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A Mathematical Modelling Unit for First-Year Engineering Students

Unidad de Modelización Matemática para estudiantes de primer año de Ingeniería

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Abstract

This paper is practice-oriented and reports on a mathematical modelling unit specifically developed for first-year engineering students in a South African context. The main idea with the unit was to foster students' mathematical modelling competency development. This idea supports an essential goal of mathematics teaching, that is to enable students to solve real - world problems by means of mathematics. The unit consists of five lessons and several tasks, carefully planned to consider students' mathematical pre-knowledge, the demands of the first-year mathematics (calculus) curriculum and the intended competency development. The unit was linked to the mathematical topic of functions and taught for different groups of students according to two different teaching designs, similar to the designs used in the German DISUM project. 144 first year engineering students from the extended curriculum programme of a large public university were divided in three groups and exposed to the unit. An empirical evaluation of the intervention (with a pre-post-test design) showed a significant competency growth for all groups, with substantial differences, dependent on the teaching design. Some strengths and shortcomings of the unit will be identified and implications for future practice will be discussed.

Este artículo está orientado a la práctica e informa sobre una unidad de modelización matemática desarrollada específicamente para estudiantes de primer año de ingeniería en un contexto sudafricano. La idea principal de la unidad era fomentar el desarrollo de la competencia de modelización matemática de los estudiantes. Esta idea apoya un objetivo esencial de la enseñanza de las matemáticas, que es permitir a los estudiantes resolver problemas del mundo real por medio de las matemáticas. La unidad consta de cinco lecciones y varias tareas, cuidadosamente planificadas para tener en cuenta los conocimientos matemáticos previos de los alumnos, las exigencias del plan de estudios de matemáticas de primer curso (cálculo) y el desarrollo de competencias previsto. La unidad se vinculó al tema matemático de las funciones y se impartió a distintos grupos de estudiantes según dos diseños didácticos diferentes, similares a los utilizados en el proyecto alemán DISUM. Se dividieron en tres grupos 144 estudiantes de primer año de ingeniería del programa curricular ampliado de una gran universidad pública y se les expuso la unidad. Una evaluación empírica de la intervención (con un diseño pre-post-test) mostró un crecimiento significativo de las competencias en todos los grupos, con diferencias sustanciales, dependiendo del diseño didáctico. Se identificarán algunos puntos fuertes y deficiencias de la unidad y se discutirán las implicaciones para la práctica futura.

Palabras clave: First-year engineering students; mathematical modelling competency; mathematical modelling unit; real-world problems; teaching design

Keywords: Estudiantes de primer año de ingeniería; competencia en modelización matemática; unidad de modelización matemática; problemas del mundo real; diseño de enseñanza

1. Introduction

The formal education of engineering students requires the development of mathematical competency and specifically higher cognitive skills such as arguing or modelling. However, the South African assessment framework has a greater focus on knowing and solving routine problems, and there is limited emphasis on applying and reasoning. According to the latest TIMSS findings (Reddy, Winnaar, Juan, Arends, Harvey, Hannan et al., 2020), the school and national assessments should include more items at higher cognitive levels. As a result of school mathematics, students entering engineering programmes at universities often lack higher cognitive skills and therefore have to be educated accordingly. One way is to expose students to a series of mathematical modelling activities (De Villiers and Wessels, 2020; Durandt, 2018). Mathematical modelling is in many countries an integral part of the curriculum at all levels of education, also at the tertiary level. The inclusion of a modelling unit in the formal education of engineering students seems important as they will be expected to apply higher cognitive skills and solve real - life problems in their profession. The question arises as to how mathematical modelling can be incorporated in the formal education of engineering students and in particular how such a unit should be constructed. This paper reports on the design of a mathematical modelling teaching unit specifically developed for first-year engineering students in a South African context.

2. Mathematical modelling

Mathematical modelling usually means solving real - world problems by means of mathematics. This involves translating the problem situation into mathematics, working within the resulting mathematical model of the situation, and interpreting the mathematical outcomes in the given situation. An example which is well-known from the literature (see Blum and Leiß, 2007, and Niss and Blum, 2020, chapter 3, for details to this example) is the question as to whether it is worthwhile to drive to a remote petrol station to fill up one's car when the petrol is cheaper there than at a nearby station. Typical steps when solving this problem are: first to understand the given problem, to imagine the situation (two petrol stations with different distances and different prices) and to construct a mental model of the situation; second to structure the situation by identifying relevant parameters (besides the distances and the prices at least also the tank volume and the consumption rate of the given car) and relations between them; third to set up a mathematical model of the given situation, that means to write down terms for the different prices for filling up; fourth to do some calculations; fifth to interpret the results of these calculations in the light of the initial problem of which station to choose; sixth to check whether the result makes sense and if there may be additional variables to consider (such as the time that it costs to drive to the stations, the air pollution by the trip or the risk of an accident on the trip), and if necessary to go once again through steps three to five with a refined model; and seventh and finally to present the whole solution together with a recommendation of what to do. These seven steps can be visualised by the *modelling cycle* shown in Figure 1 (where, according to Pollak, 1979, "rest of the world" denotes the whole extra-mathematical world).

