# Random vibration fatigue analysis with the method of Isogeometric Analyses (IGA)

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**Abstract:** *At present, Finite Element Analysis (FEA) is indispensable in the field of simulation technology, as this kind of numerical analysis method can help engineers to predict results difficult to obtain from experimental tests. However, the mesh generation process in FEA is time-consuming. It is estimated that about 80 percent of analysis time is devoted to mesh generation in some fields, such as automotive or shipbuilding industries. On the other hand, the imperfections of mesh models can lead to inaccurate results. In this study, we adopted a new numerical analysis method, Isogeometric Analysis (IGA) to develop a random vibration fatigue analysis on a wind turbine tower model. From the mesh generation process, it can be observed that the NURBS mesh creation is far more convenient and time-efficient than the finite element counterparts. From fatigue analysis results, we can conclude that IGA can predict fatigue damage using fewer mesh elements and integration points, corresponding very well with the finite element results.*

## 1 Introduction

It is necessary to predict the fatigue life of a structure during the design stage. In the numerical simulation, the fatigue analysis can be developed both in the time and frequency domain. However, compared with frequency domain fatigue analysis, the time domain fatigue analysis is computationally expensive. So, in this studying, we adopted the frequency domain fatigue analysis method to calculate the cumulative damage ratio based on Dirlik's approach, in which the input random vibration load and output stress are described by Power Spectrum Density functions (PSD).

At present, there are several disadvantages to FEA. The most significant one is to spend a long time in mesh generation. For example, it is estimated that about 80% of overall analysis time has been applied to the mesh creation process in automotive, aerospace, and shipbuilding industries [1]. In 2005, T.J.R. Hughes proposed a method, which is named Isogeometric analysis (IGA) to mainly solve the problems derived from the classical FEA.

IGA with NURBS basis function has been applied in various engineering problems, including contact mechanics [2, 3, 4], fluid mechanics [5, 6, 7], structural optimization [8, 9, 10, 11], shell analysis [12, 13, 14, 15], beam analysis [20, 16, 17] damage and fracture mechanics [18, 19], and structural vibration analysis [16, 20, 21], etc. In this paper, we mainly investigate the performance of the NURBS-based IGA LS-DYNA on a wind turbine tower model. Results are verified by classical FEA and matlab code.

The originality of this paper is that the isogeometric random vibration fatigue analysis is firstly employed on an industrial model. The structure of this article is as follows. In section 2, we briefly review some theoretical backgrounds. In section 3, isogeometric random vibration

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fatigue analysis is applied on a wind turbine tower model and the results are verified by the FEA and own developed Matlab programming. In section 4, we conclude on the present studies.

