Book of practical laboratory sessions

Fundamentals of Physics for Biotechnology

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Introduction

This is a necessary document for the development of the laboratory classes of Physics of the first course in Biotechnology degree at the Universitat Politècnica de València.

In this book we have tried to gather the concepts, images, scheems and diagrams needed to understand some of the physical phenomena and physical properties described in this course. It offers the tables, graphs and spaces needed for it to become a tool that favors the autonomous and orderly work of the student in the laboratory.

This book can be complemented with the following videos:

Medidas directas e indirectas. Incertidumbres http://hdl.handle.net/10251/49374
Medida de la constante elástica de un resorte http://hdl.handle.net/10251/53811
Medida de la temperatura. https://riunet.upv.es/handle/10251/66590
Medida de la densidad de un líquido. http://hdl.handle.net/10251/66003
Medida de la viscosidad de un líquido. http://hdl.handle.net/10251/65079
Caracterización de señales eléctricas. PoliformaT F. Físicos para la Biotecnología.
Composición de señales eléctricas. PoliformaT F. Físicos para la Biotecnología.

We hope it is a useful and suitable document for working at the laboratory.

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Chapter 1

ELASTIC CONSTANT OF A MEMBRANE

1.1 ABSTRACT

A circular elastic membrane is a physical system that can be modeled by infinite diametral elastic threads that intersect at the central point. When all the ends are held in a horizontal plane and a force is applied on the central point, hanging a mass for example, the system is deformed by lengthening its threads and it is said that the threads are tensioned. The system returns to its original shape, when force is zero. When threads are tensioned by a mass and an additional force is applied, the tensile strength rises in the threads. If this additional force suddenly disapears, the membrane will vibrate and the mass will move around its equilibrium position (harmonic motion). Both in static deformation and dinamic deformation, the membrane behaves in agreement with an intrinsic property of the material it is made of. This elastic property of materials is the Young modulus. In this lab, the period of oscillation in a elastic band is measured. From this direct measurement the transversal elastic constant is indirectly obtained (k) that is related to the Young modulus. The objective of this lab is to obtain the uncertainties of every measurement: direct and indirect (Appendix A, page 73) which are obtained with less than 5 % (relative uncertainty).

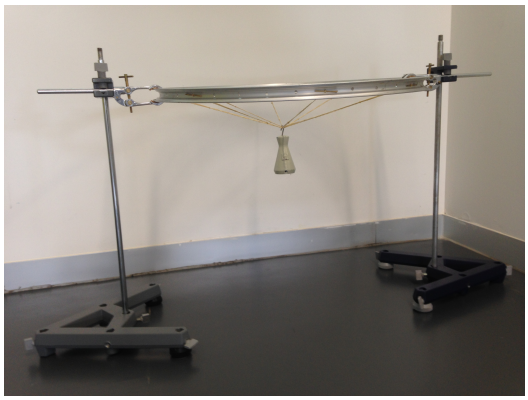

Figure 1.1: Membrane model simulation

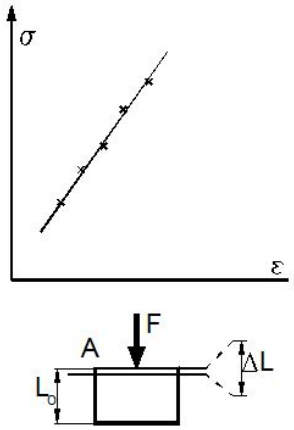

Figure 1.2: Tensile-deformation curve on a membrane

Materiales biológicos	Y (Pa)
Caucho	$30 \cdot 10^6$
Cartílago	$24 \cdot 10^6$
Tendón	$600 \cdot 10^6$
Hueso	$21 \cdot 10^9$

Alimentos	Módulo de Young aparente (Pa)
Pan	0,1-0,3
Gelatina	2
Plátano	8-30
Melocotón	20-200
Patata	50-140
Manzana	60-140
Zanahoria	200-400

Figure 1.3: Young modulus for some food and biological materials

1.2 INTRODUCTION

As long as materials do not exceed the elastic limit (maximum value), the majority of materials have an elastic behaviour when a force acts on them (tension and compression). A perfect elastic body deforms instantly when a force is applied, and it recovers its shape when the force is removed. A unitary tensile force (σ) is the perpendicular force (F) applied per unit transversal area (A):

$$\sigma = \frac{F}{A}$$

The change in length gained under a tensile force is known as deformation. The unitary deformation is the change in length (ΔL: deformation) per unit length (L_0: original lenth):

$$\epsilon = \frac{\Delta L}{L_0}$$

A linear elastic body behaves as Hooke's law, which states that tensile force is proportional to the unitary deformation, therefore, there is a linear relationship between deformation and tensile force. The slope of this straight line is the Young modulus [Y($N/m^2 = Pa$]. This Young modulus measures the stiffness of a material.

$$\mathbf{Y} = \frac{\sigma}{\epsilon} = \frac{\mathbf{F/A}}{\mathbf{\Delta L/L_0}} = \mathbf{k_L} \cdot \frac{\mathbf{L_0}}{\mathbf{A}} \tag{1.1}$$

k_L (N/m) being the axial elastic constant of the elastic body. The Young modulus can be used in biological materials as long as they are considered as homogeneous, continuous and with uniform cross sectional area. In Figure 1.3 Young modules for diferent biological and food materials are shown. Elasticity, as defined by Hooke's law, is not fulfilled in many food materials which are visco-plastic. In such a case, the sum of elastic and plastic deformation in the force-deformation curve is used in order to calculate the unitary deformation with the apparent Young modulus, known as deformability modulus.

Figure 1.4: Elastic band with a weight in the middle point

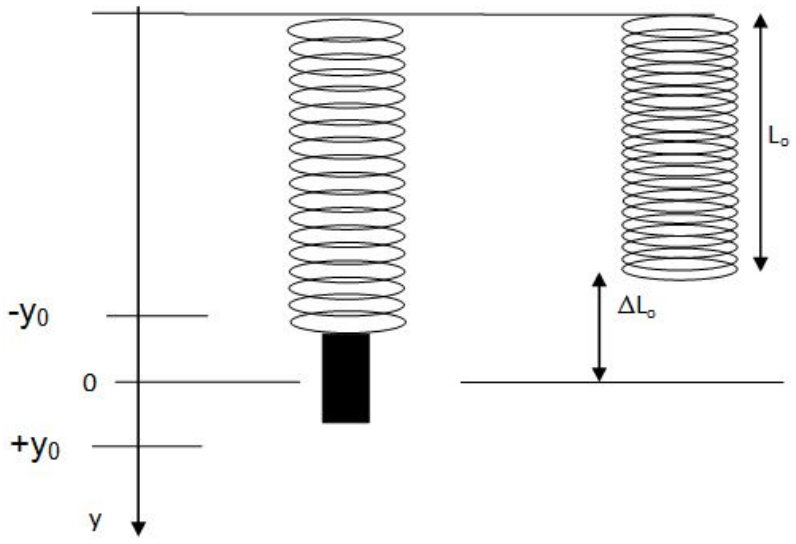

Figure 1.5: Mechanical analogy of a stretched spring at rest, in order to study the harmonic simple motion of a mass which is hanging in the center of an elastic system

OBJECTIVE

In this practice we study a very simple mechanical model, based on a single body joined by a spring to a fixed point, which represents the system of a horizontal tense elastic band at whose center a mass is located. We will measure the period of oscillation, calculate the elastic constant of the system and its uncertainty.

1.3 MATERIALS AND METHODES

1.3.1 Materials

In Figure 1.4 is shown the measured system which consist of two elastic bands that hold a metallic bar of mass m located in their center. Additionally a chronometer/timer is needed to measure the period of harmonic movement of the metallic bar.