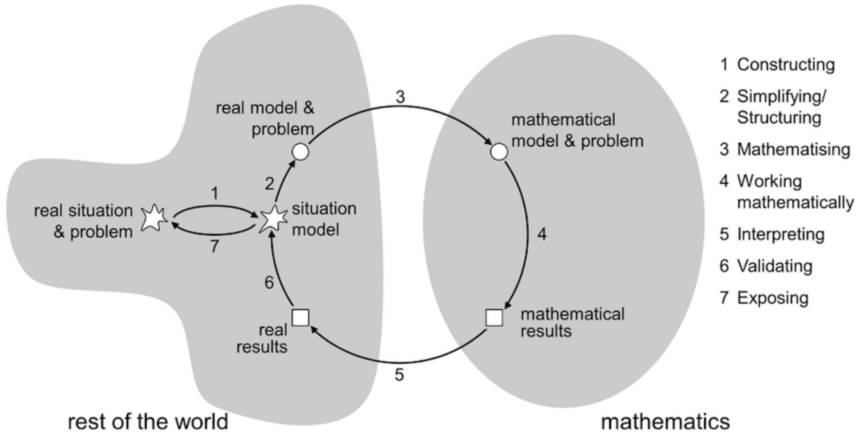


Figure 1 – Seven step modelling cycle according to Blum and Leiß (2007)

Mathematical modelling is a compulsory topic in many mathematics curricula around the world, from primary to tertiary education. There are several *justifications* for the inclusion of modelling (see Blum, 2015, for an overview of such arguments): first to help students to understand certain problems in the real world better; second to advance mathematical competencies such as mathematising, problem solving or communicating; third to contribute to an adequate picture of mathematics as a science; and fourth to support students' learning of mathematics (motivation and interest for mathematical activities, and deeper understanding of mathematical topics). In particular for mathematics as a service subject at the tertiary level, its vital role is to contribute to a better understanding and mastering of situations and problems stemming from the serviced discipline.

Mathematical modelling can be included in the learning and teaching of mathematics by appropriate tasks. No other subject in education is shaped by tasks to the same extent as mathematics. A *mathematical modelling task* is a task which requires genuine mathematical modelling activities to solve it. By "genuine" we mean that the real - world situation has to be taken seriously and is not only a mere, easily recognisable dressing-up of a mathematical task; that the answer to the given question is not obvious from the beginning; that relevant parameters have to be identified and assumptions have to be taken; that some mathematical model has to be chosen; and that the mathematical result has to be translated back into the real world. The real - world situation does not have to be authentic in the sense that it is directly taken from a context where mathematics is actually applied in industry, business, science, society or everyday life. However, the situation has to be credible so that it might occur in practice (see Niss and Blum, 2020, chapter 5, for a discussion of authenticity), or it has to be presented honestly as deliberately constructed for educational purposes. Appropriate modelling tasks for certain educational levels have, of course, to be accessible at that level, that is both the real - world context and the mathematics necessary for the solution have to be comprehensible.

The abovementioned aim to advance students' competencies presupposes a transfer from the competencies needed to solve a particular problem (such as the *Filling Up* problem) to more general competencies which can be applied to solve other problems. In this sense, an important goal of teaching modelling is to develop students' *mathematical modelling competency*, that is their ability to construct and to use mathematical models for solving real - world problems by carrying out appropriate steps as well as to analyse or to compare given models (for a detailed discussion of modelling competency and its sub-competencies see Niss and Blum, 2020, chapter 4). As is well-known from learning theories and empirical findings, transfer cannot be expected to happen automatically. The essential reason is the situatedness of all learning (see Brown, Collins and Duguid, 1989). This means that the modelling competency which a student acquires is usually restricted to the specific mathematical area and the specific real - world context in which it was needed while solving a specific task, and transfer to other tasks, areas and contexts has to be carefully organized by the teacher, especially by pointing to similarities between different tasks. One similarity which can be emphasized are the solution steps through the modelling cycle. Seven steps (like in Figure 1) will generally be too complicated for learners, but a four step cycle like the one developed in the DISUM project ("Solution Plan", see Figure 2) has proven to be helpful for learning modelling (see Schukajlow, Kolter and Blum, 2015).

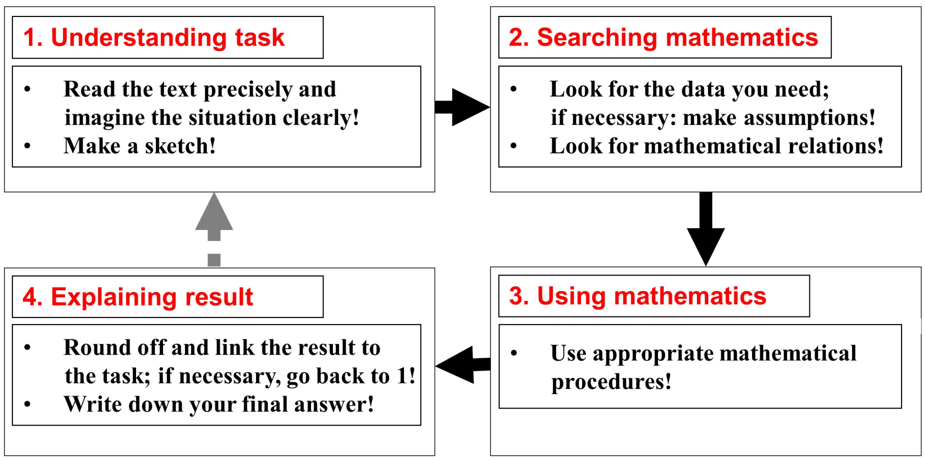


Figure 2 – Four step modelling cycle for learners, developed in the DISUM project