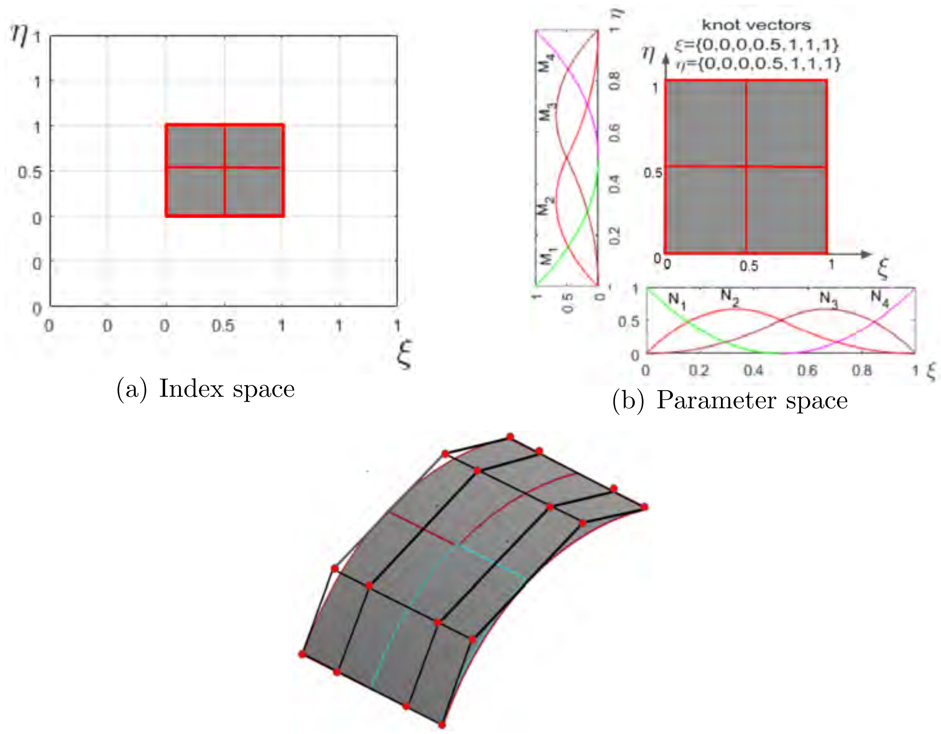
## 2 IGA modelling

At present, most IGA is developed, based on NURBS basis function, as it not only has a wide application in CAD systems but also possesses powerful capability in describing complex geometric models. The NURBS basis functions are defined by the B-spline basis function built from knot vectors. Details can be found in [1].

### 2.1 Some basic concepts of IGA

- I) Different spaces

The index space in two dimensions is an equally divided domain, no matter with the knot values of knot vectors. For example, in the case where the knot vectors are respectively $\Xi = \{0, 0, 0, 0.5, 1, 1, 1\}$ and $\eta = \{0, 0, 0, 0.5, 1, 1, 1\}$, the index space ranges from $[0, 1]$ (figure 1 $(a)$). The parameter space in two dimensions is the $[0, 1] \otimes [0, 1]$ domain where the NURBS basis functions are defined (figure 1 $(b)$). And the control points, physical mesh, and control mesh are defined in physical space (figure 1 $(c)$).




(a) Index space (b) Parameter space

(c) Physical space

**Figure 1**: Schematic illustration of different spaces

- II) Knot vector

A knot vector in one dimension is defined as a series of non-decreasing coordinates in the parametric space, denoted by $\Xi = \{\xi_1, \xi_2, ..., \xi_{n+p+1}\}$, where $\xi_i \in R$ is the $i$th knot (or coordinate), and $i$ is the knot index from $1, 2, \ldots, n + p + 1$, in which $n$ is the number of B-spline basis function along $\xi$ parametric direction, and $p$ is the polynomial order of B-spline basis function. In the construction of B-spline surface and solid, it is necessary to use 2 and 3-knot vectors, which are respectively directed along $\xi$ and $\eta$ directions. Each knot or coordinate of a knot vector is used to divide the parametric space of a geometrical model to obtain elements, meaning that all of the mesh elements can be selected by knot values of the knot vectors. In terms of the space between different knots, a knot vector can be referred to as a uniform or non-uniform knot vector. In a uniform knot vector, the knots are equally spaced in the parametric space, such as $\Xi = \{1, 2, 3, \ldots, \xi_{n+p+1}\}$. Similarly, in a non-uniform knot vector, the knots are unequally spaced in the parametric space, such as $\Xi = \{1, 1.5, 2.5, 3, \ldots, \xi_{n+p+1}\}$. In a knot vector, there can be repeated knots, and a knot vector is said to be open if its first and last knots repetition are equal to the $p + 1$, in which $p$ is the polynomial order of the basis function. In one dimension, the basis functions constructed by an open knot vector interpolate the ends of parametric space.

