

Chaos synchronization for a class of uncertain chaotic supply chain and its control by ANFIS

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Abstract:

In this paper, modeling of a three-level chaotic supply chain network. This model has the uncertainty of the retailer in the manufacturer. An adaptive neural fuzzy method has been proposed to synchronize the two chaotic supply chain networks. First, a nonlinear feedback control method is designed to train an adaptive neural fuzzy controller. Then, using Lyapunov theory, it is proved that the nonlinear feedback controller can reduce the synchronization error to zero in a finite time. The simulation results show that the proposed neural fuzzy controller architecture well controls the synchronization of the two chaotic supply chain networks. In the other part of the simulation, a comparison is made between the performance of the nonlinear controller and the adaptive neural fuzzy. Also, in the simulation results, the controller signal is depicted. This signal indicates that the cost of implementation in the real world is not high and is easily implemented.

Key words:

Supply chain, chaotic, synchronization, ANFIS.

1. Introduction

In the last twenty years, the control and synchronization of the chaotic dynamic model have attracted many scientists. The phenomenon of chaos occurs in many natural and industrial phenomena. The source of the undesirable behavior of chaotic dynamic systems is simple differential equations. A chaotic dynamic system is a nonlinear dynamical system having properties such as being very sensitive to initial conditions and sustained irregularity. The well-known Lorenz chaotic attractor was introduced in a three-dimensional autonomous system in 1963 (Lorenz, 1963). The Lorenz system has been applied to atmospheric science. There are many practical models in various fields of engineering such as aerospace (Hamidzade and Esmaelzadeh, 2014; Kumar et al., 2020), electric motors (Hamidzadeh and Yaghoobi, 2015), guidance and navigation (Aghababa and Aghababa, 2013; Yu et al., 2020),

secure communications (Ouannas et al., 2020; Abdullah et al., 2019) [7,8] laser (Mu et al., 2020), and mechanics (Mu et al., 2020; Dantas and Gusso, 2018).

Two goals have been studied in the control of chaotic dynamical systems. First, the stability and elimination of undesirable behavior, which was the first study by OGY in 1990, and second, the synchronization of master-slave chaotic systems via different initial conditions. The idea of synchronization to identical chaotic dynamics with different initial conditions was first studied by Pecora et al. in 1997 (Pecora & Carroll, 1997). The synchronization concept applies force to the dynamics of the slave system so that it can track the dynamics of the master system and reduce the error to zero in a limited time. Synchronization means, controlling the output of the slave system so that it tracks the output of the master system, while the system error moves asymptotically to zero.

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Several methods have been introduced for the stability and synchronization of chaotic dynamic systems the most recent of which are nonlinear (Korneev et al., 2020), adaptive (Mohadeszadeh & Pariz, 2020), fuzzy (Chen et al., 2020), impulsive (Li et al. 2020), robust (Khan et al., 2020), backstepping (Shukla et al., 2017), predictive (Sadaoui et al., 2011), and also a combination of fuzzy impulsive (Behinfaraz et al., 2020) and robust adaptive (Ahmad & Shafiq, 2020).

Supply Chain Management aims at improving competitiveness of the supply chain as a whole, by integrating organizational units along the supply chain and by coordinating material, information and financial flows in order to fulfill (ultimate) customer demands (Kilger, 2000). Due to the complexity and breadth of supply chain networks, controlling the supply chain is always very difficult. In some supply chain networks, where the time from request to order is constantly changing, the complexity will be even greater.

Lu Yingjin et al. in the year 2004, studied the complexity of the effect of the bullwhip in the supply chain (Yingjin et al., 2004). Bullwhip Effect is explained as the internal nonlinear behavior and study on the complexity of the bullwhip effect for orders up to policy under demand signal processing. They try to show a mathematical relationship between the bullwhip effect in the supply chain network and fractal and chaotic dynamics. Zhang Lei et al. modeled a three-tier supply chain network based on the dynamic chaotic Lorenz (Lei et al., 2006). They investigated a nonlinear three-level supply chain model, which under certain conditions can appear in the form of Lorenz chaotic equations. Synchronization in a chaotic supply chain network using the RBF neural network method has been proposed. also, is External perturbation investigated in this model. The main purpose of this research is chaos modeling and synchronization. But the state of control policy is not illustrated. On the other hand, the cost of the proposed method and its ability to be implemented in the real world is unclear. But oscillations at every level of the supply chain are examined. Analyzed the dynamic behavior of a three-echelon supply chain network (Anne et al., 2009). In their modeling, there is the effect of a bullwhip effect in the supply chain network. After analyzing the supply chain model, chaotic synchronization is discussed, but the controller behavior is not depicted. Alper Göksu et al. presents the synchronization and control of a chaotic supply chain identical with

different initial condition (three-level) using the equations of Lie et al., 2006 (Göksu et al., 2014). They have synchronized two chaotic supply chain networks by active control method. The behavior of the nonlinear active controller is not discussed in this study.

In the year 2016, described are robust control designs for chaotic behavior in supply chain networks by H. Norouzi Nav et al. (Norouzi Nav et al., 2016). This paper examines the modeling, analysis, and control of a chaotic supply chain network. Their model has a control center that determines the order of inventories and their inventory is controlled based on customer demand. Uğur Erkin Kocamaz et al, have addressed control and synchronization of chaotic supply chains via intelligent control (Kocamaz et al., 2015). Their intelligent method is the use of artificial neural networks (Adaptive Neuro-Fuzzy Inference System, which is an abbreviation ANFIS). Using the equations of Lie et al., and it is assumed that two chaotic supply chain systems are taken where the initial conditions are different. The proposed control policy is not illustrated to control and synchronization of the chaotic supply chain. Hamid Norouzi Nav et al. explained are modeling and analysis of the chaotic behavior in supply chain networks with a control theoretical approach (Norouzi Nav et al., 2018). Sayantani Mondal in the year 2019, offers A new supply chain model and its synchronization behavior (Mondal, 2019). This paper assumed that demand for the product does not increase monotonically with the increase of inventory and also, it is assumed that the demand has a saturation level and it does not increase monotonically with the inventory. The synchronization of the two coupled identical supply chain are investigated and sufficient conditions using Unidirectional and bidirectional coupling. The control policy used is not illustrated to synchronize the two chaotic supply chains. Yang Peng et al. studied a new model for a supply chain system and they performed the synchronization by the impulse control method (Peng et al., 2020). Firstly, they have presented a new supply chain system that is sensitive to various uncertainties along with exogenous disturbances. the next discussed impulsive control for the synchronization behavior of two supply chain systems via the identical model. The impulsive control behavior is not discussed for the synchronization of two chaotic supply chains. Management, optimization, and control of chaotic supply chain system with adaptive sliding mode control presented by Xiao Xu et al. (Xu et al., 2020). Their idea is an adaptive super-twisting (STW)

sliding mode control (SMC) algorithm to manage a chaotic supply chain system. also, investigated external disturbance. The main activity of this article has been the elimination of chaotic behavior and chaotic synchronization. This study depicts the sliding mode control behavior, in which an attempt is made to reduce the fluctuations of the proposed control policy. This can indicate the cost of the method in a real-world implementation.