With the difficulty of transfer we have mentioned one of many aspects which make the teaching and learning of mathematical modelling demanding for learners and for teachers (for an overview on empirical findings concerning modelling see Blum, 2015; Kaiser, 2017; Stillman, 2019; Niss and Blum, 2020, chapter 6). We know from several empirical studies that each step in the modelling process is a cognitive challenge for students and may be a barrier which may even lead to a breakup of the solution process if students are working completely alone, without teacher support. There is a fundamental difference between students working alone, and students working independently but supported, if needed, by a teacher. It is crucial that teachers give minimal support, aiming at a permanent balance between students' independence and teachers' guidance. A key element to achieve such a balance is to use adaptive teacher interventions which allow students to continue their work without losing their independence (Blum, 2011; Stender and Kaiser, 2016), in particular strategic interventions (such as "Read the text carefully!", "Imagine the situation clearly!", "What do you aim at?", "What is missing?", "Does this result make sense for the real situation?"). Another key element are meta-cognitive tools like the Solution Plan just mentioned. Many empirical results have shown positive effects of meta-cognitive activities during modelling (see Vorhölter, Krüger and Wendt, 2019, for an overview). There are several more aspects which ought to be taken into account for teaching modelling. Basic criteria of quality teaching such as an effective classroom management, an orientation towards the students' pre-knowledge or a permanent cognitive activation of the students have proven important also for teaching modelling.

3. Study context and aims

In 2018, the idea arose to carry out a research study analogous to the German DISUM[1] project, but for South African engineering students at the tertiary level. The main aim of this study was to compare the effects of different teaching designs for a mathematical modelling unit on the development of students' modelling and mathematical competencies. To monitor the effectiveness of the intervention as well as the influence of the respective teaching design, the participants' competencies were assessed in a pre-post-test design and their attitudes towards mathematical modelling were also measured after the unit.

The unit was designed by the first two authors of this paper and developed for initial implementation in an extended curriculum programme at the University of Johannesburg. In this programme, the first year of the mainstream programme is split over two consecutive years to allow students with lower grades in key subjects (such as mathematics, science, and English) to develop competencies and adequate learning strategies to successfully adapt to the tertiary environment. Hence, the modelling unit was constructed as a part of formal education and during a scheduled weekly tutorial session in students' first semester. During February and March 2019, a sample of 144 first-year engineering students participated in such a curriculum programme and thus in our study. Their participation was voluntary, and standard departmental ethical matters were addressed. The students were automatically assigned to three distinct class groups by the university's registration system, not more than 50 each, according to their focus of study (physical or extraction metallurgy, or construction).

These three groups were taught according to two different teaching styles which fulfil certain criteria of quality teaching (see Blum, 2015), a "method-integrative" and a "teacher-directive" style, analogous to those styles in the DISUM project (for details see Blum and Schukajlow, 2018). The construction group (labelled "MI") was taught according to the method-integrative style which aims essentially at students' independent work on tasks, adaptively supported by the teacher. Both the construction and the physical metallurgy groups (labelled "TD1" and "TD2" respectively) were taught according to the teacher-directive style where the students are to work on tasks guided by the teacher (for more details of the teaching design see Durandt, Blum and Lindl, 2021). The language of instruction was English for all groups, although English was not the home language for the lecturers and for most students. In both the TD1 and TD2 group approximately 16 % of students speak English as their home language, while in the MI group approximately 12 % of students have English as their home language (other languages spoken are mostly African such as Zulu or Northern Sotho). The two lecturers who implemented the modelling unit were the first author of this paper for the groups TD1 and MI, and a colleague from the same department for the group TD2. Both lecturers were experienced in teaching mathematics, but only the first lecturer had experience in teaching mathematical modelling. The researchers' intention was to evaluate the unit after implementation, especially concerning students' learning gains, and to make strategic changes to the unit for further implementation in consecutive years (see Section 6).

In the present, practice-oriented paper we mainly report on the design of the mathematical modelling teaching unit and its contents, not on details of the teaching methods. Also, we will

[1] Didaktische Interventionsformen für einen selbständigkeitsorientierten aufgabengesteuerten Unterricht am Beispiel Mathematik–in English: Didactical intervention modes for mathematics teaching oriented towards students' self-regulation and guided by tasks. The project was directed by W. Blum (mathematics education), R. Messner (pedagogy, both University of Kassel), and R. Pekrun (pedagogical psychology, University of München) and was carried out 2002–2013.

only briefly report on the empirical evaluation of the modelling unit and the corresponding development of students' competencies and attitudes (for more details see Durandt, Blum and Lindl, 2021). Therefore, the leading question for this paper is: *How can a mathematical teaching unit be constructed to support first–year students' development of modelling competency?*

