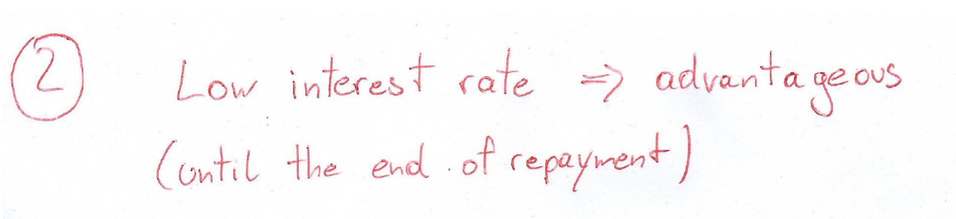

(2) Low interest rate => advantageous (until the end of repayment)

Figure 6 – Student response 4 - worksheet 2, task 2 (translated)

However, two students only focused on the beginning of the repayment period, see for example Figure 7. Their response is correct in itself, but it does not cover the whole period. Their statements would need to be expanded.

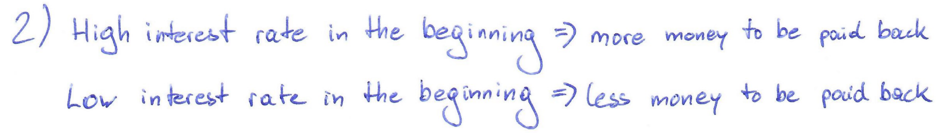


Figure 7 – Student response 5 - worksheet 2, task 2 (see Dorner, 2017, p. 276, translated)

Finally, task 4 of worksheet 2 is of special interest. How did the students improve the modelling of the interest rate? All students improved the model of the interest rate by implementing dependencies as intended. Eight students programmed the interest rate of a certain year as a uniformly distributed random number from the closed interval with lower bound - interest rate of the previous period minus 1 - and upper bound - interest rate of the previous period plus 1 -. Some of these students used a different constant here, for example 2. Additionally, some described the resulting interest rate curve as less jagged and therefore more realistic, while others criticized the possibility of negative interest rates, which seemed unrealistic to them, see Figure 8.

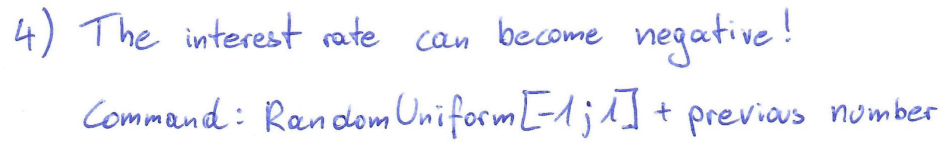


Figure 8 – student response 6 - worksheet 2, task 4 (see Dorner, 2017, p. 276, translated)

There are students whose observations do not feature any “complaints”. For example, one student noticed different interest rate developments in his new interest rate model.

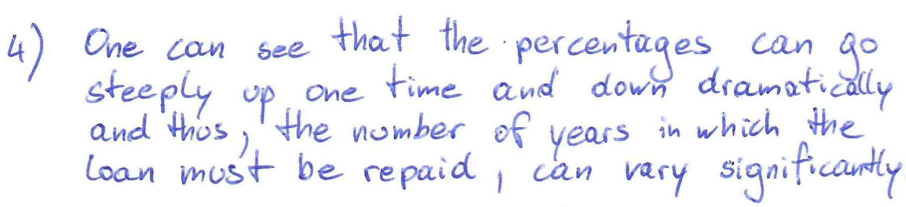


Figure 9 – Student response 7 - worksheet 2, task 4 (translated)

Two students discovered the command `RandomNormal(<Mean>,<Standard Deviation>)`, this command generates a random number from a normal distribution with given mean and standard deviation. The students took the opportunity, researched on the internet and implemented a normally distributed interest rate curve. Referring to the table in Figure 2, the two students entered an initial value in the cell C2, e.g. 2, then they inserted the command RandomNormal(C2,0.2) in the cell C3, RandomNormal(C3,0.2) in C4 and so on. One result of the development of the interest rate when using the above-described normal distribution model can be seen in Figure 10.

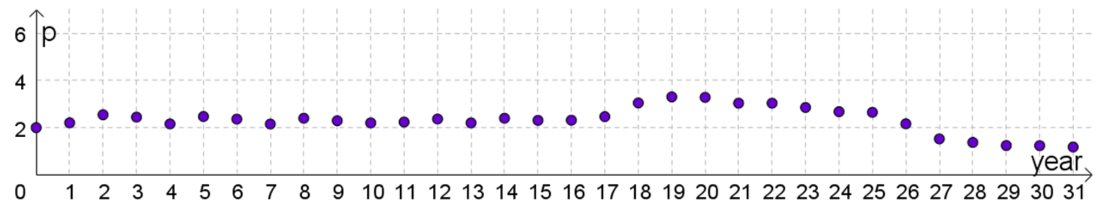


Figure 10 – Development of the interest rate when using normally distributed random numbers

However, their response turned out to be rather short, see Figure 11.