- III) B-spline basis function and B-spline curve

The B-spline basis functions are defined by the following equation 1 and 2.
For $p = 0$, it is defined by:

$$N_{i,0}(\xi) = \begin{cases} 1 \quad if \quad \xi_i <\leqslant \xi < \xi_{i+1} \\ 0 \qquad otherwise \end{cases} \tag{1}$$

For $p$ = 1, 2, 3,..., they are defined by

$$N_{i,p}(\xi) = \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi) \tag{2}$$

B-spline curves are defined by the linear combination of B-spline basis functions and the corresponding control points, the vector - valued coefficients of the basis function $\mathbf{P}_i \in R$, $i = 1, 2, ..., n$, as in equation 3.

$$C(\xi) = \sum_{i=1}^{n} N_{i,p}(\xi)\mathbf{P}_i \tag{3}$$

- Non-Uniform Rational B-Spline (NURBS) basis function, NURBS curve, and NURBS surface

The univariate NURBS basis function is described by the rationale of weighted B-spline basis functions as:

$$R_{i,p}(\xi) = \frac{\omega_i N_{i,p}(\xi)}{W(\xi)} = \frac{\omega_i N_{i,p}(\xi)}{\sum_{i=1}^{n_{cp}} \omega_i N_{i,p}(\xi)} \quad 1 \leq i \leq p+1 \tag{4}$$

Where $\omega_i$ denotes the weight value of the control point $\mathbf{P}_i$, and $W(\xi)$ is the weighted linear combination of B-spline basis functions. Here, $n$ denotes the total number of NURBS control points. The NURBS curve is defined by the linear combination of univariate NURBS basis function $R_{i,p}(\xi)$ and control point $\mathbf{P}_i$ by the following expression [1]:

$$C(\xi) = \sum_{i=1}^{n} R_{i,p}(\xi)\mathbf{P}_i \tag{5}$$

And the NURBS surafce is defined by:

$$C(\xi,\eta) = \sum_{i=1}^{n}\sum_{j=1}^{m} R_{i,j}^{p,q}(\xi,\eta)\mathbf{P}_{i,j} \quad (6)$$

where $R_{i,j}^{p,q}(\xi,\eta)$ is bivariate NURBS basis functions, which are defined by:

$$R_{i,j}^{p,q}(\xi,\eta) = \frac{N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}}{\sum_{i=1}^{n}\sum_{j=1}^{m} N_{i,p}(\xi)M_{j,q}(\eta)w_{i,j}} \quad (7)$$

where $N_{i,p}(\xi)$ and $M_{j,q}(\xi)$ are $p$th and $q$th order B-spline basis function, which are defined in $\xi$ and $\eta$ parametric directions, respectively.

## 3 The IGA and FEA on a wind turbine tower model

In this section, isogeometric and finite element random vibration fatigue analysis are developed on a wind turbine tower model created based on the reference[22].

### 3.1 The analysis preparation

#### 3.1.1 The geometric model, material properties

As shown in figure 2, the wind turbine tower model is assembled by a series of different thickness cylinders and conical shell sections, in which the geometry parameters like the height, thickness, etc are displayed in the form of $mm$. The tower model consists of 3 flange connections, whose base, middle and top flange thicknesses are respectively 300, 200 and 200 $mm$.

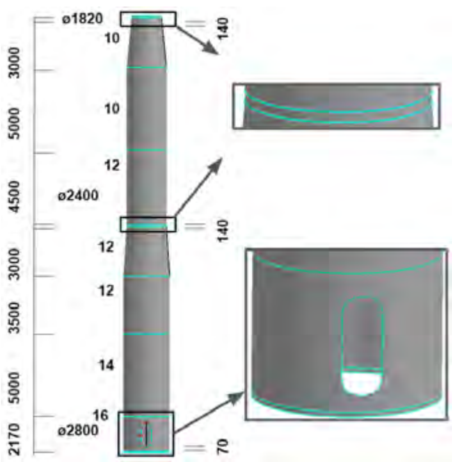



**Figure 2**: geometry model of the tower

The material properties are shown in table 1. And the material constants of the S-N curve are respectively $\beta = 9.82$ and $C = 4.0641 \times 10^{88}$ [23].