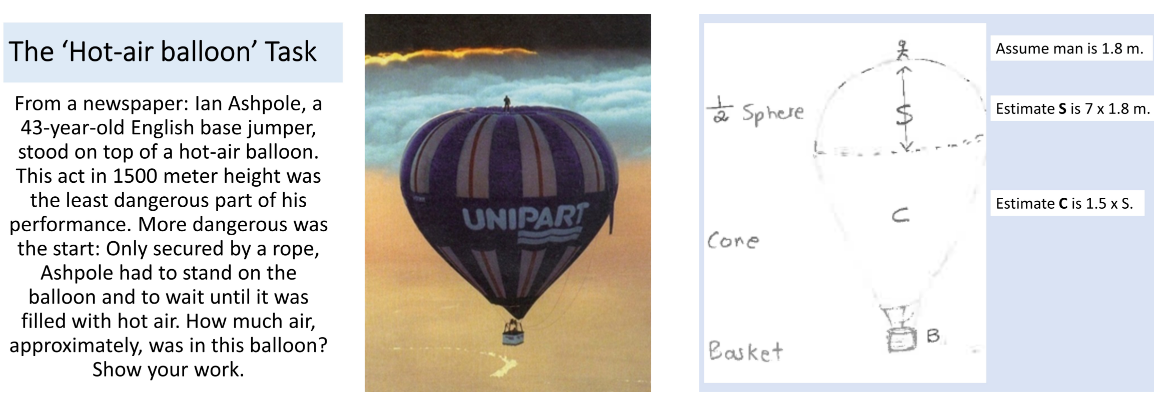
4. Description of the unit

The mathematical modelling teaching unit was constructed over five lessons with ten different tasks, all carefully planned considering students' mathematical pre-knowledge from the pre-tertiary phase, the demands of the first-year engineering mathematics curriculum and the intended development of students' mathematical and modelling competency. All tasks were developed with a focus on the mathematical content area of functions. The last two tasks in lesson 5 were not modelling tasks as characterised in Section 2 but mere word problems, primarily serving for deepening the understanding of proportions (a topic which is needed in several of the modelling tasks). In the following, we describe the content of the individual lessons (45 minutes each; for an overview see Table 1).

Lesson	Number of tasks	Tasks
1	3	*Hot-air Balloon, Hot-water Tap, and Weight of Person*
2	2	*Age of Trees,* and *Evacuation of an Aircraft*
3 & 4	1	*Traffic Flow*
5	4	*Giant's Shoe*, *Statue in Germany*, and two situations with direct and indirect proportions

Table 1 – Tasks included in the five lessons of the modelling unit

Lesson 1: This lesson consists of three tasks, from which the first requires most of the available time. The first task, *Hot-air Balloon*, contains the problem of how much air is in a balloon shown on a picture with a base jumper on top, see Figure 3a (from Herget and Torres-Skoumal, 2007). When solving the task, students are expected to model the balloon (e.g., as a half sphere and a cone), make some approximations (related to the man, the half sphere, and the cone), then proceed with calculations, and finally interpret the mathematical results. For one such example see Figure 3b; here the height of the half sphere was estimated as 12.6 meters and the height of the cone as 18.9 meters. Consequently, the total volume was estimated as approximately 7000 cubic meters.



(a) (b)

Figure 3 – (a) The *Hot-air Balloon* task from lesson 1 (b) A possible solution approach for the *Hot-air Balloon* task from lesson 1

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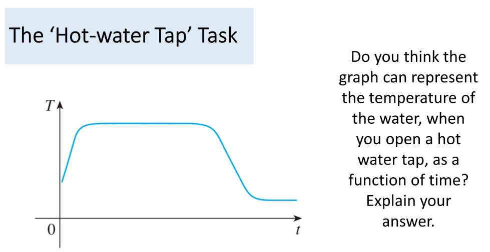
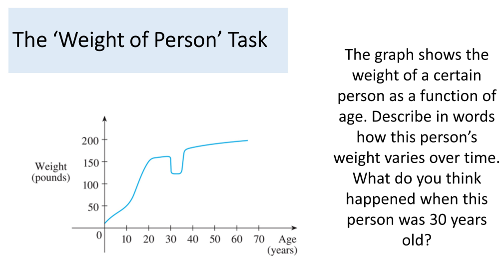
The other two tasks included in this lesson were interpretations of given real - life graphs (both tasks from Stewart, 2016). With the *Hot-water Tap* task (see Figure 4a) students are expected to explain their view on how the graph represents the temperature of the water (T), when a hot water tap is opened, as a function of time (t). One possible solution is to explain that the initial temperature of the running water is close to room temperature because the water has been sitting in the pipes. When the water from the hot-water tank starts flowing from the geyser, T increases quickly. In the next phase, T is constant at the temperature of the heated water in the tank. When the tank is drained, T decreases to the temperature of the water supply. With the *Weight of Person* task (see Figure 4b) students are expected to describe in words how a person's weight varies over time as represented by the graph. Then an interpretation is expected particularly for when the person is 30 years old, with possible reasons such as diet, exercise, or health problems.