1.3.2 Physical model: Mass-spring mechanical model. Period of oscillation

The harmonic oscillator is a mechanical system consisting of a spring fixed at one end and a body with mass m attached to the other end, as shown in Figure 1.5. The spring without a mass attached has a length of L_o. If you hang up the body of mass m, the spring is stretched to length ΔL_0. When the mass is at rest (v = 0) and at equilibrium (F = 0) the spring force and weight are equal and opposite. If a mass m is hanged up, the spring is stretched to length ΔL_0. The force with which the spring pushes back obeys Hooke's law, which states that the force is proportional to the distance that the spring stretches and to the spring constant. Since the mass is stationary, the downwards force (weight) is exactly balanced by an upwards force provided by the tension in the stretched spring. The stiffness of the spring is characterized by its elastic constant k which relates the force applied and the lengthening ΔL_0 of its lenght L_0. The balance equation is:

$$\mathbf{m} \cdot \mathbf{g} - \mathbf{k} \cdot \mathbf{\Delta L_0} = \mathbf{0} \tag{1.2}$$

$$\mathbf{k} = \frac{\mathbf{m} \cdot \mathbf{g}}{\mathbf{\Delta L_0}} \tag{1.3}$$

If the spring departs from its initial equilibrium position (y = 0) a distance $y = y_0$ (initial time) then the mass ranges from y=+y_0 to y=-y_0 with a period (and frequency) that depends on the elasticity of the spring and the mass of the body. The dynamic equation is:

$$\mathbf{mg} - \mathbf{k}\,(\mathbf{y} + \mathbf{\Delta L_0}) = \mathbf{m} \cdot \ddot{\mathbf{y}} \tag{1.4}$$

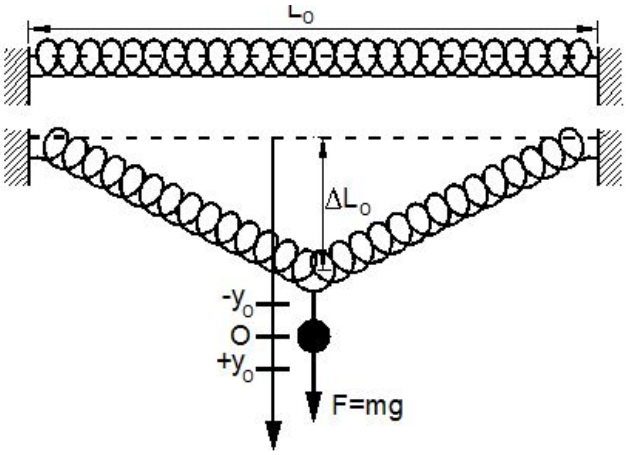

Figure 1.6: Transversal streching of a spring due to a central weight

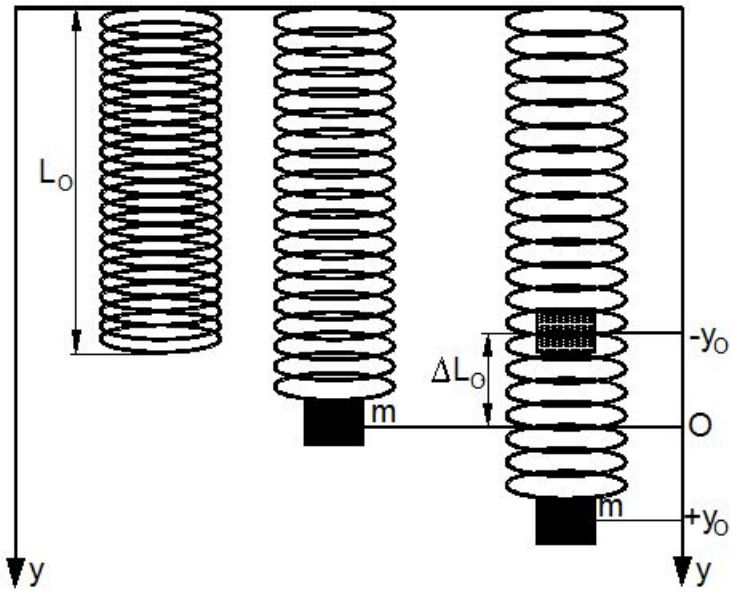

Figure 1.7: Mechanical analogy of a stretched spring at rest, in order to study the harmonic simple motion of a mass which is hanging in the center of an elastic system

Substituting in Equation 1.4 the value of k according to Equation 1.3, the simplified dinamic equation remains as:

$$\ddot{\mathbf{y}} = -\frac{\mathbf{k}}{\mathbf{m}} \cdot \mathbf{y} \tag{1.5}$$

The movement equation that satisfies Equation 1.5 is of the following type:

$$\mathbf{y} = \mathbf{A}\cos\left(\omega_0 \mathbf{t} + \varphi_0\right) \tag{1.6}$$

Armonic Simple Motion equation (ASM) of a body. Taking into account the velocity and acceleration definitions, the following equations can be deduced from Equation 1.6.

$$\dot{\mathbf{y}} = -\mathbf{A} \cdot \omega_0 \,\mathrm{sen}\left(\omega_0 \mathbf{t} + \varphi_0\right) \tag{1.7}$$

$$\ddot{\mathbf{y}} = -\mathbf{A} \cdot \omega_0^2 \cos\left(\omega_0 \mathbf{t} + \varphi_0\right) = -\omega_0^2 \cdot \mathbf{y} \tag{1.8}$$

Considering the initial conditions of the movement: $\dot{y}(t=0) = 0$ it can be deduced $\varphi_0 = 0$ and $y(t=0) = 0$ it can be deduced $A = y_0$. Equating Equation 1.5 with Equation 1.8 it can be obtained:

$$\omega_0 = \sqrt{\frac{\mathbf{k}}{\mathbf{m}}} \tag{1.9}$$

From the Equation 1.9 it can be deduced that the elastic constant is related to the mass and the angular frequency of the SAM by:

$$\mathbf{k} = \mathbf{m} \cdot \omega_0^2 \tag{1.10}$$

The period t_0 is the time it takes to complete a cycle of the SAM and the phase is: $[2 \cdot \pi = \omega_0 \cdot t_0]$, from which it is posible to write:

$$\mathbf{t_0} = \mathbf{2}\pi \cdot \omega_0^{-1} = 2\pi \cdot \sqrt{\frac{\mathrm{m}}{\mathrm{k}}} \tag{1.11}$$

Substituting Equation 1.11 in Equation 1.10 the equation which allows for determining the elastic constant is obtained:

$$\mathbf{k} = \mathbf{4}\pi^2 \mathbf{m} \mathbf{t}_0^{-2} \tag{1.12}$$

Therefore measuring the period t_0 and the mass m, the elastic constant can be calculated.

Table 1.1: Period measurements

Medida	10·t_{oi}	t_{oi} 1cicle	t_{oi} order	Intervals	Repetition	$(t_{oi} - < t_o >)^2$
1						
2						
3						
4						
5						
6				X	X	
7				X	X	
8				X	X	
9				X	X	
10				X	X	
11				X	X	
12				X	X	
13				X	X	
14				X	X	
15				X	X	
16				X	X	
17				X	X	
18				X	X	
19				X	X	
20				X	X	
X	SUM =		X	X	SUM =	
X	$<t_o>$ =		X	X	$u(t_0)$	

Figure 1.8: Frecuency histograms

1.3.3 Experimental method

Period measurement

1. The metallic bar must be moved upwards around 1 cm.
2. Let the bar go (Figure 1.6).
3. Use the chronometer to measure the time it takes to complete 10 cycles of movement [10·t_{01}]. You can count 10 flashes in the LED lamp to know when to start and stop the chronometer.
4. This procedure must be repeated 20 times for obtaining 20 values of t_0, t_{0i} (i = 1,2,....., 20).
5. The period of each measurement is obtained from [10·t_{01}] divided by 10.
6. Periods must be ordered from least to greatest and are grouped into 5 intervals of equal amplitude between the minimum and maximum values of the period.
7. The number of measurements in each interval must be writen down in Table 1.1.
8. A frequency histogram with five equidistant intervals can be represented in Excel spreadsheet and it must be shown in Figure 1.8.
9. An analysis of the distribution must be done (Appendix A, page 76).
10. The average value of the period can be calculated as:

$$\langle \mathbf{t_0} \rangle = \frac{\mathbf{1}}{\mathbf{20}} \cdot \sum_{\mathbf{i=1}}^{\mathbf{20}} \mathbf{t_{0i}} \tag{1.13}$$

11. The square difference between each t_{0i} and the average value t_0 will be used when estimating the uncertainty of the period.