In recent years, the supply chain model based on hyper-chaotic dynamic systems has also been proposed. Control and synchronization of hyper-chaos in digital manufacturing supply chain described by Yan et al. (Yan et al., 2020). They constructed a mathematical model for a supply chain with a computer-aided digital manufacturing process.

A hyper-chaotic four-layer supply chain model is presented based on dynamic hyper-chaotic system (Hamidzadeh et al., 2022a), Hamidzadeh et al. have reviewed and presented a hyper-chaotic closed-loop supply chain model (Hamidzadeh et al., 2022b).

Further limitations in the above studies indicate that the examination of control costs of the supply chain network should be specified. This issue can be discussed in the simplest terms by depicting the behavior and number of controllers. Another very important issue is the time to reach stability. When a control system is designed, we expect to achieve our goal. But the time to reach it is also very important. (Of course, Hamidzadeh et al. have investigated this issue in two cases.)

Our goal in this paper is to analyze a class of supply chains based on chaotic systems. Then, will be analyzed the proposed controller behavior. This behavior can express how much it costs to implement in the real world. This behavior can indicate what

the cost of implementing the proposed controller for the real world is, the above authorities have not mentioned this issue.

This paper has been organized as follows. Section 2 described modeling for a class of uncertain chaotic supply chains. Section 3 discussed the Synchronization of a chaotic supply chain network with an adaptive neural fuzzy controller method. Section 4 will be illustrated Numerical simulations, which in its subsection, is a numerical simulation of the neural fuzzy controller with different initial conditions and a comparison between the performance of two methods of controlling nonlinear and fuzzy adaptive neural feedback. The conclusion is given in section 5.

2. Modeling for a class of uncertain chaotic supply chain

In this section, a model of the supply chain with uncertainty based on chaotic dynamics will be introduced. Consider a three-tier supply chain network that retailers, distributors, and manufacturers. The route of transfer of products from right to left as well as the route of transfer of supply chain network information from left to right.

Assumption 1: The requests of the retailer and distributor are transferred in period i in the supply chain network. Orders are processed in the $i+1$ period. Hence, the information sent may be accompanied by deviations.

Assumption 2: The manufacturer receives information from the distributor and retailer to check product satisfaction.

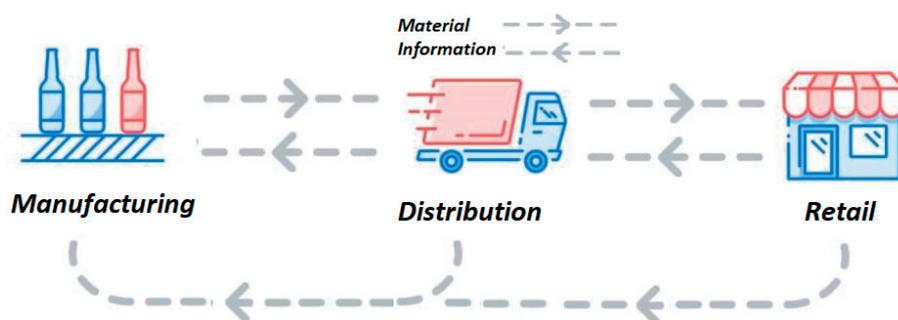


Figure 1. Three-level supply chain model.

Scenario 1: Modeling the behavior of retailers that are affected by their sales and the distributor's response to these requests. The retailer must send his sales order to the distributor according to the customers' request. Therefore, in this section, only the retailer is affected by the distributor's response to requests. The equation of this section is as follows.

$$x_i = ay_{i-1} \quad (1)$$

In Equation (1), a is the delivery factor of products from the distributor to the retailer.

Scenario 2: In the supply chain network, the distributor mainly seeks to control its inventory level. Therefore, it will be constantly ordering from the factory, and will also process retail orders. The equation is as follows:

$$y_i = bz_{i-1} \quad (2)$$

In Equation (2), b is the delivery factor of the producer to the distributor. In other words, distributors' requests to control their inventory level always depend only on the manufacturer's response rate.

Scenario 3: To measure the satisfaction of their products, the manufacturer directly contacts the retailer and distributor and receives the information. So, the satisfaction of the retailer and the distributor is always changing. The manufacturer produces its products with safety factors. The equations of this section will be as follows:

$$z_i = mx_{i-1} - sy_{i-1} - cz_{i-1} - x_{i-1}^2 \quad (3)$$

In the latter equation, m and s are the satisfaction of the retailer and distributor of the manufacturer's products, respectively. c as the production safety factor and x^2 as the model uncertainty seen from the retailer in the manufacturer.

Finally, if Equations 1 to 3 are written integrally, will have:

$$\begin{aligned} x_i &= ay_{i-1} \\ y_i &= bz_{i-1} \\ z_i &= mx_{i-1} - sy_{i-1} - cz_{i-1} - x_{i-1}^2 \end{aligned} \quad (4)$$

If it is assumed that the order period in the supply chain network path is very short, the discrete differential equation (4) will become the continuous differential equation (5).

$$\begin{aligned} \dot{x}(t) &= ay(t) \\ \dot{y}(t) &= bz(t) \\ \dot{z}(t) &= mx(t) - sy(t) - cz(t) - x^2(t) \end{aligned} \quad (5)$$

Where a, b, c, m, s are the parameters of the supply chain network, if their value is equal to $m=7.5, s=3.8, c=2, a=1, b=1$.