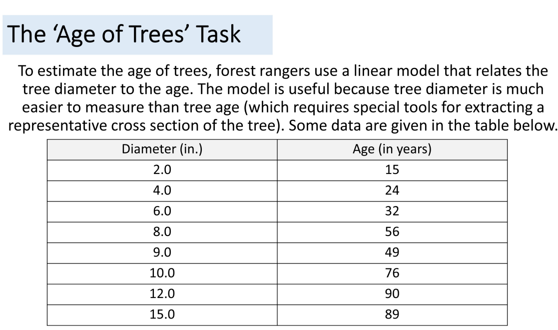
(a)



(b)

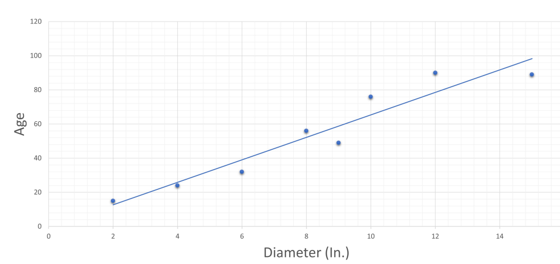
Figure 4 – (a) The *Hot-water Tap* task from lesson 1 (b) The *Weight of Person* task from lesson 1

Lesson 2: This lesson consists of two tasks. In both, data is provided regarding the situation. In the first example, the *Age of Trees* task, a linear model is used that relates the tree diameter to the age of the tree (from Stewart, Redlin and Watson, 2012; see Figure 5a). In real situations, it is much easier to measure the diameter of a tree than the age of a tree, which requires special tools for extracting a representative cross section of the tree. Based on a table of data values, students are expected to construct a linear model that relates the variables. A possible representation of the data and the linear function (given by $\hat{y} \approx -0.5 + 6.6x$) is presented in Figure 5b.



Diameter (in.)	Age (in years)
2.0	15
4.0	24
6.0	32
8.0	56
9.0	49
10.0	76
12.0	90
15.0	89

(a)



(b)

Figure 5 – (a) The *Age of Trees* task from lesson 2 (b) A possible solution for the *Age of Trees* task from lesson 2

The second example in this lesson deals with the evacuation of an aircraft, a modelling multiple choice task taken from Haines, Crouch and Davis (2001), where the best out of five given sets of relevant variables has to be identified (see Figure 6).

The time required to evacuate an aircraft after an emergency landing at an airport needs to be known by the emergency and safety services. There are conflicting needs of aircraft construction, safety, access and ease of exit.

In a simple mathematical model, an aircraft fuselage wide enough for two seats either side of a central aisle is considered with passengers exiting singly at the front and the rear of the aircraft. Which one of the following options contains parameters, variables or constants, each of which should be included in the model?

A. Time elapsed after the emergency landing; Number of people evacuated at time *t*; Time of day at which the landing occurred.

B. Speed of people leaving their seats; Initial delay in unbuckling seatbelts before the first person can leave; Amount of personal items carried out.

C. Number of people evacuated at time *t*; Time of day at which the landing occurred; Width of the emergency exits.

D. Total time to evacuate everyone; Space between passengers leaving; Width of the emergency exits.

E. Number of people in the aircraft; Time elapsed after the emergency landing; Number of people evacuated at time *t*.

Figure 6 – The multiple-choice modelling task from lesson 2 (Haines, Crouch and Davies, 2001)

Students are expected to work through the multiple options and indicate which one of the options contains parameters, variables or constants which should be included in the model. An expected solution (presented by Haines, Crouch and Davies, 2001) is:

A. "Time of day at which the landing occurred" is irrelevant, so A is not appropriate.

B. "Amount of personal items carried out" is not important, so B is not appropriate.

C. "Time of day at which the landing occurred" is hardly relevant, so C is not appropriate.

D. All three variables are relevant, so D is appropriate, but one of the most important variables, number of people in the plane, is missing, that means D brings partial credit only.

E. All three variables are relevant, so E is appropriate, and these three variables are sufficient for a simple model, so this is the best option, thus full credit.

Lessons 3 and 4: Only one, more complex task, *Traffic Flow*, is treated extensively over two lessons (for details of this example see Niss and Blum, 2020, chapter 3). The situation of dense traffic on a single-lane road is presented and students are asked to find the speed at which cars should go to maximize the flow rate. An obvious answer seems to be *as quickly as possible*, but the faster the cars go, the bigger the distance between two cars has to be, for safety reasons, so it is not immediately clear what an optimal balance between velocity and safety would be. In order to be accessible, the situation has to be simplified and structured. The students are expected to:

1. *Imagine the situation* – dense traffic in a single lane travelling at a specific speed; which speed, which car length and which distance?

2. *Look for a suitable mathematical model* – draw a diagram; define "traffic flow rate" as number of cars per time; define distance rules and thus specify the flow rate function.

3. *Do mathematics* – analyse the flow rate function and find the maximum if there is one (calculating values, graphing, finding possible maximum values).

4. *Explain the solution* – optimal speed for maximising flow.