$$u\left(t_0\right) = \sqrt{\frac{\sum_{i=1}^{20}\left(\langle t_0\rangle - t_{0i}\right)^2}{20\left(20-1\right)}} \tag{1.14}$$

$$\Delta m = \frac{1}{100}\cdot m + 1\cdot 0,1(gramos)$$

$$u(m) = \frac{\Delta m}{\sqrt{3}} \tag{1.15}$$

Table 1.2: Direct measurements and their uncertainties

X	Medida	ΔX	u(X)
$<t_0>$(s)		—	
m (g)			
m (kg)			

$$u\left(k\right) = \sqrt{(u_{t0}(k))^2 + (u_m(k))^2} \tag{1.16}$$

$$u_{t0}\left(k\right) = \frac{\partial k}{\partial t_0}u\left(t_0\right) =$$

$$u_m\left(k\right) = \frac{\partial k}{\partial m}u\left(m\right) =$$

Table 1.3: Indirect measurement and partial uncertainties

$k\ (kgs^{-2})$	$u_{to}(k)$	$u_m(k)$	$u(k)$

Table 1.4: Elastic constant

$k \pm u(k)\ (kg\cdot s^{-2})$	$u(k)/k\ (\%)$

Mass measurement

The mass m of the metallic bar is measured by using a precision digital scale that weighs in grams with a centesimal approximation. (Table 1.2).

Elastic constant

Using equation Equation 1.12 we can calculate the spring constant k in an indirect way (mathematical operations and direct measurements of mass in kilograms and period in seconds). Its units in the SI (International system) are $kg \cdot s^{-2}$.

1.4 RESULTS AND DISCUSSION

1.4.1 Uncertainties

Period

Taking into account that the uncertainty of the chronometer is much less than the measured time interval and the error that can happen by touching the buttons of the chronometer, it is calculated as a type A standard uncertainty (Appendix A, page 76). This type of uncertainty is the result of many measurements in a normal distribution (Equation 1.14).

Mass

The mass of the metallic bar is measured using a precision electronic scale whose precision is given by the manufacturer (it measures in grams with 1% precision, two decimals on the screen, and the reading uncertainty is one unit of the last figure of the reading. If there is not fluctuation in the measurement, we only take one measurement, and the only standard uncertainty will be of type B (Equation 1.15, Appendix A, page 75).

Elastic constant

Being an indirect measurement, its uncertainty is calculated by Equation 1.16, by partially deriving with respect to its variables: mass and period, (number π is considered without uncertainty by having many decimal figures)(Appendix A, page 77). In order to write the uncertainty properly follow rules in Appendix A, pages 78 and 79.

Chapter 2

MEASUREMENT OF THE TEMPERATURE. WHEATSTONE BRIDGE

2.1 ABSTRACT

The Wheatstone Bridge (WB) is a resistor network that allows us to find an equivalent electrical resistance from another unknown electrical resistance by circulating a very low intensity current. This method is useful when the unknown resistance does not admit enough current to be measured with acceptable uncertainty, while the equivalent resistance can be measured in these conditions. In this lab the WB is applied to measure the electrical resistance of a Temperature Dependent Resistor (TDR) whose electrical resistance varies linearly with the temperature. WB is applied because electric current should not pass through the TDR so it does not alter the system temperature. We will calibrate the resistive sensor 0°C and at 100°C so that we will have a useful thermometer for measuring temperature in the range of calibration, by measuring electrical resistance that is easy to control.

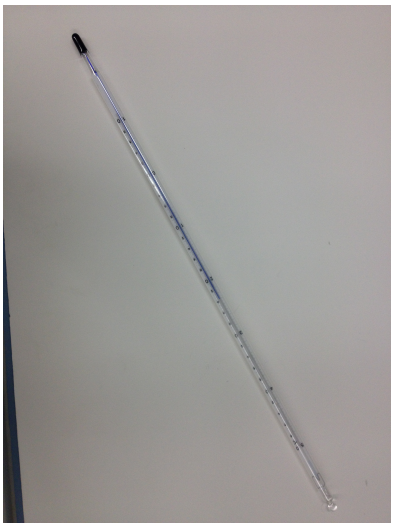

Figure 2.1: Mercury termometer

Figure 2.2: Termometer TDR-Pt

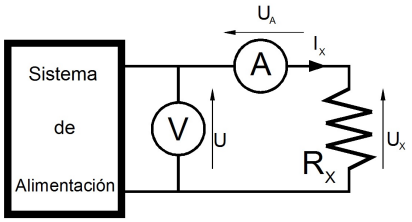

Figure 2.3: Measurement of Rx. Long circuit

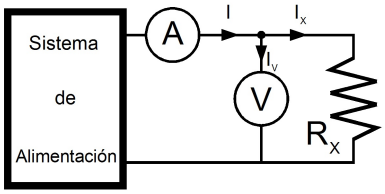

Figure 2.4: Measurement of Rx. Short circuit

2.2 INTRODUCTION

Measuring the temperature of a body requires four steps:

1. A thermodynamic system has to be chosen with any measurable macroscopic property (thermometric property) which varies linearly (linear thermometer) and reproducibly with temperature (mercury, alcohol, (Figure 2.1); electric resistance (Figure 2.2)).
2. The system has to be put in touch with the body, until both reach the same temperature (thermical balance).
3. The thermometric property has to be measured with the thermometer.
4. The thermometer has to be calibrated, to do so it is necessary to measure the themometric property of the thermometer in two systems of reference: 0 and 100°C.

If the thermometer, in the range 0-100°C, varies linearly with temperature, it will be already available to measure.

To measure the electrical resistance R, a current generator is needed in order to circulate a current I through the resistance R, the current has to be measured by an ammeter and also the potential difference V has to be measured at both ends of the resistance R. The resistance value is R=V/I. In addition to the systematic error, that is gained by placing the ammeter and the voltmeter (2 possibilities: long circuit and short circuit, Figure 2.3 y Figure 2.4 respectively), which has to be corrected, the relative uncertainty of R is the square root of the sum of the squares of the relative uncertainties of the voltmeter and ammeter. An ohmmeter measures the value of R directly through this procedure. If we measure an R of 100 ohms, 0.45 mA are circulating in the scale of 200 Ω of our ohmmeter and the semi-interval of error is $\Delta R = 1.1\Omega$. If we do it on the scale of 2 kΩ, the intensity decreases to 0.28 mA and $\Delta R = 1.8\Omega$ is greater. The lower the intensity the greater the uncertainty. However, when it comes to measuring the temperature, the current I that flows through R has to be the less as possible.

Wheatstone Bridge (WB) is an electrical method for obtaining equivalent resistance measured with a very low current intensity, which decreases electrical power in the measurement process and reduces the systematic error. In a separate measurement we measure the equivalent resistance with the ohmmeter, with the intensity that is needed and without affecting the unknown original resistance value.