Then Equation (5) will be simplified as follows.

$$\begin{aligned} \dot{x}(t) &= y(t) \\ \dot{y}(t) &= z(t) \\ \dot{z}(t) &= 7.5x(t) - 3.8y(t) - z(t) - x^2(t) \end{aligned} \quad (6)$$

The differential equation (6) with the values of the expressed parameters is known as Arneodo chaotic equations (Arneodo et al., 1981). These equations were first introduced by Arneodo et al. These equations will behave chaotically with any initial condition in real space.

This model of the supply chain can be in the following general form:

$$\begin{aligned} \dot{x}(t) &= y(t) \\ \dot{y}(t) &= z(t) \\ \dot{z}(t) &= f(x(t), y(t), z(t), \Delta\{x(t), y(t), z(t)\}) \end{aligned} \quad (7)$$

See Figure 2, the initial conditions for drawing Figure 2 for the supply chain differential equations (6) are $[x(t_0), y(t_0), z(t_0)]^T = [1, 1, 1]^T$.

Figure 2 shows that the simple differential equations expressed in (6), can have very undesirable behavior. In terms of supply chain network costs, this situation is very costly. It may even cause the supply chain network to collapse.

3. Synchronization of chaotic supply chain network with general adaptive neural fuzzy controller method

The use of fuzzy systems to make decisions and select the appropriate solution is very common. Combining fuzzy models with adaptive neural

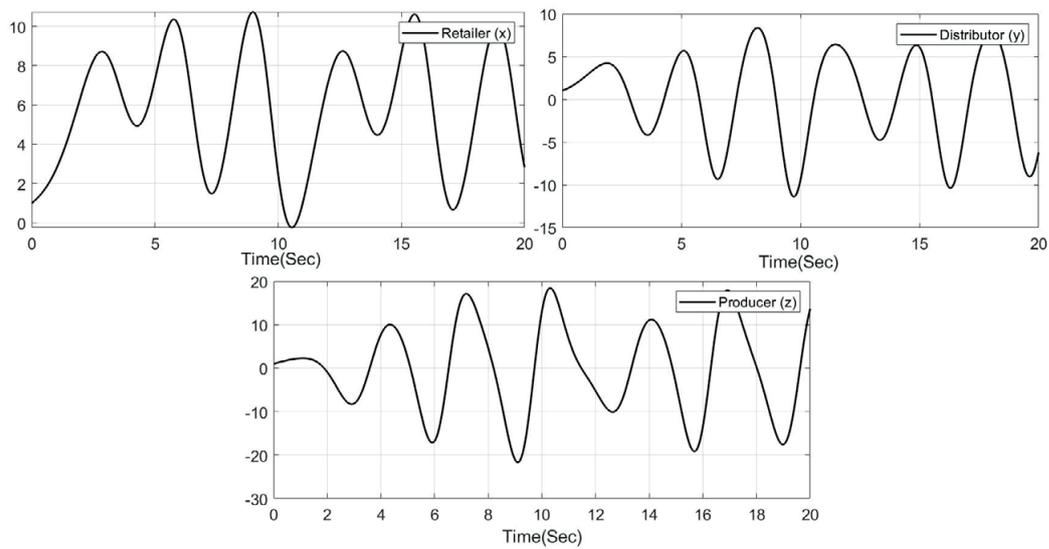


Figure 2. Time response uncertainty chaotic supply chain behavior.

networks can lead to highly optimal decisions. To design an adaptive neural fuzzy controller, input vectors must first be prepared for training. Adaptive fuzzy neural networks with these vectors can learn the dynamics of a chaotic supply chain. Thus, when the best response is obtained in adaptive neural fuzzy training, we will use it to synchronize two chaotic supply chain networks. Of course, the initial conditions for training and testing the neural network will be different. Therefore, first, the supply chain network must be synchronized with a control method. Error and control vectors will be used to train the adaptive neural fuzzy network. Figure 3 shows the fuzzy neural network training architecture of the adaptive neural network.

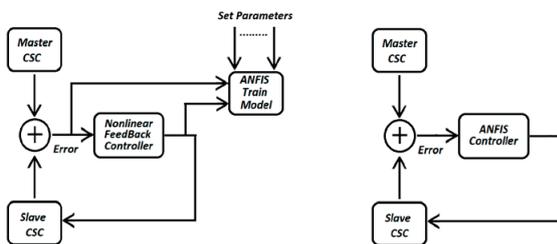


Figure 3. Adaptive fuzzy neural network training model with nonlinear controller.

3.1. Nonlinear Control design

In this section, using a nonlinear method, the chaotic supply chain network will be synchronized first.

Then, using the appropriate data obtained from this method, an adaptive neural fuzzy network model will be proposed. Training should be done to synchronize the two chaotic supply chain networks using the adaptive neural fuzzy control method. A nonlinear feedback design method is designed to access training data.

The adaptive neural fuzzy model is supposed to make decisions instead of the nonlinear controller. Therefore, it must be able to learn well the nature of the dynamics of the chaotic supply chain model. For this purpose, the controller input and output will be considered adaptive neural fuzzy network inputs.

The nonlinear feedback control method will be designed to synchronize the two chaotic supply chain networks introduced. To synchronize the two chaotic supply chain networks with different initial conditions, consider the main supply chain model and the follower according to equations (8) and (9). Index m means master and index s means slave.

$$\begin{aligned} \dot{x}_m &= y_m \\ \dot{y}_m &= z_m \\ \dot{z}_m &= 7.5x_m - 3.8y_m - z_m - x_m^2 \end{aligned} \tag{8}$$

$$\begin{aligned} \dot{x}_s &= y_s + u_1 \\ \dot{y}_s &= z_s + u_2 \\ \dot{z}_s &= 7.5x_s - 3.8y_s - z_s - x_s^2 + u_3 \end{aligned} \tag{9}$$

In Equation (9), u_1, u_2, u_3 are nonlinear controllers that must be designed. These controllers force the follower supply chain dynamics to move towards the main supply chain dynamics, thus synchronizing. If the basic conditions of the main system and the follower are equal $[x_m(t_0), y_m(t_0), z_m(t_0)]^T = [1, 1, 1]^T$ and $[x_m(t_0), y_m(t_0), z_m(t_0)]^T = [6, -3, 7]^T$. Figure 4 shows the behavior of two turbulent supply chain networks (8) and (9). As can be seen, if the initial conditions are different, the system behavior will be different.