By a few obvious assumptions, that all cars have the same speed v, all cars have the same length l, and the distance d between two cars (dependent on v) is the same everywhere, a model for traffic flow rate can be constructed ($F = v/(l+d)$, where F is the number of cars per time at a fixed point on the road). Although several possibilities exist, the intention is to define common distance rules such as the "half speed rule" and the "driving school rule" and then specify the flow rate function. The "half speed rule" is given by $d = \langle v \rangle/2$ meters, and the "driving school rule" is given by $d = 3 \cdot \langle v \rangle/10 + (\langle v \rangle/10)^2$ meters, where $\langle v \rangle$ means the absolute measure of the velocity in km/h. The graph of the flow rate function determined by the "half speed rule" (see Figure 7) is strictly increasing. This can be easily proved by looking at the term of this function which is a quotient of two linear terms. Thus, the driver can drive as fast as he can in order to maximize the flow rate. In reality, this rule is by no means safe for high velocities (because the braking distance varies, for physical reasons, quadratically with velocity). The graph of the flow rate function determined by the "driving school rule" (see Figure 7 where it is assumed that all cars have a length of 5 meters) has a maximum value. This can also be easily proved by noticing that the term, a quotient of a linear and a quadratic term, tends to 0 both for very low and for very high velocities. The optimal velocity is approximately 20 km/h, a surprisingly low value. In reality this rule is more cautious than physically necessary because it presupposes that the car in front stops immediately to 0.

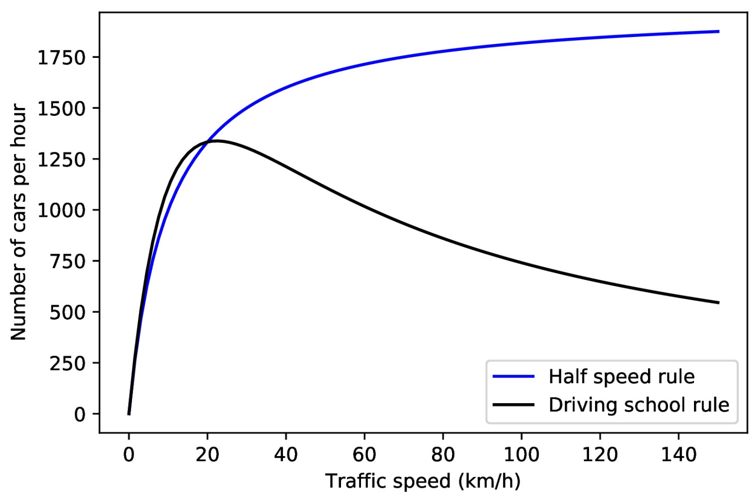
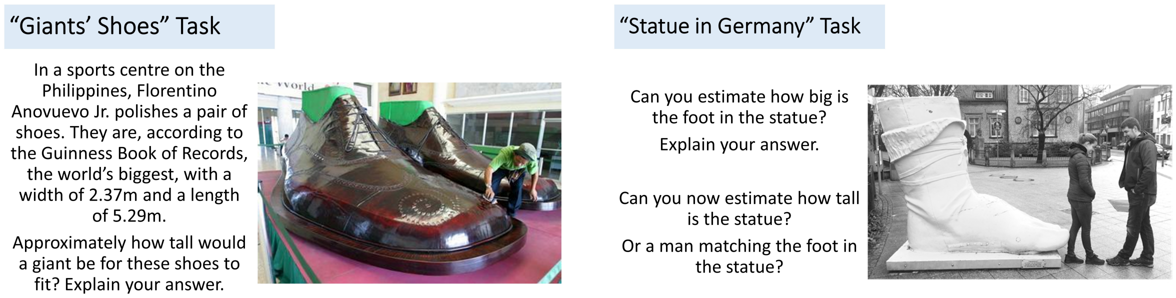


Figure 7 – Graphs of flow rate functions from the "Traffic Flow" task in lessons 3 and 4

Lesson 5: In this lesson four tasks are treated: two picture tasks similar to the first task in lesson 1, and two tasks written in words related to proportions ("direct" and "indirect" proportion). In the first task students are expected to approximate how tall a giant would be in order to fit the world's biggest shoes (2.37 meters by 5.29 meters) given on a photo (see Figure 8a). A possible solution assumes the length of a human foot as, for instance, 0.25 meters, which leads to a shoe length of approximately 0.30 meters. Assuming a human is approximately 1.80 meters tall, and applying the factor 6 ($1.80 \div 0.30 \approx 6$) to the length of the giant's shoe, we can estimate the giant is approximately 32 meters tall ($5.29 \times 6 \approx 32$). Similarly, in the second task students are expected to estimate the height of a man's statue whose foot is shown (see Figure 8b). One possible solution is to assume the four human feet are altogether roughly 1 meter in length, so the foot in the statue is approximately 2 meters. Hence, a man matching the statue is approximately $2 \times 7 \approx 14$ meters, using the ratio $1.80 \div 0.25 \approx 7$.



(a) (b)

Figure 8 – (a) The *Giants' Shoes* task from lesson 5 (b) The *Statue in Germany* task from lesson 5

The remaining two tasks deal with proportions. To practice the concept of "direct proportion" a word problem is given: *A student on a bicycle rides at a constant speed to cover a distance of 12 km in 45 minutes. If he is able to maintain this speed for 1 hour and 15 minutes, how far will he be able to travel?* A straightforward solution is possible by multiplying 12 km by the factor 75/45, thus the distance is 20 km. To practice the concept of "indirect proportion", an exemplary fictitious situation is described: *Two students sharing the costs of a holiday home means 510 (South African) Rand per day for each, three students sharing the costs means 340 Rand per day for each, and four students sharing the costs means 255 Rand per day for each.* The situation is visualised by a hyperbola.