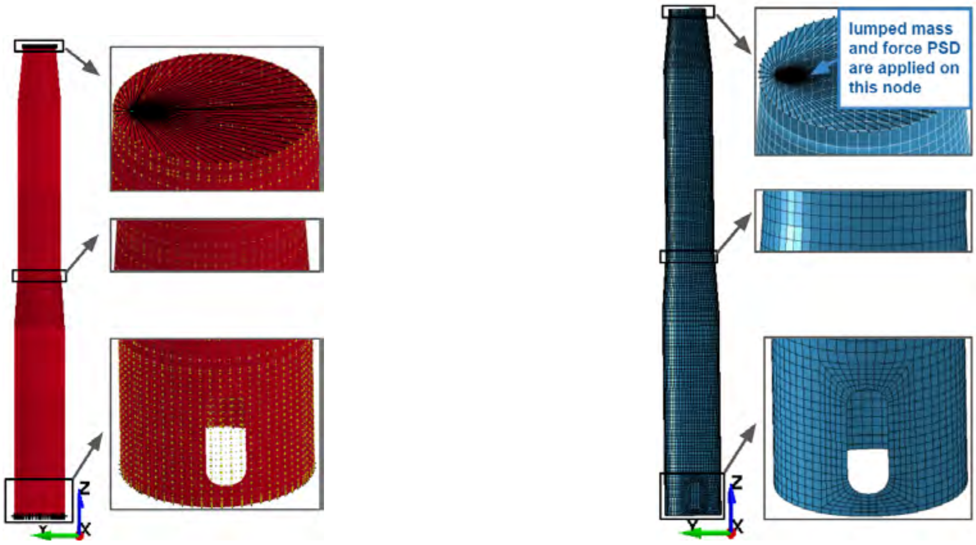
**Table 1**: Material properties

| Mass density | Young's modulus | poisson's ratio |
|---|---|---|
| 3.81e-3 $g/mm^3$ | 3.1e+11 $Pa$ | 0.33 |

### 3.1.2 Mesh models and boundary condition

The isogeometric and finite element mesh models are presented in figure 3, in which the number of control points and nodes are respectively 7639 and 12969. The finite element mesh model is created by quadrilateral 4 nodes mesh elements, and the shell element formulation of Belytschko-Tsay is chosen to develop fatigue analysis. For IGA, we used the isogeometric NURBS element, and adopted Hughes-Liu with rotational DOFs shell formulation; the polynomial order of univariate shape functions in $s$ and $r$-directions in the parametric space are respectively 2, and in LS-DYNA, the mesh refinement method, SUBDIVISION, is used to create more isogeometric mesh elements. After mesh generation on each section, the keyword, NODE DUPLICATION, is used to merge duplicate control points (nodes for FEA) to assemble the different sections.

To simulate the weight effects of blades, turbines, and other parts on the top of the wind turbine tower, at the height of $Z$ = 26460 and X= -750, Y = 0 $mm$, a node is created to substitute the concentrated mass element of 4.023e+7 $g$. Then the node is connected with all control points of the top flange edge, and the weight direction is set to in negative $z$-direction. During analysis, the base flange of the tower model is clamped in the translational and rotational local $x$, $y$, $z$-directions.




(a) (b)