First, calculate the error.

$$\begin{aligned} e_1(t) &= x_s(t) - x_m(t) \\ e_2(t) &= y_s(t) - y_m(t) \\ e_3(t) &= z_s(t) - z_m(t) \end{aligned} \quad (10)$$

The objective function for this design will be as follows.

$$\lim_{t \rightarrow \infty} \|e_i(t)\| = 0, \quad i = 1, 2, 3, \quad e(t_0) \in \mathbb{R}^n \quad (11)$$

It is then derived from Equations (10), and equations (8) and (9) are placed in them.

$$\begin{aligned} \dot{e}_1(t) &= \dot{x}_s(t) - \dot{x}_m(t) = y_s + u_1 - y_m \\ \dot{e}_2(t) &= \dot{y}_s(t) - \dot{y}_m(t) = z_s + u_2 - z_m \\ \dot{e}_3(t) &= \dot{z}_s(t) - \dot{z}_m(t) = 7.5x_s - 3.8y_s - z_s - x_s^2 + u_3 - 7.5x_m + 3.8y_m + z_m + x_m^2 \end{aligned} \quad (12)$$

Theorem 1: Consider the dynamic equations of a chaotic supply chain network (8) and (9). The

synchronization error will move asymptotically to zero if the nonlinear control rule is as follows.

$$\begin{aligned} u_1 &= e_2 + \lambda_1 e_1 \\ u_2 &= e_3 + \lambda_2 e_2 \\ u_3 &= 7.5e_1 - 3.8e_2 - e_3 - x_s^2 + x_m^2 + \lambda_3 e_3 \end{aligned} \quad (13)$$

Where $\lambda_1, \lambda_2, \lambda_3$ are the gains control.

Proof 1: Considering Lyapunov function candidate such as:

$$V(e) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \quad (14)$$

The derivative of the Lyapunov function candidate is as follows.

$$\begin{aligned} \dot{V}(e) &= e_1(y_s + u_1 - y_m) + \\ &e_2(z_s + u_2 - z_m) + \\ &e_3(7.5x_s - 3.8y_s - z_s - x_s^2 + u_3 - \\ &7.5x_m + 3.8y_m + z_m + x_m^2) \end{aligned} \quad (15)$$

If Equations (13) are placed in Equations (15), then

$$\begin{aligned} \dot{V}(e) &= e_1\lambda_1 e_1 + e_2\lambda_2 e_2 + e_3\lambda_3 e_3 \\ &\Rightarrow \lambda_1 e_1^2 + \lambda_2 e_2^2 + \lambda_3 e_3^2 \end{aligned} \quad (16)$$

Equation (16) is always positive if they $\lambda_1, \lambda_2, \lambda_3$ have values less than zero ($\lambda_1, \lambda_2, \lambda_3 < 0$). Thus, the proof was complete.

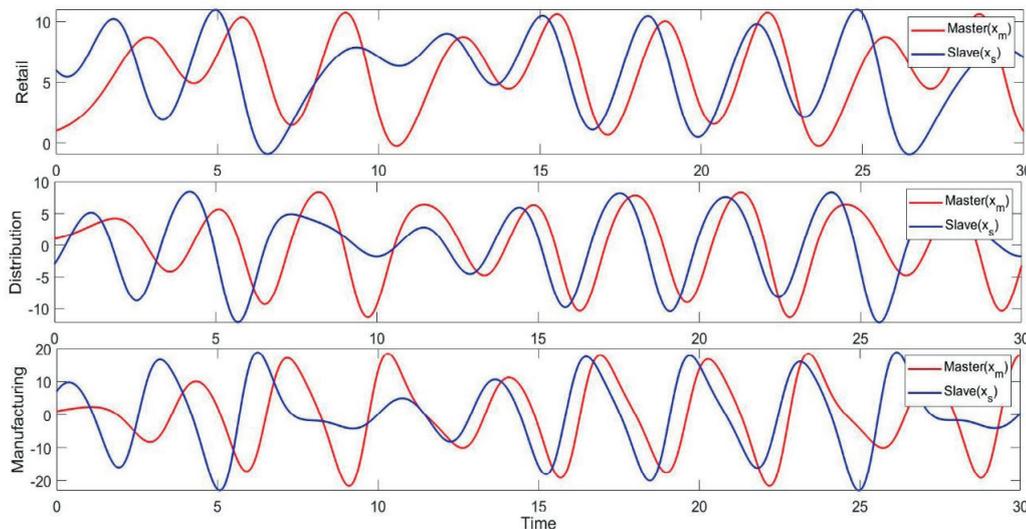


Figure 4. Time response of two chaotic supply chains when different initial conditions.

3.2. Nonlinear control design simulation results

In this section, the results of the nonlinear control design will be illustrated. The basic conditions of a chaotic supply chain network are equal to (8) and (9) $[x_m(t_0), y_m(t_0), z_m(t_0)]^T = [1, 1, 1]^T$ and $[x_s(t_0), y_s(t_0), z_s(t_0)]^T = [6, -3, 7]^T$. Controller gain values equal to $\lambda_1 = \lambda_2 = \lambda_3 = -4$ was selected. Figure 5 shows the synchronization of two chaotic supply chain networks with different initial conditions. Controller added to the chaotic supply chain slave of $t = 5$.

Figure 6 illustrates the synchronization error. As can be seen, the synchronization error tends to zero asymptotically. The time to reach the objective function of Equation (11) is approximately $t < 1.5$. Figure 8 shows the behavior of the nonlinear controller.

3.3. Training of adaptive neural fuzzy controller

Now, according to Figure 9, nonlinear control data can be used to train the neural fuzzy network.