5. Some empirical results

To monitor the effectiveness of the modelling unit described in section 4 and students' progress in mathematics and mathematical modelling after its implementation, a pre-test and a post-test were administered to all three groups ("MI", "TD1", and "TD2") before and after the intervention (see section 3). In these two tests, different but comparable tasks were used, which were directly aligned with the learning content of the intervention (see Table 2, and for a detailed description of the test design Durandt, Blum and Lindl, 2021). Pre-test and post-test thus consisted of three sections each: A) modelling tasks with pictures (such as a beer container or a straw roll) or data tables, B) mathematical tasks (with proportional, linear, and rational functions), and C) multiple-choice modelling tasks similar to the task in lesson 2.

Depending on the number and type of tasks per test section, a different maximum number of marks could be achieved (A: 2 tasks, 6 marks; B: 3 tasks: 7 marks; C: 3 tasks, 6 marks). The answers of the participants were evaluated by two independent raters, marks were assigned for partially or completely correct solutions of the tasks, and the marks were summed up per test section and for the overall test. Table 2 provides an overview of the marks per test section and overall, obtained on average by each group.

Here, we report only briefly on the essential results of the evaluation of these data; further details can be found in Durandt, Blum and Lindl (2021). If we compare the maximum possible marks with the average results achieved by each group in the tests, these are certainly unsatisfactory from a normative point of view in the individual test sections as well as overall. Nevertheless, all groups have reached on average higher values in the post-test than in the pre-test, both with respect to each test section and overall. The learning gains (i.e., mean differences between pre-test and post-test) are descriptively highest for group MI (method-integrative tea-

Test section	Alignment with modelling lesson(s)	Number of tasks / maximum possible marks	Pre-test (*N* = 139) TD1 *M* (*SD*)	TD2 *M* (*SD*)	MI *M* (*SD*)	Post-test (*N* = 141) TD1 *M* (*SD*)	TD2 *M* (*SD*)	MI *M* (*SD*)
A: *Modelling tasks* (with pictures resp. with given data)	1 & 5 resp. 2	2 / 6	1.09 (1.25)	.45 (.71)	1.07 (1.31)	2.65 (1.77)	1.10 (1.29)	2.64 (1.63)
B: *Mathematical tasks* (including proportional, linear and rational functions)	2, 3 & 4	3 / 7	3.41 (1.54)	3.61 (1.26)	3.67 (1.52)	4.35 (1.34)	4.98 (1.26)	5.17 (1.42)
C: *Multiple-choice modelling tasks* (version 1 and 2)	2	3 / 6	2.59 (1.53)	2.53 (1.32)	2.48 (1.50)	3.11 (1.23)	3.00 (1.38)	3.09 (1.28)
Total test		8 / 19	7.09 (2.86)	6.59 (1.84)	7.22 (2.65)	10.11 (2.68)	9.08 (2.73)	10.89 (2.69)

Note. TD1 and TD2: teacher-directive design groups, MI: method-integrative teaching design group; M = mean; SD = standard deviation. For Section C, the results of test versions 1 and 2 are summarised here and, for psychometric reasons, only the results of three (of the original five) items are presented.

Table 2 – Pre- and post-test design, alignment with the modelling unit, and means and standard deviations per test (section) and teaching design group (teacher-directive vs. method-integrative)

ching design), in all test sections and overall, although in the modelling tasks section they are almost equivalent to group TD1 (teacher-directive design).

In deepened analyses (using so-called linear mixed regression models; see Hilbert, Stadler, Lindl, Naumann and Bühner, 2019), which among other things can also consider influential factors such as the entrance selectivity of groups, it can even be shown that the performance increase in the section with modelling tasks is significantly higher in group MI than in group TD2 and that all groups show similar significant performance gains regarding mathematical tasks. While there are no significant differences concerning the multiple-choice tasks, MI performs significantly better than TD2 and descriptively better than TD1 in the overall test.

We also measured the participants' *attitudes* towards mathematical modelling after the intervention, using an internationally well-established instrument, the Survey of Attitudes Towards Statistics (SATS-36, Schau, Stevens, Dauphinee and Del Vecchio, 1995; Schau, 2003), adapted towards mathematical modelling. Analogous to the original instrument, six dimensions are differentiated: Affect (6 items, e.g., "I am scared of mathematical modelling"), Cognitive competence (6 items, e.g., "I can learn mathematical modelling"), Value (9 items, e.g., "Mathematical modelling should be a required part of my professional training"), Difficulty (7 items, e.g., "Mathematical modelling is highly technical"), Interest (4 items, e.g., "I am interested in using mathematical modelling"), and Effort (4 items, e.g., "I plan to/did attend every mathematical modelling class session"). For organisational-administrative reasons, it was only possible to distinguish between the teacher-directive groups (TD1 and TD2) and the method-integrative group (MI) when surveying attitudes. Figure 9 visualises the differences between the groups for all six dimensions.

As can be seen in Figure 9, neutral attitudes exist for both groups in terms of affect, cognitive competence, and value. The two groups show more negative attitudes for difficulty, and more positive attitudes for interest and effort. In five out of six aspects (except affect), the MI group has on average descriptively more positive attitudes than the TD groups.