**Figure 3**: Mesh models (a) IGA (b) FEA

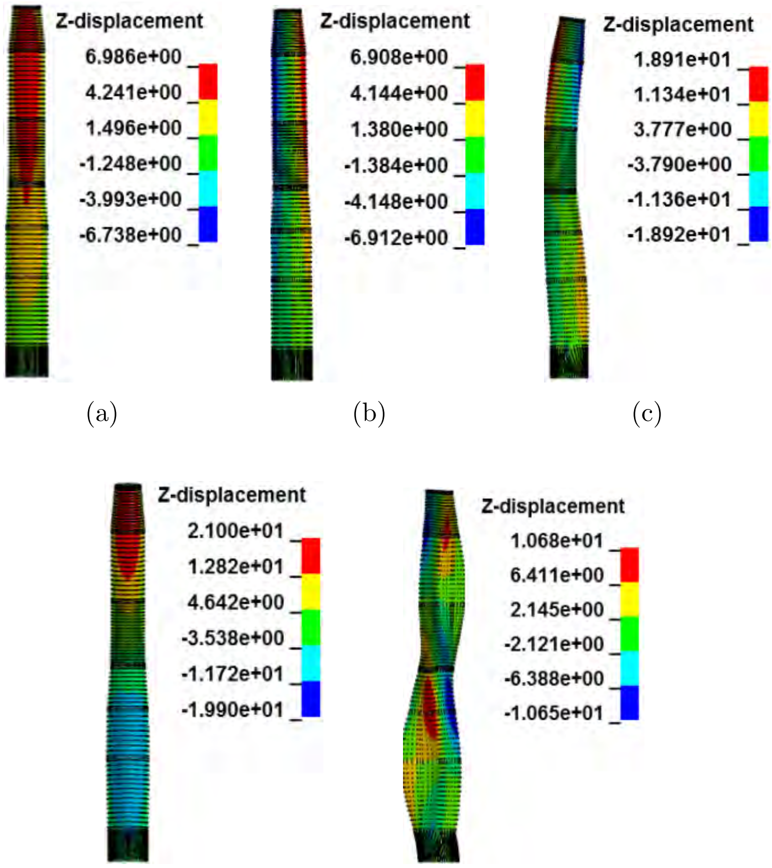
## 3.2 Analysis results

### 3.2.1 Modal analysis results: the first five natural frequencies and vibration mode

Table 2, figure 4 and 5 respectively show the first five natural frequencies and corresponding vibration modes obtained from IGA and FEA, from which it can be observed that the frequencies and the vibration modes have a good agreement.

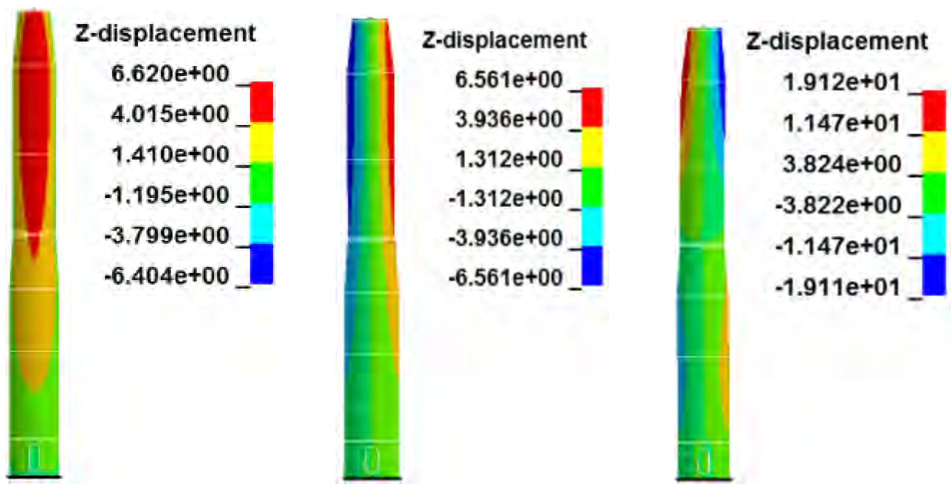
**Table 2**: The first five natural frequencies(Hz)

| Method | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| IGA | 4.47 | 4.55 | 26.83 | 27.26 | 30.47 |
| FEA | 4.47 | 4.54 | 27.15 | 27.21 | 30.48 |

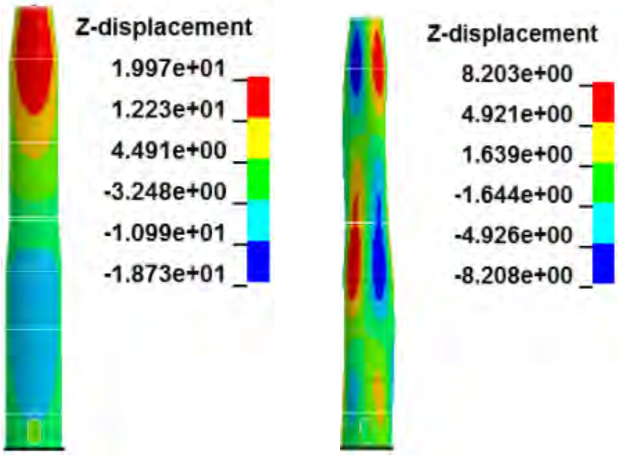



(a) (b) (c) (d) (e)