Figure 11 – Student response 8 - worksheet 2, task 4 (see Dorner, 2017, p. 276, translated)

The participating students had mixed feelings about the idea of using random numbers. Many students complained about the possibility of getting a negative interest rate. Some wrote that in most cases, one would agree on a fixed interest rate, so random numbers make no sense. Four students emphasized its usefulness considering the impossibility of forecasting. Moreover, they put themselves in the position of a borrower and argued that, by using random numbers, they are able to consider many eventualities and are therefore better prepared in advance, see for example Figure 12.

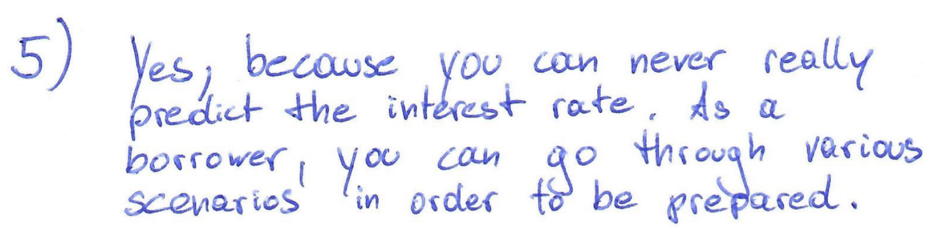


Figure 12 – Student response 9 - worksheet 2, task 5 (translated)

The teacher of this class wrote in his feedback about the worksheets that he appreciates that only few prerequisites are needed for these worksheets, specifically, that there is needed no specific knowledge about loans and that students can work individually. Also, there were a few questions concerning technical issues with GeoGebra. Due to the fact that the students did not have much experience with modelling tasks, some students posed questions about the non-realistic assumptions in worksheet 1 task 5 and some students did not have the courage to set the size of the interval on their own, see worksheet 2 task 4. They asked the teacher to give them suitable numbers, but he always responded: “Choose the number that makes sense from your point of view”.

5. Discussion

The goal was for the two worksheets to contribute to the development of financial literacy and mathematical modelling skills. In order to confirm the successful achievement of these goals, the two topics will be considered separately.

Both worksheets focus, above all, on the sub-competencies interpreting and validating. The modelling of loan repayments initiated through the instructions on the worksheets seems to be suitable for this intention. The outcomes of the teaching trial show that this environment puts students in a position that allows them to interpret fairly complex relations between interest rates and repayments. While some responses turned out rather short or not precise enough, most students considered the important issues of each task. Furthermore, students were quite critical and recognized flaws in the modelling process, such as negative interest rates, growing debt although installments were being paid, as shown in more detail in section 4. Interestingly, the teacher's feedback reveals that some students struggled with validating model 1. But in the end, all students validated both models reasonably and even though not all of their work encompassed every detail, they recognized unrealistic assumptions. Maybe this can be traced back to the prescribed instructions during the first steps of the modelling process, which might have led them to the easier position of validating an already outlined model instead of their own model. It must be emphasized that the teacher has to collectively discuss and compare the different student answers at the end of this teaching sequence. The responses of the trial show high potential for fruitful discussions. During these, teachers can deepen the coverage of issues such as the mathematical background, appropriate interpretations and limitations of the models.

From the students' responses, one can deduce that the presented task set contributes to financial literacy. While working through the exercises, students deal with the dynamics of interest rates and their impact on the repayment schedule. Hence, they consider sources of risks and equip themselves with knowledge that is useful for borrowings. Therefore, students have a more solid foundation for deciding between fixed interest rates and floating interest rates when raising a loan, as can be seen in Figure 12. Furthermore, the interplay of mathematical literacy and financial literacy is observable in the students responses: interpreting diagrams or charts requires mathematical skills that can be placed in a financial context, see figures 3, 4, 5, 6 and 7 and in order to validate models, one needs financial literacy that can be transferred into a mathematical language, see figures 8, 9 and 11. Altogether, the tasks presented above prepare students for making effective decisions in the financial world, which is one of the main parts of the definition of financial literacy.

These worksheets can be considered as a starting point for further (mathematical) activities focusing on topics such as geometric series, recurrence relations, probability calculations, normal distribution, paper and pencil calculations and, of course, interdisciplinary teaching. Further financial topics for mathematics that can contribute to mathematical literacy and financial literacy are for example: discounts (How can algebra be used to shop intelligently?), diversification (Is it wise to put all of your money into a single stock?), randomness of stock prices (Can one predict stock prices in the future?).

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