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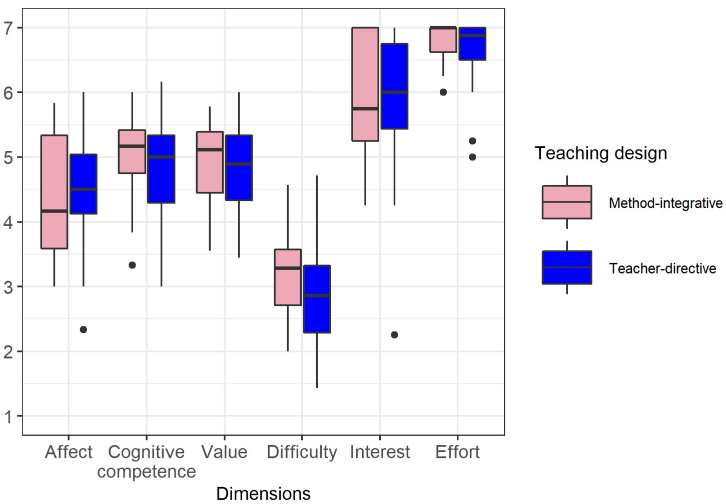

Figure 9 – Attitudes towards mathematical modelling with six dimensions (7-point rating scale: 1 = strongly disagree,..., 7 = strongly agree)

6. Conclusion and perspectives

On a practical level, the intention was to construct a mathematical modelling teaching unit to support first year engineering students' competency development. To document this process and thus to verify the effectiveness of the intervention, it was evaluated with a pre-post-test design. The empirical results show an encouraging competency gain both for mathematics and for mathematical modelling, for all groups, with advantages (also in terms of attitudes) for the group with the more independence-oriented teaching design. Despite the relatively short duration of the intervention, this indicates that the intervention succeeded in substantially increasing the students' competencies, although both the results of the pre-test and the results of the post-test were not satisfactory from a purely normative point of view. Information about the student profiles made us realise some important factors that might have influenced the results independently of the design of the modelling unit. One such factor could be that the students had to work in English, which for most of them is not their home language (see in Section 3). We know from several studies how important language proficiency is for academic achievement at the tertiary level (see, e.g., Du Plessis and Gerber, 2012) and for understanding the context of a modelling task (see, e.g., Plath and Leiß, 2018).

Certainly, the design of the teaching unit can be further improved. The students' progress during the unit was rather small as five lessons can only offer limited possibilities. De Villiers and Wessels (2020) report a similarly slow progress of South African engineering students' competency development over an intervention with six modelling tasks. One obvious possibility for improvement is therefore to extend the duration of the teaching unit and to include both more tasks and more phases for individual practising, with and eventually without teacher support. Yet another possibility is to link the modelling examples more closely to engineering topics and to South African students' life contexts. One concrete idea along these lines is to include an additional task in lesson 4, *Combined Resistors* (adapted from Stewart, Redlin and Watson, 2012). This task is linked to the same content of rational functions as the *Traffic Flow* task, and in addition to the engineering context and to the South African context. In the task a situation is created, and students would be asked to give immediate advice to the technician and to use technology at hand as an aid for solving the task.

Combined Resistors: Theory on electricity explains that electrical resistance of two resistors R_1 and R_2, connected in parallel, gives a combined resistance R by $R = (R_1 R_2)/(R_1 + R_2)$. In a nearby building in Johannesburg a technician connects a fixed 8 Ohm resistor in parallel with a variable resistor. For safety reasons the technician must know precisely how the combined resistance depends on the resistance of the variable resistor, in particular, how big the combined resistance may become.

Another idea is to make a change to one picture task in lesson 5, namely to replace the "Statue in Germany" with a picture of a statue in South Africa (see figure 10). In this task, developed by the first two authors, students are asked to estimate the real volume of Nelson Mandela's upper body (covered by his shirt). Thus, by asking for a volume and not for a length, this task aligns better with the photo tasks contained in the pre- and the post-test.

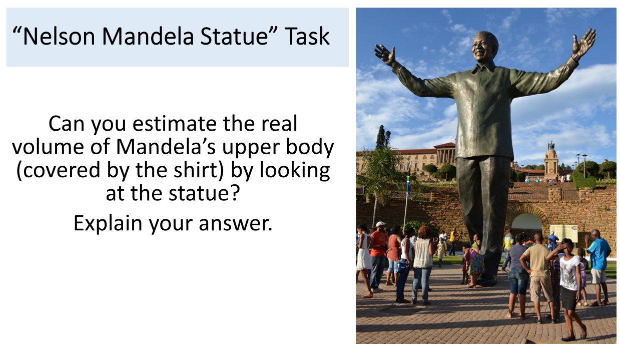

Figure 10 – The *Nelson Mandela Statue* task planned for lesson 5

Our goal is to develop a mathematical modelling teaching unit suitable for first-year engineering students to support their competency development. The unit developed in 2019 and presented here effectuated a significant competency growth, but can certainly be further improved, both concerning the content and the teaching method. The effects of changes to the existing unit can only be evaluated after implementation. We will report on effects of the refined unit in due course.

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