**Figure 4**: Isogeometric first five vibration mode




(a) (b) (c)




(d) (e)

**Figure 5**: Finite element first five vibration mode

### 3.2.2 Fatigue analysis results: effective stress PSD, RMS and cumulative damage ratio

The force PSD load, as shown in figure 6 is applied on the node substituting the element concentrated mass in the $x$-direction. The random vibration fatigue analysis of unit second, in which the damping ratio is set to 0.01, is developed to calculate the effective stress PSD, RMS, and cumulative damage ratio in Ls Dyna. Then based on obtained PSD, the cumulative damage ratio is validated in Matlab using Matlab program.

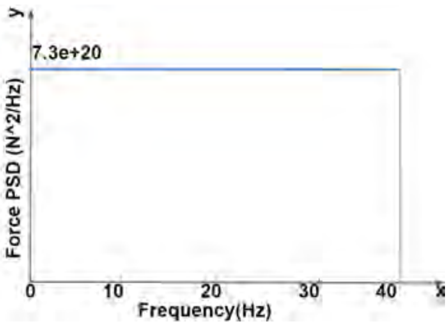



**Figure 6**: Applied load PSD

Figure 7, and 8 show the calculated isogeometric and finite element effective stress PSD and RMS, in which only the first natural frequency is excited by the applied force PSD. It is observed that isogeometric and finite element PSD and RMS display a good agreement, in which the maximum effective stress RMS from IGA and FEA is 3.151e+8 and 3.125e+8 $pa$ respectively, leading to the relative error of 0.83%, based on the equation 8. From figure 9, it can be seen that the obtained isogeometric and finite element cumulative damage ratios are respectively 2.678e-4 and 2.638e-4, with a relative error of 1.52%, and the maximum damage ratios are located on similar elements close to the door edge. According to the equation 9, the expected isogeometric and finite element fatigue life $E[T_f]$ are 3.7341e+04, and 3.7908e+04 seconds respectively. Based on the Matlab program, the isogeometric and finite element damage ratios are respectively 2.6204e-04 and 2.6406e-04, which are in a good accordance with the damage ratios computed from Ls Dyna.

$$Relative\ error = \frac{IGAresult - FEAresult}{FEAresult} \tag{8}$$

$$E[T_f] = \frac{T}{E[D]} \tag{9}$$

Where $T$ is the duration time (1 seconds in these analyses), $E[D]$ is the obtained cumulative damage ratio.

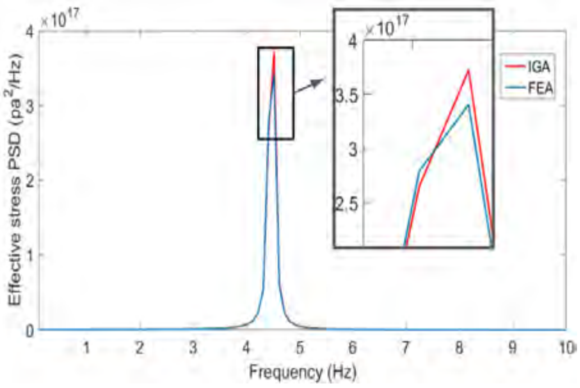
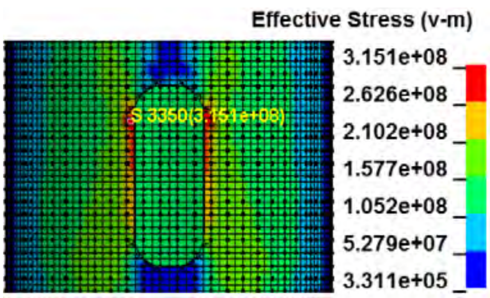
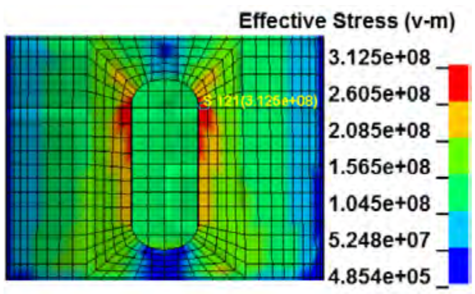