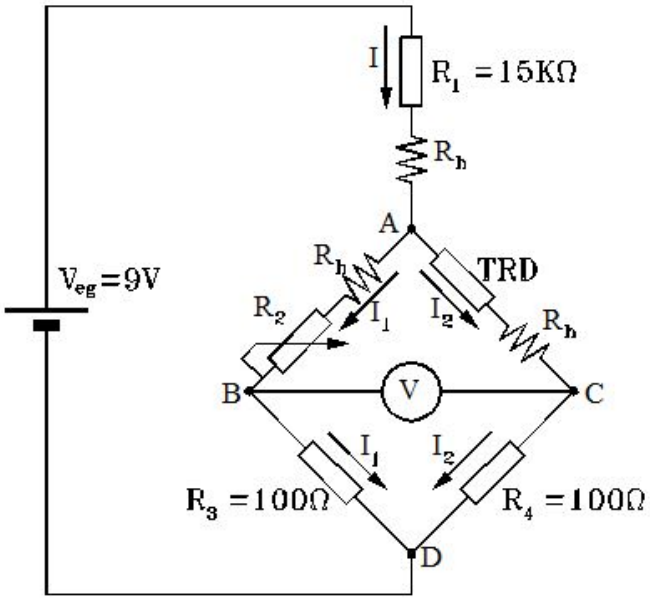

Figure 2.5: Wheatstone Bridge

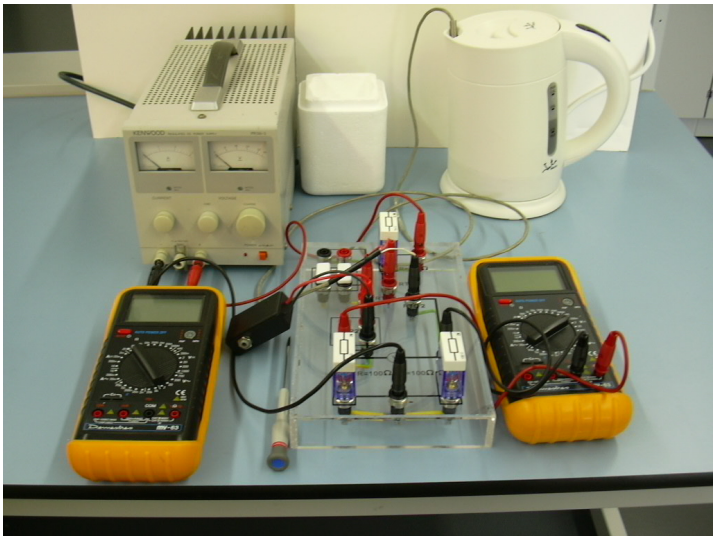

Figure 2.6: Used materials in the measurement of the temperature

OBJECTIVES

In this lab we will calibrate a Temperature Dependent Resistor (TDR), commercial sensor that consists of a piece of platinum whose electrical resistance is 100 ohms at 0 ° C (Pt-100) and varies linearly in the range 0-100 ° C, to measure temperatures in this range. To measure the resistance in a TDR sensor and find its equivalent resistance, the minimum intensity will pass through it so that it doesn?t alter the system?s temperature, therefore a Wheatstone Bridge (WB) is used. After this, resistance will be measured without intensity limit with an ohmmeter. The WB has an additional advantage, and it is that the electrical resistance of the connecting cables is removed. WB circuit and method used in this lab are commonly used in digital thermometers (Figure 2.2).

2.3 MATERIALS AND METHODS

2.3.1 Physical model: Wheatstone Bridge

In Figure 2.5 the scheme of the electrical circuit that is known as the wheatstone bridge is shown. The current reaches point A and it is divided into two current flows I_1 which is on the branch and I_2 by the branch AC. The resistor R_2 is variable (potentiometer) it is varied until the voltmeter V between B and C mark zero. In this situation, the bridge is balanced. No current flows between B and C because V_{BC}=0. So the same I_1 passes for AB than through BD. The same is true of the current I_2 passes through AC and CD. As the resistance of the branches CD and BD are equal (100 Ω) and $V_B = V_C$ the two intensities are equal to $I_1 = I_2$. Thus the resistance of the branch AB is the same as the AC branch.

The TDR has 3 connected wires, two at end A and one at end C. Each of them has a Rh resistance which varies with temperature, whose value we have to remove by measuring only TDR. In the circuit shown one of the three wires is connected to R_1 and the current passing through it is I. The other two wires are located respectively in the section AB and AC. So in the two branches there are resistances:

R_2+R_h=R_{TRD}+R_h, which it involves: R_2=R_{TRD}

2.3.2 Materials

The picture in Figure 2.6 shows the materials needed to obtain the temperature through the measuring of the electric resistance in a TDR. Electrical devices: Temperature Dependent Resistance (TDR) of three-wire, voltage generator of 1.5 Volts, a resistor of 15 kΩ, 2 resistor of 100 Ω , a variable resistor or potentiometer, a voltmeter and connecting wires Thermal equipment: glass with ice and water (melting point), and electric boiler to carry water to the boiling point.

Table 2.1: Measurement of R_2

	0°C	100°C	$T_{room}(°C)$
R_2 (Ω)			

Table 2.2: Measurement of R_2 in several measurement devices

Measurement of R_2	0°C	100°C
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

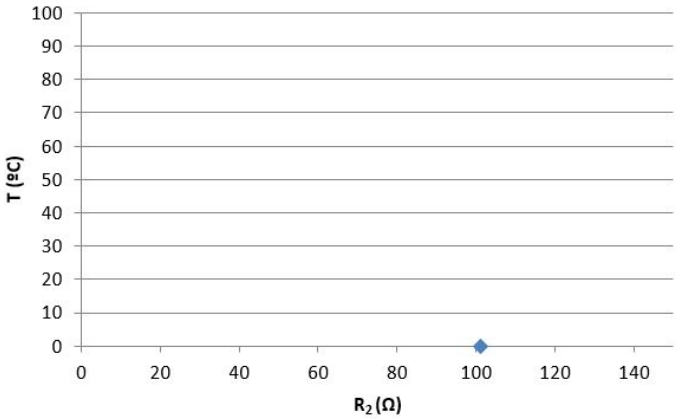

Figure 2.7: Graph T (°C) vs $R_2(\Omega)$

2.3.3 Experimental method

The experiment consists of comparing the electrical resistance of the TDR by WB, with the equivalent resistance of the potentiometer, and then measuring it with the ohmmeter. There will be 3 TDR measurements in water: at room temperature, at point of fusion of ice (0 °C) and at boiling point of water (100 °C).

1. The TDR is put into a glass of water at room temperature (T_{amb}).
2. The WB circuit has to be assembled like Figure 2.5. The current of the source is very small, I = 0.1 mA. The intensity is divided between the two branches of the circuit, $R_3 - R_2$ (potentiometer) and $R_4 - R_{TRD}$. Hay que montar el circuito PW que se representa en la . La intensidad de la fuente es muy pequeña, I=0,3 mA. La corriente se divide entre las dos ramas del circuito, $R_3 - R_2$ (potenciómetro) y $R_4 - R_{TRD}$. The potential difference between points B and C has to be read in the voltmeter, which changes by varying the potentiometer.
3. The potenciometer (R_2) is tuned with a screwdriver until the reading in the voltmeter is zero, in this situation the WB is balanced, the resistance of the two branches are equal and therefore $R_{TRD} = R_2$.
4. The potentiometer is taken out the circuit and resistor R_2 is measured with an ohmmeter. The value of R_2 is written down in Table 2.1.
5. This procedure of measuring the R_2 is repeated, when the bridge is balance in ice water conditions temperature is T_{0^oC} and in boilling water conditions temperature is T_{100^oC}. Write the results down in Table 2.1.

Calibration

In order to obtain the equation of the straight line that relates the temperatures of 0°C and 100°C with the values of the electrical resistance of the TDR, we use the measured $R_2(0)$ y $R_2(100)$. The equation of a straight line is: y = a + b·x

$$T = a + bR_2 \tag{2.1}$$

Two point are not enough to do a good calibration, due to the fact that the equation that better fits to two experimental points will always be a straight line with determination coefficient of R^2=1. Therefore, we will take every points obtained in the different measuring devices (Table 2.2), because although they are concentrated in two conditions of known temperature, they will provide us the most unfavorable uncertainty for the type of TDR sensor used. Graph T (°C) vs R_2 is represented in Figure 2.7.

Table 2.3: Results of the linear fitting of the experimental data T (°C) vs R_2

Parameters X	X	u(X)	Parameters	
b (°C·Ω^{-1})			R^2	
a (°C)			T_{room} (2.1)	

$$\Delta R_2 = \frac{0,8 \cdot R_2(T_{amb})}{100} + 3 \cdot 0,1 =$$

$$u(R_2) = \frac{\Delta R_2}{\sqrt{3}} \tag{2.2}$$

Table 2.4: Direct measurement and uncertainty

R_2 (Ω)	ΔR_2	u(R_2)

$$u\left(T\right) = \sqrt{(u_{R_2}(T))^2 + (u_a(T))^2 + (u_b(T))^2} \tag{2.3}$$

$$u_{R_2}\left(T\right) = \frac{\partial T}{\partial R_2} u\left(R_2\right) =$$

$$u_a\left(T\right) = \frac{\partial T}{\partial a} u\left(a\right) =$$

$$u_b\left(T\right) = \frac{\partial T}{\partial b} u\left(b\right) =$$

Table 2.5: Measurement of the room temperature

$T_{room} \pm u(T)$ (°C)	$u(T)/T_{room}$ (%)

If the Excel spreadsheet is used, the next steps must be followed to determine the characteristic parameters of the linear fitting and their uncertainties:

1. Two columns are created for data: R_2(x)-T(y).
2. Ten empty boxes of the spreadsheet have to be highlighted (two columns and five rows)
3. Then it has to be written down "= estimación.lineal (y numbers: T; x numbers: R_2; verdadero; verdadero)".
4. Finally, click simultaneously: Ctrl-Mayus-Enter.