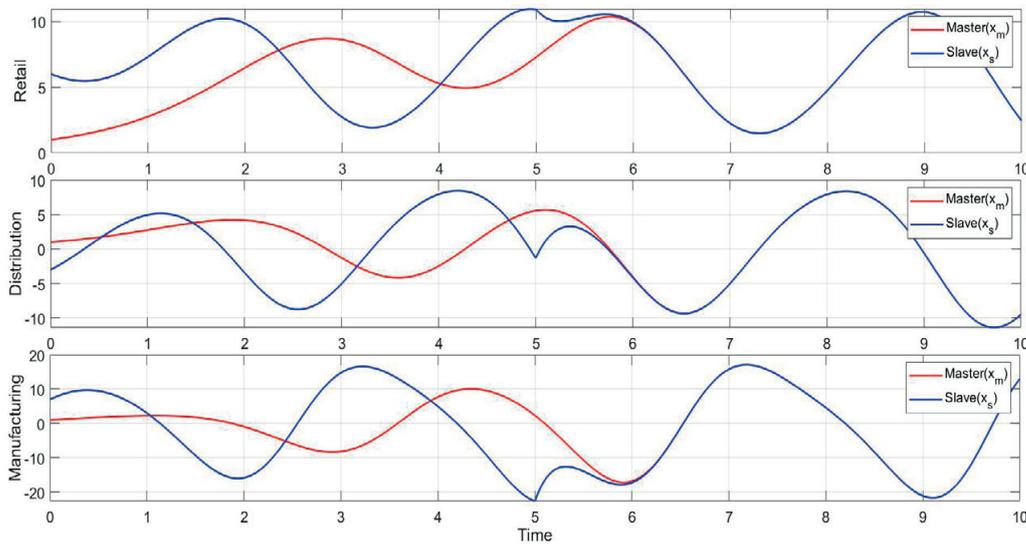


Figure 5. Synchronization of two chaotic supply chain networks by a nonlinear control method.

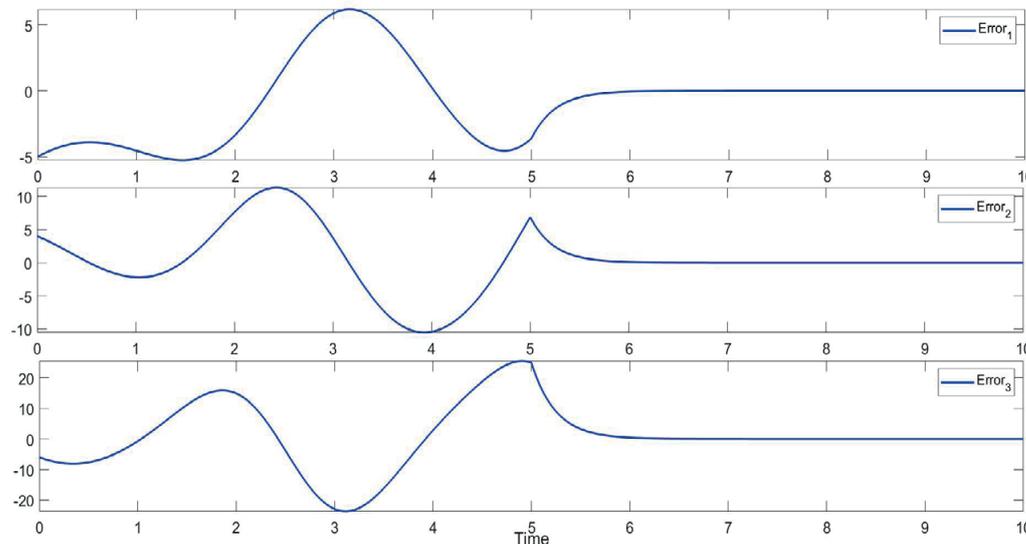


Figure 6. Synchronization error of two chaotic supply chain networks using a nonlinear controller.

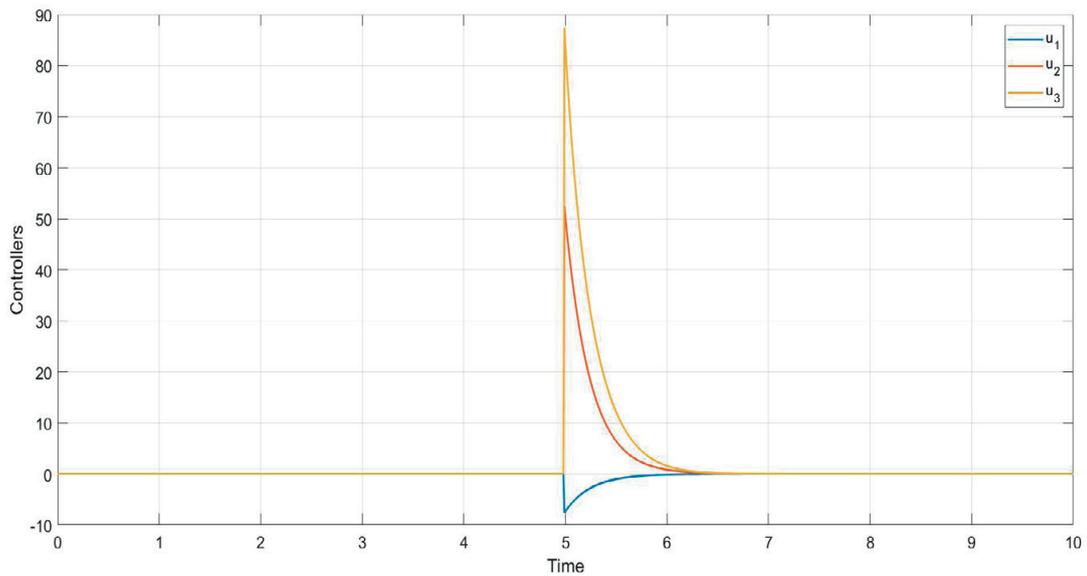


Figure 8. Signal controller of nonlinear feedback control method.

Because of the strong dependence on chaotic supply chain dynamics, training must be very precise and in-depth. The number of training data can be obtained from Figure 6. Given that the controller starts at $t = 5$ and ends at $t = 7$. So, the total number of training data in steps of 0.01 will be equal to 200. The proposed neural fuzzy architecture is given in Table 1.

In Figure 9, P_s are adjustable parameters of the fuzzy neural network. The training will be done in three parts for each controller separately. To deeply learn the dynamics of the chaotic supply chain, all three error vectors have been used in each part of the training.

Table 1. General adaptive neural fuzzy architecture.

Total number of training	201
Number of training pairs	67
Number of check pairs	67
Number of test pairs	68
Number of input/output membership functions (P1)	5
Input / Output Membership Function Type (P2)	'gbellmf'
epoch (P3)	80
Training error (P4)	0

The total number of training data is 201. This number will be used equally to train and check the fuzzy neural network. After training the network, with the check data, the constructed network will be evaluated.