**Figure 7**: The effective stress PSD




(a) IGA




(b) FEA

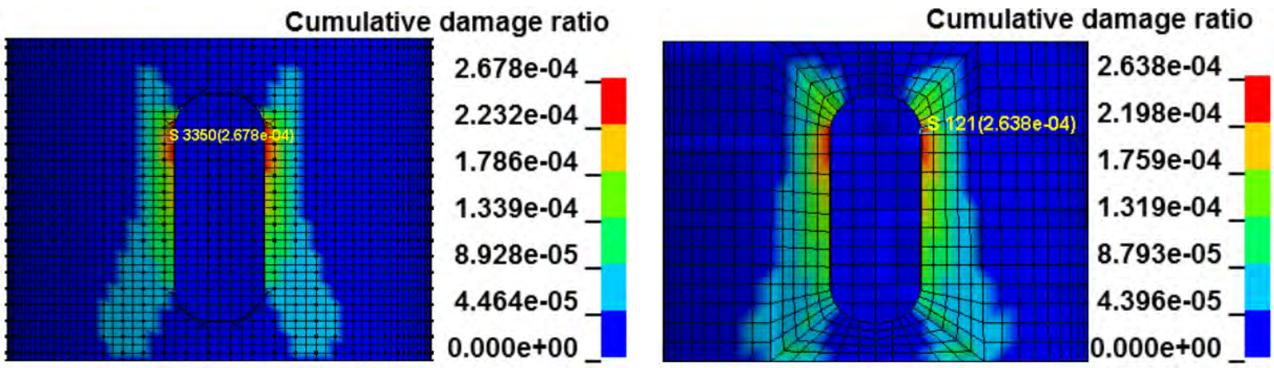
**Figure 8**: The effective stress RMS

## 4 Conclusion

In this studying, we considered random vibration fatigue analysis on a tower model using IGA and FEA, in which the isogeometric and finite element damage results are validated by the Matlab program.

During the analysis, the tower model is clamped on the base flange, and random force PSD in a vertical direction to the tower surface is applied to the concentrated mass element. From modal analysis, it can be found that the obtained first five natural frequencies and vibration modes from IGA and FEA have a good agreement. Fatigue analyses show that the obtained isogeometric and finite element maximum effective stress RMS are 3.151e+8 and 3.125e+8 $pa$ with a relative error of 0.83%, and cumulative damage ratios are 2.678e-4 and 2.638e-4 with a relative error of 1.52%. Based on the Matlab program, the isogeometric and finite element damage are respectively 2.62e-4 and 2.64e-4, leading to the relative error of -0.76%.

On the other hand, in the aspect of the mesh refinement process, for IGA, it is not necessary to create mesh elements on the original geometry model. it is sufficient to develop mesh elements




(a) IGA (b) FEA

**Figure 9**: The cumulative damage ratio

on the previous mesh model, and so the mesh refinement time can be largely saved. However, for the FEA, the refinement process is mandatory to communicate with the original geometric model, and so this process is more time-consuming in LS Dyna software.

In addition, IGA can predict the fatigue life using fewer NURBS elements and integration points in the thickness direction, which correspond very well to the fatigue life computed by FEA, with the relative errors of 0.68% . Through the comparison of numerical analysis results, it can be observed that the obtained isogeometric, finite element PSD and RMS have a good agreement, leading to conclude that IGA is suitable for the random vibration fatigue analysis.

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