2.4 RESULTS AND DISCUSSION

2.4.1 Calculation of the room temperature

Assuming that the electrical resistance TDR varies linearly in the range 0-100°C, following the Equation 2.1, mesuring $R_2(T_{amb})$(value of TDR when the PW is balance at room temperature) and taking the coeficients of Table 2.3, room temperature can be obtained. Compute the room temperature and write it down on Table 2.3.

2.4.2 Calculation of the uncertainties

Resistance R_2

The uncertainty of $R_2(T_{amb})$ is calculated, taking into account that the semi-interval error of the ohmmeter is 0.8% of the reading plus the reading error that it is 3 digits of the precision in the range of 200 ohms. (Table 2.3.3). Therefore, its uncertainty as direct measurement is calculated by Equation 2.2 (Appendix A, page 75).

Temperature

In order to calculate the uncertainty of room temperature u(T), it will be considered the only known uncertainty is due to R_2 (so, the uncertainty related to PW is neglected because it is considered much lower). It is calculated as an indirect uncertainty considering Equation 2.1, and by using the Equation A.7 in page 77.

Compute the uncertainty of the room temperature and its relative uncertainty. Complete Table 2.5. Remember the writing rules for the uncertainty, Appendix A, page 78.

Table 2.6: Room temperature measurements as a common thermal system for all lab devices

Measurement	$T_{room}(^oC)$	$(Ti- < T >)^2$
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
SUM=		
$< T >=$		XXXXXXXXXX
$u_A(T)=$		XXXXXXXXXX

Table 2.7: Measurement of the room temperature as a direct measurement

$< T > \pm u_A(T)$ (°C)	$u_A(T)/ < T >$ (%)

Temperature as a result of N repetitions, type A

The uncertainty calculated in Table 2.5 has been evaluated considering the errors made in the estimated parameters for the room temperature in single measurement device. Considering all the measurement devices (same type) used in the lab, the uncertainty can be calculated as if it is a measurement of type A. To do this we will take a temperature measurement of the same thermal system with each of the measuring devices of the laboratory (each group takes its own temperature). We will put in common all the data and we will calculate the average temperature and the direct uncertainty of type A (Table 2.6 and Table 2.7 according to Appendix A, page 76).

Chapter 3

MEASURING THE DENSITY OF A LIQUID

3.1 ABSTRACT

Density of a liquid is an intensive property that expresses the relationship between mass and volume of a body. This is an important property that affects to the force exerted by fluids on submerged particles and on the walls of the reservoir which contains them. The Archimedes' Principle gives the relationship between the buoyant force and the density of a liquid, and is one of the most commonly used to measure density. In this lab, buoyant force of a body immersed in a liquid is measured by using a scale. This experimental method measures the density of water with a relative standard deviation lower than 5 %.

Figure 3.1: Pycnometer

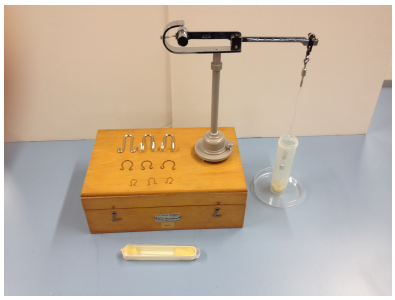

Figure 3.2: Balance of Westphal

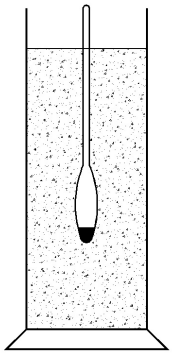

Figure 3.3: Hydrometer

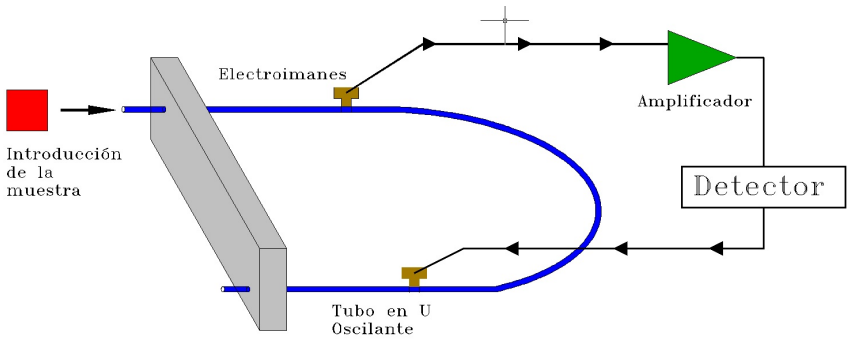

Figure 3.4: Measuring tube

3.2 INTRODUCTION

Density of liquids increases when temperature decreases. However, water is an exception when it is cooled from 4°C, (maximum density) to 0 °C. Ice is less dense than liquid water, so it floats on the surface, it is a thermal isolator so it can keep the temperature of liquid water above 0 °C although outside the temperature is lower. Different physical laws are used to measure the density of liquids. The Archimedes' Principle states that: **A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid**.

Some of the most important methods used to measure density are described in the following paragraphs:

1. The pycnometer (Figure 3.1) is a simple tool which consists of having accurate volume of liquid in a container of known weight. By weighing the filled pycnometer we can calculate the mass of liquid whose volume is known. Precision pycnometers have two fine graduated capillaries above the fluid reservoir. The pycnometer is filled so that the fluid reaches both capillaries and the volume can be measured. The error in this method varies between 10^{-2} and 1 (kg/m^3) depending on the pycnometer. The error is mainly due to variations in the volume of the reservoir and diameter of the capillary.

2. b) The Balance of Westphal (Figure 3.2) consists of immersing a lead weight in a liquid. The lead is suspended with a string from the scale and it allows us to measure the balance force (tension of the string) when the weight and buoyant force are applied to the lead. Density of a fluid is calculated as an indirect measurement of buoyant force and density of the lead with an error of 0.2 (kg/m^3).

3. A third method called the Hydrometer (Figure 3.3) is based on the principle of Archimedes, which states that the buoyant force of a liquid varies directly with the submerged volume of a body. The device consists of a graduated glass stem in which end there is a bulb filled of lead or mercury, this device is left on a liquid and the density value can be read on the graduated scale at its point of buoyancy. This method is subject to surface tension which limits the accuracy of the results, in ideal situations the results can have errors of 0.1 (kg/m^3).

4. d) Finally a very accurate method that requires little quantity of liquid is the Oscillating U-Tube (Figure 3.4) it is based on the principle that the density of a liquid sample in a test tube is related to the difference of the acoustic resonance frequencies when the cylinder is full and when it is empty.