3.4. Test of general adaptive neural fuzzy controller

According to Table 2, the values obtained in the training and check section indicate that the test stage can be entered. It should be noted that at this stage, the fuzzy neural network test is performed with one-third of the data obtained from the design of the nonlinear feedback controller. In different architectures, training and check errors never reached zero. Therefore, we stop using different architectures for training and checking. Although the results obtained are also acceptable. Now using the isolated data to test the adaptive neural fuzzy controller, the controller will be evaluated. Figure 10 shows the test results of an adaptive neural fuzzy controller. As can be seen, the adaptive neural fuzzy controller was able to train the nonlinear controller behavior well. Thus, an adaptive neural fuzzy controller was constructed.

4. Numerical simulation

In this section, the results of the performance of the adaptive neural fuzzy controller will be depicted. It is reminded that the initial conditions of the two chaotic supply chains are not the same, so their behavior will be different. In part 2 of numerical simulation, a comparison between adaptive and nonlinear neural fuzzy controllers will be discussed. These results show how much choosing a control policy for the supply chain network can reduce real-world implementation costs.

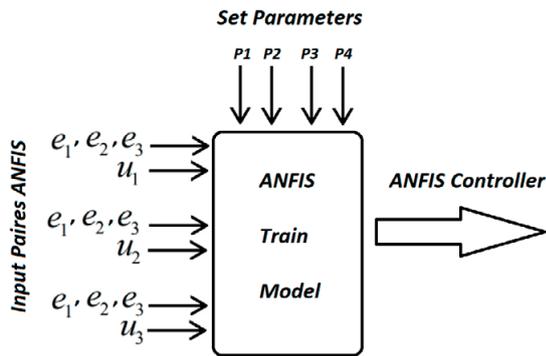


Figure 9. Training architecture for adaptive neural fuzzy controller.

4.1. Numerical simulation of the adaptive neural fuzzy controller with different initial conditions

In this section, using the adaptive neural fuzzy controller obtained from the previous section, the synchronization will be investigated of two chaotic supply chains when different initial conditions. In adaptive neural fuzzy network training, the controller application time was $t=5$, which will be different here as well. The basic conditions for the main chaotic supply chain are equal to and $[x_s(t_0), y_s(t_0), z_s(t_0)]^T = [2, 2, 2]^T$. The application

time of the adaptive neural fuzzy controller is $t=8$. Figure 11 shows the synchronization of two chaotic supply chain networks using a neural fuzzy controller. Figure 12 shows the synchronization error and Figure 13 shows the adaptive neural fuzzy control signal. As can be seen from Figure 11, synchronization is complete.

4.2. Comparison between the performance of two methods of controlling nonlinear and fuzzy adaptive neural feedback

In the second part of the simulation, the aim is to compare the performance of nonlinear and adaptive neural fuzzy feedback controllers. The controller at time $t=5$ will be considered in both simulations. Initial conditions will change. For the main chaotic supply chain equal to $[x_m(t_0), y_m(t_0), z_m(t_0)]^T = [3, 7, 9]^T$ and $[x_s(t_0), y_s(t_0), z_s(t_0)]^T = [-1, 5, 5]^T$. These initial conditions will be applied once to the nonlinear controller and again to the neural fuzzy controller.

As can be seen from Figure 14, the synchronization error of the two chaotic supply chain networks has reached zero at approximately $t < 1$. It is reminded again that all conditions are the same, even the initial conditions for the analysis of two controllers. In Figure 14 a, the neural fuzzy controller error tends to

Table 2. Calculates MSE and RMSE for training and checks data.

	MSE Error1	RMSE Error1	MSE Error2	RMSE Error2	MSE Error3	RMSE Error3
Train	6.6049e-08	0.000257	6.2410e-07	0.000790	2.8571e-04	0.016903
Check	1.3253e-07	0.000364043	1.3421e-06	0.00115847	2.9767e-04	0.0172532

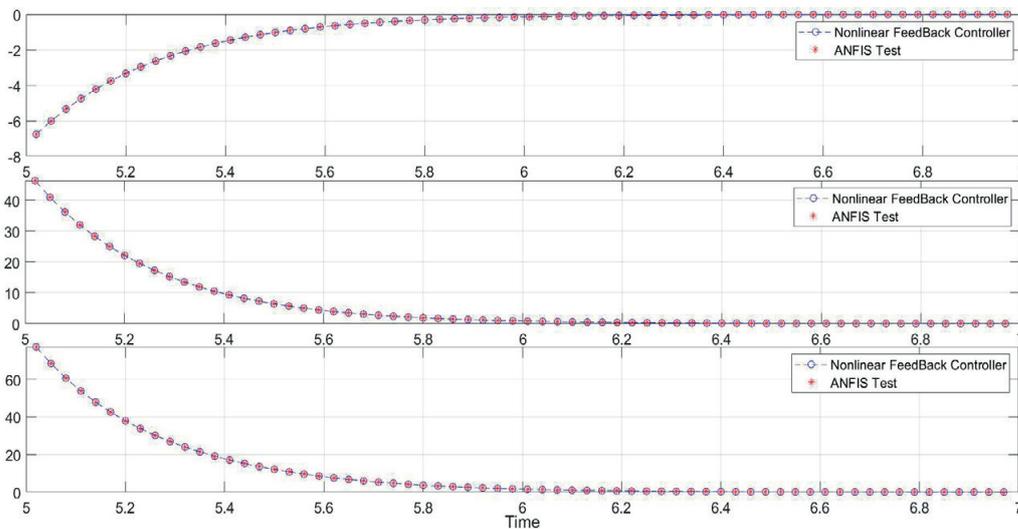


Figure 10. Fuzzy neural network test results with training data.

zero faster. In Figures 14 for error 2 and error 3, this is violated and the nonlinear feedback controller moves to zero faster. Figure 15 illustrates the behavior of two controllers. On the other hand, it's the cost of implementing it in the real world. In Figure 15 policy control 1 and 3 for the adaptive neural fuzzy controller, the cost of this implementation in the retailer and manufacturer is much lower than for the nonlinear feedback controller. But in Figure 15 policy control 2, the opposite has happened. This means that the cost of implementation in a nonlinear feedback controller is very low in the distributor compared to adaptive neural fuzzy. Another very important point in both designs is the issue of no

fluctuations in the controllers. This means that after reaching zero error, there is no need to implement control policies.

In Table 3, a comparison between the error performance of the system is shown. The number of controllers injected into the chaotic supply chain model is also observed.