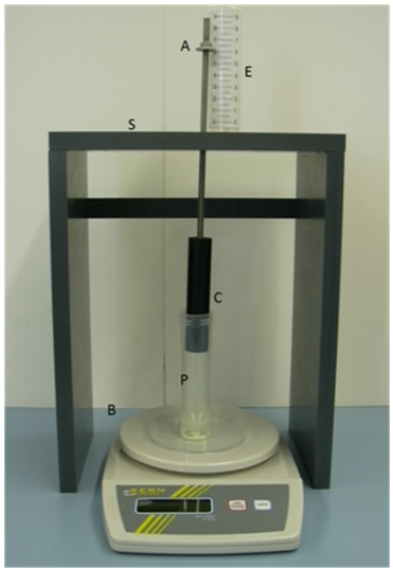

Figure 3.5: Measuring device

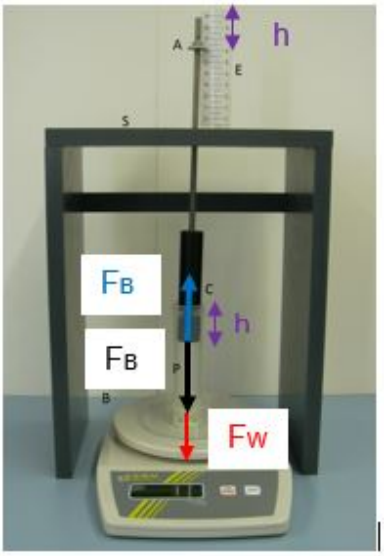

Figure 3.6: Physical model

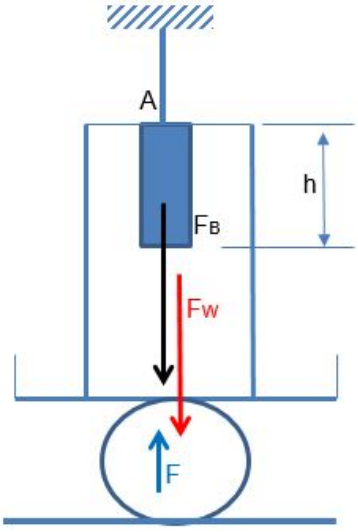

Figure 3.7: Diagram of physical model

OBJECTIVES

This chapter presents a method of measuring the density of a fluid using Archimedes' principle, in which a body immersed in a liquid applies a downward buoyant force proportional to the density of the liquid and the volume submerged. The buoyant force upon the liquid is measured by a scale that holds the deposit and the liquid. The measure of density of water and its uncertainty are shown in the following paragraphs.

3.3 MATERIALS AND METHODS

3.3.1 Physical model

Archimedes' principle states that the buoyant force exerted by a liquid on an immersed body equals the weight of the liquid displaced by the body of volume V.

$$\mathbf{F_E} = \rho \cdot \mathbf{g} \cdot \mathbf{V} \tag{3.1}$$

being **g** free fall acceleration and ρ the density of the liquid.

This upward buoyant force is the same as the downward buoyant force exerted by the body on the liquid, due to the action-reaction principle. The scale measures the weight of the liquid inside the test tube, $\mathbf{F_W}$ which is constant, since the overflowed liquid falls into the plastic tray that is located on the scale, and the downward buoyant force, $\mathbf{F_B}$ which increases with the immersed length of the cylinder. If the cylinder is immersed a length of **h**, the scale will receive a force:

$$\mathbf{F} = \mathbf{F_W} + \mathbf{F_B} = \mathbf{F_W} + \rho \cdot \mathbf{g} \cdot \mathbf{A} \cdot \mathbf{h} \tag{3.2}$$

Being **A** the cylinder base and **h** the cylinder depth.

Measuring **F**, **h** and **A** density ρ can be obtained.

3.3.2 Materials

Figure 3.5 shows a picture where a test tube P is full to the brim with the tested liquid, the test tube rests on the plate of the scale B, the plastic cylinder is immersed into the test tube and a rod for supporting S where the cylinder is fixed by a threaded nut. The threaded rod attached to the cylinder has a washer A which measures the depth by using a millimeter scale E.

Table 3.1: Data on weight, depth and force for calibrating with distilled water. T (°C)=

	h(mm)	B(g)	h(m)	F(N)
1st	0			
2nd	10			
3th	20			
4th	30			
5th	40			
6th	50			
7th	60			
8th	70			

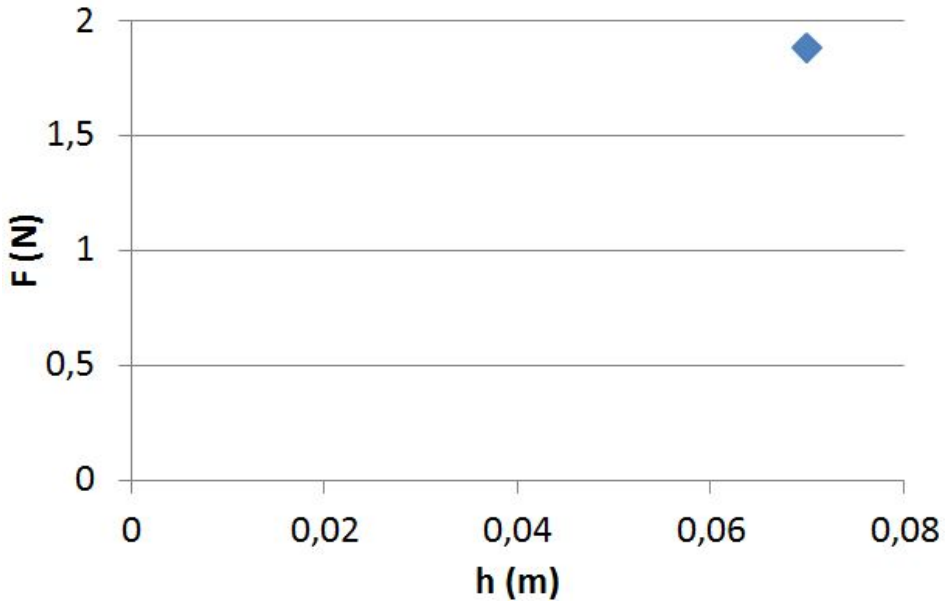

Figure 3.8: Graph F(N) vs h(m) and linear fitting of experimental data

3.3.3 Experimental method

1. Measure the diameter of the cylinder, D.
2. Tare the scale empty. Then, put the test tube on the plate of the scale B just under the cylinder C.
3. Fill the test tube P with liquid to the brim, and write down the initial mass.
4. Screw the threaded rod and the cylinder will go down until it touch the free surface of the liquid. Stop screwing when the reading of the initial mass is in the display of the digital scale again. The weight in the scale B(0) in grams corresponds to the immersed length of the cylinder h(0)=0.
5. Adjust the washer A of the threaded rod in the zero of the millimeter scale E without moving the cylinder. Visual-washer-millimeter scale.
6. Screw the threaded rod so as the cylinder goes down and dips into the liquid. Be careful, the washer should not turn with respect to the threaded rod. The liquid overflows and falls into the plastic tray that is sticked to the test tube.
7. Complete Table 3.1: h(i) and B(i) each 10 mm 10 (mm).

Calibration

Distilled water is the liquid used in the calibration because its density is known at different temperatures. In an Excel spreadsheet two first columns have to be filled up and depth h in meters and force F in Newtons have to be calculated: $F = B \cdot 10^{-3} \cdot 9,81$. Plot the line that best fits to the set of points result of graph F vs h, the equation of the straight line obtained is like this:

$$\mathbf{F = a + b \cdot h} \tag{3.3}$$

The characteristic parameters **a** and **b** and their uncertainties can be obtained by following the next steps:

1. Two columns are created for data: h_2(x)-F(y).
2. Ten empty boxes of the spreadsheet have to be highlighted (two columns and five rows)
3. Then it has to be written down "= estimación.lineal (y numbers: T; x numbers: R_2; verdadero; verdadero)".
4. Finally, click simultaneously: Ctrl-Mayus-Enter.

Table 3.2: Density of distilled water depending on the temperature. T(°C)=..... ρ=.....

T (°C)	ρ (kg/m^3)	T (°C)	ρ (kg/m^3)	T (°C)	ρ (kg/m^3)
15	999,19	20	998,29	25	997,13
16	999,03	21	998,08	26	996,86
17	998,86	22	997,86	27	996,59
18	998,68	23	997,62	28	996,31
19	998,49	24	997,38	29	996,02

Table 3.3: Calibration results

b (kg/s)	u (b)	R^2 (-)	g (m/s^2)	Δg	u (g)

Table 3.4: Data on A of different devices

Measurement	A_i	$(A_i - < A >)^2$
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
$< A >=$		XXXXXXXXX
SUM=	XXXXXXXX	
u(A)=		XXXXXXXXX

Table 3.5: Effective area (A) calculated by calibration and uncertainty

A (m^2)	u(A)(m^2)

Density of distilled water changes with temperature. Measure the room temperature and read the value of density in Table Table 3.2.

After obtaining the linear fitting with $R^2 > 0.98$, the slope **b** contains information about the effective area **A** of our measuring device. Complete Table 3.3

Comparing Equation 3.2 to Equation 3.3 it is obtained:

$$\mathbf{b} = \rho \cdot \mathbf{g} \cdot \mathbf{A} \tag{3.4}$$

The effective area of the cylinder **A** used in the measuring device can be calculated by the following equation:

$$\mathbf{A} = \mathbf{b} \cdot \mathbf{g}^{-1} \rho^{-1} \tag{3.5}$$

The error made in the determination of the effective area **A** will be estimated by calculating the uncertainty of type A, given the N number of data taken by different work groups (u(A)). Complete Table 3.4

Having consider the effective area of the measuring device and its uncertainty, a new experience can be carried on with the purpose of measuring the density of any other liquid, such as: milk, oil, serum.....

The effective area **A** can be compared with the area measured indirectly by the geometric area, $A_{geo} = (\pi/4)D^2$. Being the diameter of the caliper measured with a caliper. The geometric area, that will be calculated hereafter, has to be quite similar to the effective area **A**. Complete Table 3.6

Table 3.6: Geometric measurements

Parameters	D (mm)	D (m)	A_{geo} (m^2)
caliper measurements			

The relative difference between both areas (A_{geo} and **A**) does not exceed 2%, therefore the effective area **A** will be taken as the representative value of the measuring instrument for later determinations of the densities of other liquids.

Table 3.7: Direct measurements of weight (B) and depth (h) for a liquid. T (°C)= .....

	h(mm)	B(g)	h(m)	F(N)
1st	0			
2d	10			
3th	20			
4th	30			
5th	40			
6th	50			
7th	60			
8th	70			

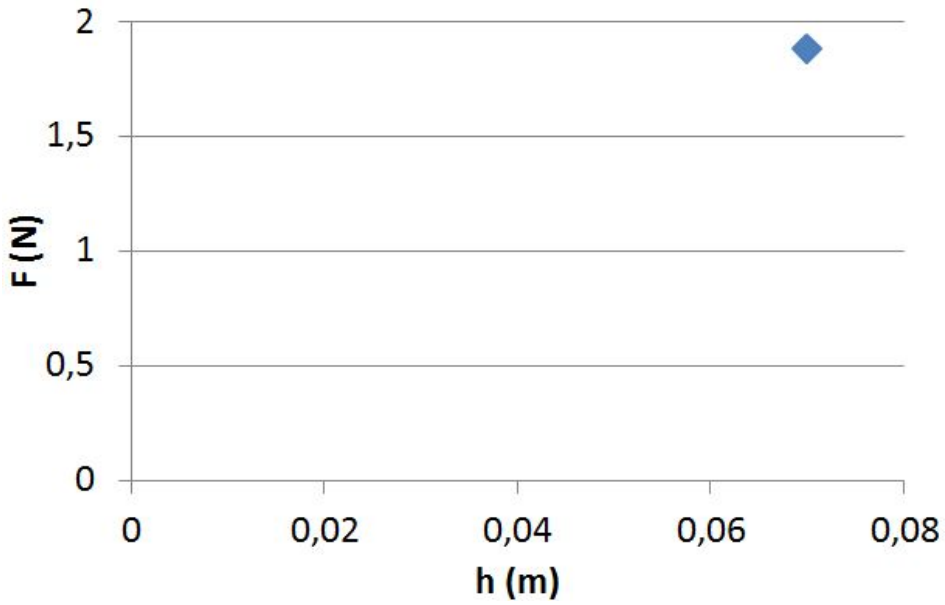

Figure 3.9: F(N) vs h(m) graph and the straight line that best fits the set of experimental data.

3.4 RESULTS AND DISCUSSION

3.4.1 Calculation of the density of a liquid

New readings are taken, following the experimental method describe in (page 31) for the new liquid whose density is going to be determined.
It is appropiate to copy the Excel sheet generated for the calibration into a new sheet called DENSITY. In this new sheet, boxes corresponding to the effective area and its uncertainty will be pasted as fixed numbers for their preservation(Table 3.6). Both A and u(A) are characteristic values of the measuring device, which have been obtained through calibration with distilled water and they keep invariable whatever the measured liquid.
New direct measurements will be typed in the data table, F will be recalculated and therefore the new set of data on F and h will be depicted in Figure 3.9. Linear estimation will be updated automatically (Equation 3.3). From which a new value of the slope (b) and its uncertainty u(b) will allow for obtaining the density of the measured liquid.

Density is obtained solving the Equation 3.4

$$\rho = \mathbf{b} \cdot \mathbf{g}^{-1}\mathbf{A}^{-1} \tag{3.6}$$

Calculated density: $\rho =$

3.4.2 Calculation of the uncertainties

1. The uncertainty of the effective area A has been determined with the calibration.

2. The uncertainty of the free fall acceleration ($u(g)$), (Appendix A, page 77):
   The acceleration is a non-exact number, if it is taken as a number with two decimal figures: g=9,81 (m/s^2).
   Its semi-interval of error is: Δg =0,01/2= 0,005 (m/s^2).
   Its uncertainty is: $u(g) = \Delta g/\sqrt{3}$

3. The uncertainty of coefficient b is given by the linear estimation.

Table 3.8: Previous measurements and their uncertainties

X	Measurement	ΔX	u(X)
A (m^2)		—	
g (m/s^2)			
b $(kg \cdot s^{-2})$		—	

$$u(\rho) = \sqrt{(u_A(\rho))^2 + (u_g(\rho))^2 + (u_b(\rho))^2} \tag{3.7}$$

$$u_A(\rho) = \frac{\partial \rho}{\partial A} u(A) =$$

$$u_g(\rho) = \frac{\partial \rho}{\partial g} u(g) =$$

$$u_b(\rho) = \frac{\partial \rho}{\partial b} u(b) =$$

Table 3.9: Indirect measurements and their partial uncertainties

ρ $(kg \cdot m^{-3})$	$u_A(\rho)$	$u_g(\rho)$	$u_b(\rho)$

Table 3.10: Measured density of a liquid and its estimated uncertainty

$\rho \pm u(\rho)$ $(kg \cdot m^{-3})$	$u(\rho)/\rho$ (%)

Uncertainty of the density ($u(\rho)$)

The uncertainty of the density is calculated by using Equation 3.7, because it is an indirect measurement.

The uncertainty of ρ is the result of the made errors in the different paramenters: during the measurement of the area of the cylinder, by taking g as a number with two decimal figures or even in the calculation of coeficient b.

To evaluate the contribution of each of these variables in the uncertainty ρ there must sum all the uncertainties of ρ with respect to each variable. The uncertainty of ρ due to a variable is the partial derivative of ρ with respect to this variable multiplied by the corresponding uncertainty of the variable.

Expand the partial derivatives and calculate the partial uncertainties. Finally calculate the uncertainty of the density as well as the relative uncertainty.

Table 3.11: Measurement table of ρ with N=12 repetitions

Measurement	$\rho(kg \cdot m^{-3})$	$(\rho_i - < \rho >)^2$
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
$< \rho >=$		——————-
SUMA=	——————-	
u(ρ)=		——————-

Table 3.12: Density of a liquid and its uncertainty calculated directly

$\rho \pm u(\rho)\ (kg \cdot m^{-3})$	$u(\rho)/\rho\ (\%)$

Uncertainty of the density considering a direct measurement

In order to determine a characteristic value of the density of the liquid, considering the random errors made by the different measuring devices, the following table can be completed, where we will calculate the error committed in the measurement of the density directly ((Appendix A, page 76)). This value will be used in the next lab.

Chapter 4

MEASUREMENT OF THE VISCOSITY OF A LIQUID

4.1 ABSTRACT

The viscosity is an index of friction between adjacent layers of a fluid, it appears when there is relative movement of fluids with respect to solids, fluids moving in pipes, or solid particles moving into fluids. In this lab we will study theoretically and experimentally the motion of water in a capillary tube (circular section of 1 mm in diameter approximately). A cylindrical tank is filled with water and then is drained through a capillary tube in other reservoir located below. Based on the experimental results we will obtain a relationship between the level of water in the lower reservoir and the time while water flows in the capillary. Using the Bernoulli equation for a real fluid, in which friction is considered as energy loss in the capillary tube (Eq. Hagen-Poiseuille) the viscosity of water is deducted with a relative uncertainty under 3%.