5. Conclusion

In this article, a three-level supply chain modeling method based on chaotic dynamics was the

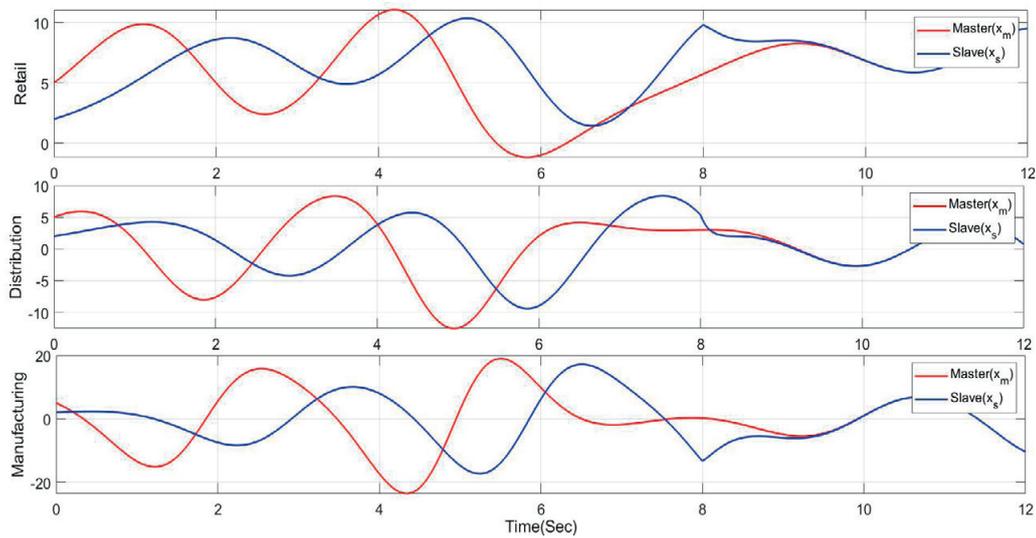


Figure 11. Synchronization of two chaotic supply chain networks with different initial conditions by a neural fuzzy controller.

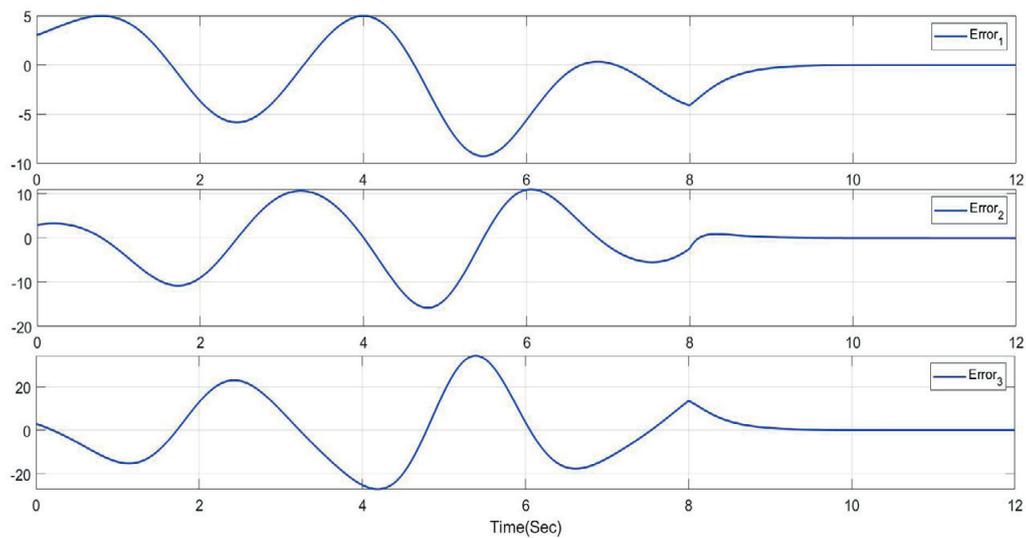


Figure 12. synchronization error of two chaotic supply chains by neural fuzzy controller.

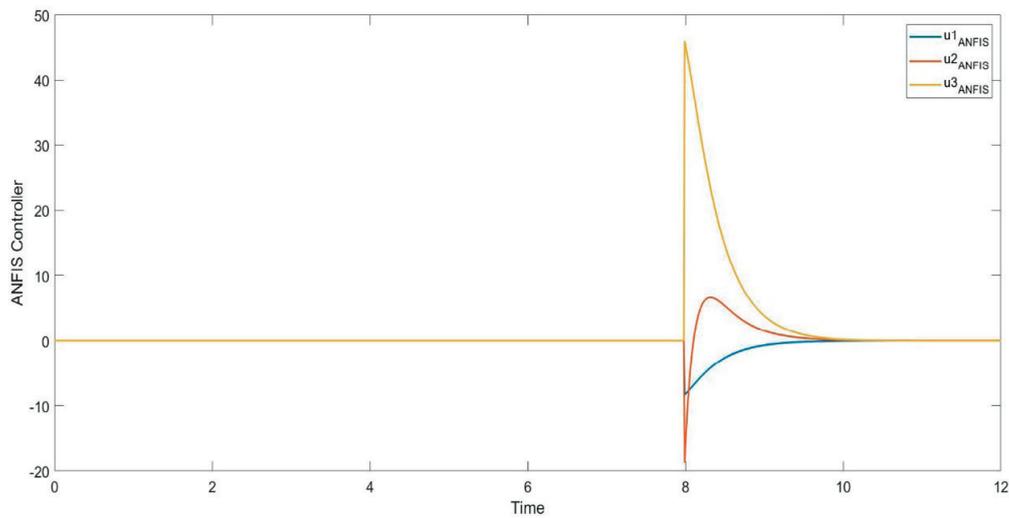


Figure 13. Adaptive neural fuzzy control signals.

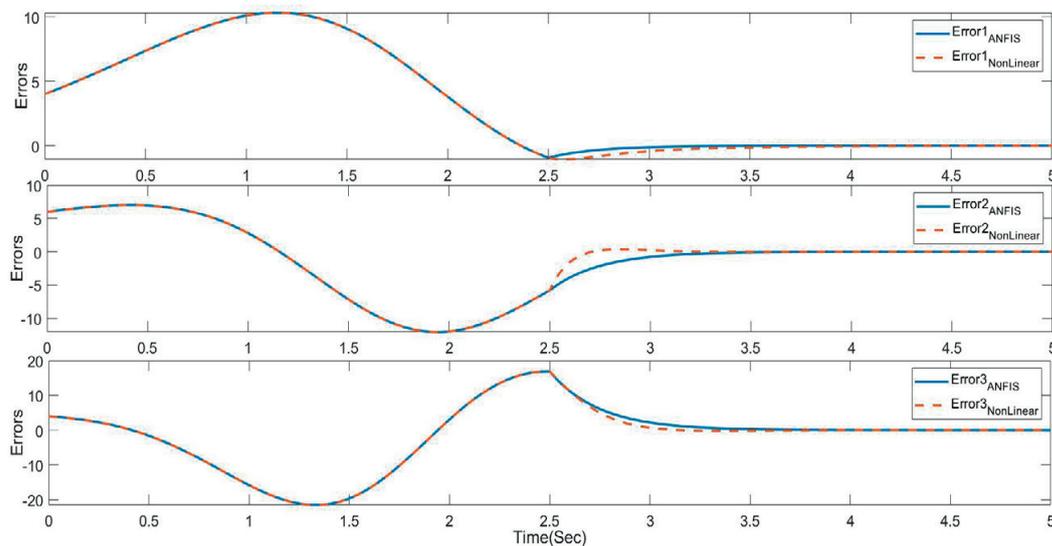


Figure 14. Error comparison between an adaptive neural fuzzy controller and nonlinear feedback.

description. In this model, there was uncertainty from the retailer to the manufacturer. Using the neural fuzzy method, the synchronization of two chaotic supply chains was investigated. First, a nonlinear feedback control method was used to train the adaptive neural fuzzy. Using Lyapunov’s theory of stability, the nonlinear feedback method was proved to synchronize the two chaotic supply chain networks.

The best architecture was selected based on the comparison of training error results and checks. It was then shown that the adaptive neural fuzzy method works well for the test data. Thus, an

adaptive neural fuzzy controller was designed. In the numerical simulation, evaluation was performed by changing the initial conditions and activation time of the adaptive neural fuzzy controller. To show that the adaptive neural fuzzy controller is well-trained and can reduce the synchronization error to zero when activated, the activation time of the controller is chosen differently in each part of the simulation.

The simulation results in this section show that the synchronization error moves asymptotically to zero. It was also shown that there is very low amplitude and fluctuation in the control signal. Therefore, the

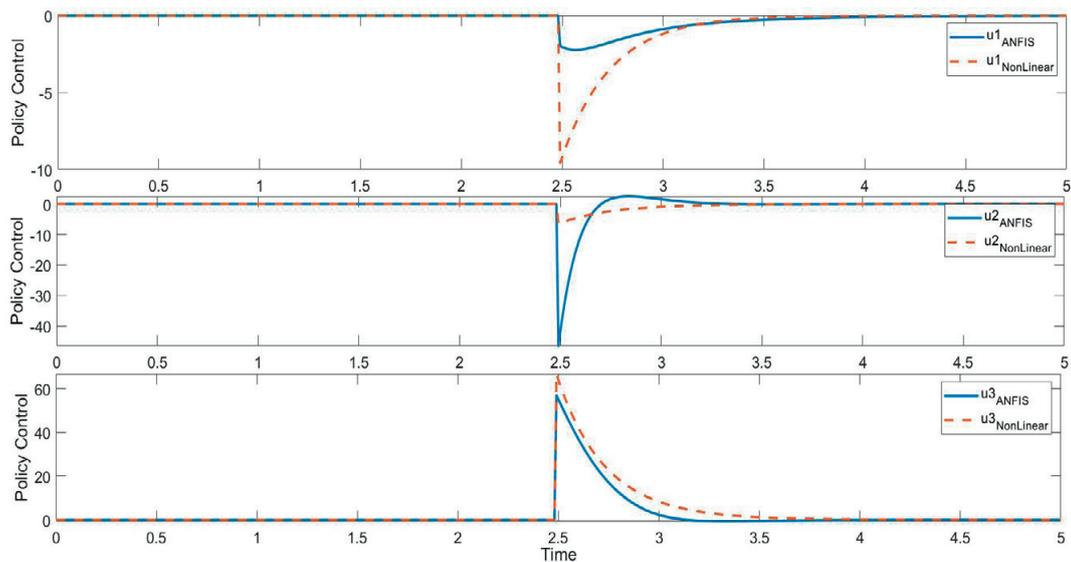


Figure 15. Comparison of adaptive neural fuzzy controller behavior and nonlinear feedback.

Table 3. Comparing the time to reach zero error and the number of controllers in different chaotic supply chain control methods.

Name and Year	Goal	Method of control	Number of controllers	Stability Time
Lie et al. (2006)	synchronization	RBF control	Three	≈ 5 sec
Göksu et al. (2014)	Synchronization, elimination chaos	Active control	Three	≈ 7 sec
Mondal, 2019	synchronization	Nonlinear control	Three	≈ 0.5 sec
Peng, et al. 2020	synchronization	Impulsive control	Three	≈ 0.7 sec
Yan et al. 2020	Synchronization, elimination chaos	Nonlinear control	Four	≈ 6 sec
Hamidzadeh et al. 2022a	Synchronization, elimination chaos	Nonlinear control	One	≈ 0.5 sec
Hamidzadeh et al. 2022b	synchronization	Sliding mode control	Four and Two	≈ 1 sec
This paper	Synchronization	ANFIS	Three	≈ 1.5 sec

proposed method can be implemented in the real world at a low cost.

In the other part of the simulation, the performance comparison of nonlinear and adaptive neural fuzzy feedback methods for the synchronization of two chaotic supply networks was evaluated. The initial conditions and activation time of the adaptive neural fuzzy controller have been re-selected to synchronize two different chaotic networks. In this comparison, it was shown that the neural fuzzy controller signal as a whole moved faster to zero.

Limitations and future research lines should be analyzed in the disturbance and unknown input for the supply chain. The number of controllers applied supply chain can be reduced by others ANFIS models.

From the point of view of supply chain network management, the following are important:

- 1- Identifying the network control cost with the number of controllers and the time to reach stability
- 2- When applying the controller to the supply chain network, if the chaos in the supply chain network is detected early, it can be brought to stability with less cost.

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