Figure 4.1: Viscosimeter of Ostwald. Source:Wikipedia

Figure 4.2: Digital rotational viscometer. Source: civiltest-aparato.com

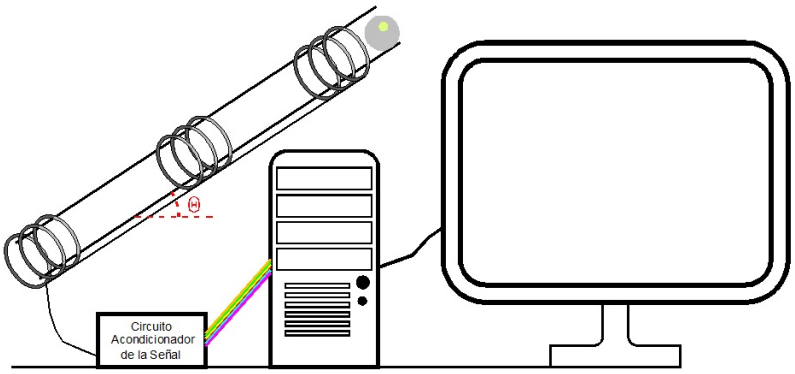

Figure 4.3: Ball drop viscometer. Coefficient of friction b=6·π·η·r

4.2 INTRODUCTION

The viscosity η relates the tangential force per unit area F/S over the surface S of a fluid and the change of velocity per unit length in the z direction perpendicular to the surface: F/S=η·(dv/dz). In recent literature, we can find several methods for measuring the viscosity of fluids, many of them based on the circulation of a fluid through a capillary tube, for example: the flow of a fluid through a capillary tube that can be done vertically or slightly inclined (Figure 4.1). Other methods are based on the tangential force it has to be done to get that some metal paddles rotate at a certain velocity inside a fluid, where definition of viscosity by Newton's law is applied, (rotational viscometer, Figure 4.2). Others are based on measuring the velocity at which a ball falls within a fluid in a straight tube that is inclined and applying Stokes' law (falling ball viscometer, Figure 4.3)

OBJECTIVES

In this lab the viscosity of liquids is measured by studying how a tank full of fluid is drained into another tank through a capillary tube by applying the laws of Bernoulli and Hagen-Poiseuille, and comparing theoretically and experimentally the filling of the lower reservoir. By means of the calibration with distilled water, a characteristic parameter of the measuring device **K** is calculated. This parameter K is used in the measurement of the unknown viscosity of other fluid.

Figure 4.4: Measuring device

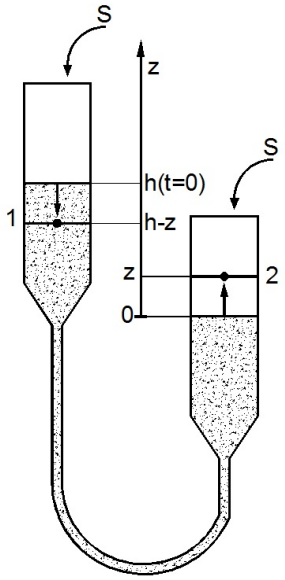

Figure 4.5: Physical model

Table 4.1: Geometric parameters of the measuring device and temperature of the distilled water

Parameters	mm	m	Parameters	mm^2	m^2
h			S		
L					
D				°C	K
R			T		

Table 4.2: Temperature-viscosity for distiled water

T (°C)	η (kg/(m·s))	T (°C)	η (kg/(m·s))	T (°C)	η (kg/(m·s))
15	0.001139	20	0.001003	25	0.000891
16	0.001109	21	0.000979	26	0.000871
17	0.001081	22	0.000955	27	0.000852
18	0.001054	23	0.000933	28	0.000833
19	0.001028	24	0.000911	29	0.000815

4.3 MATERIALS AND METHODS

4.3.1 Materials

Figure 4.5 depicts the experimental device. It consist of two calibrated test tubes, a silicon capillary tube with inside diameter of 1.5 mm, the measuring fluids, a chronometer, a measuring tape and a caliper.

4.3.2 Experimental method

1. Height h between tube 1 and 2 is measured. Inside diameter D is measured and its circular area S is calculated. Length L of the capillary tube is measured. Complete Table 4.1

2. Both tubes are placed at the same height (Figure 4.4). Fill one of the tubes with water up to the 50 mark. Put the piston into it. Push the piston in order to drag the water inside the capillary tube and remove all the air bubbles inside the capillary tube. Then, fill both syringes with water up the 0 mark level.

3. Temperature of distilled water is measured. It has to be written down in Table 4.1

4. Tube 1 has to be raised a height h with respect to the tube 2 (Figure 4.5). The chronometer will start (t=0) when the tube 1 is raised, the height z of the free surface of water in tube 2 varies over time. The time t it takes to achieve the different marks in the measuring tape in milimeters has to be written down in Table 4.2.

4.3.3 Physical Model

Applying Bernoulli's equation to the points 1 and 2, free surface, in a generic time t, taking as a height of reference z=0 in the point 2 (free surface to the fluid in the lower syringe):

$$\mathbf{P_0} + \rho \mathbf{g}(\mathbf{h} - \mathbf{2z}) + \frac{\mathbf{1}}{\mathbf{2}}\rho \mathbf{v_1^2} = \mathbf{P_0} + \frac{\mathbf{1}}{\mathbf{2}}\rho \mathbf{v_2^2} + \mathbf{\Delta P_{12}} \tag{4.1}$$

Where P_0 the atmospheric pressure, ρ density, g acceleration of gravity, h height of 1 above 2 at t=0, v the fluid velocity, and 1 and 2 are two points of the tube with the same velocity (continuity equation). ΔP_{12} is the energy loss per unit volume (pressure drop) in the capillary tube.

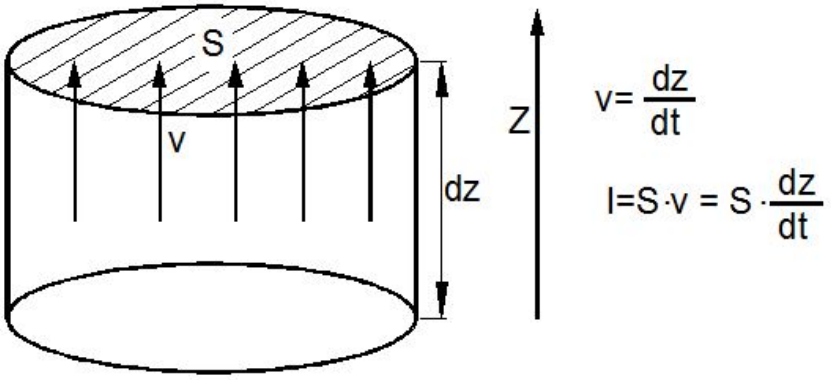

Figure 4.6: Volume flow rate

Question 1

Calculate the volume flow rate in the first interval of time (0-t_1).

$\mathbf{I_{max}(m^3/s)} = \frac{\mathbf{Volume}}{\mathbf{\Delta t}} = \frac{S\Delta z_1}{t_1} =$

If the capillary tube is 50 cm in lenght and it takes up 1 g of water which density is 997 (kg/m^3), calculate the radius of the capillary tube.

$\mathbf{r(m)} =$

Check if the fluid moves along the capillary tube in laminar flow by calculating the Reynolds number ($N_{RE} < 2000$), if viscosity of water is $\eta = 10^{-3}$ (Pa·s), its density is $\rho = 10^3$ (kg/m^3) and the radius of the capillary tube is r=0.75 (mm)

$$\mathbf{N_{RE}} = \frac{\mathbf{2} \cdot \rho \cdot \mathbf{I}}{\pi \cdot \mathbf{r} \cdot \eta} = \tag{4.2}$$

Question 2

Calculate the geometrical factor K of the measuring device, by means of Equation 4.7 from the experimental model and the geometrical data measured or estimated previously.

$\mathbf{K(m/s)^2} =$

Energy loss due to drag forces

The Hagen-Poiseuille expression allows us to calculate the energy loss per unit volume ΔP_{12} between the extremes 1 and 2 of a straight tube of length L, circular section of radius r, where a fluid of viscosity η circulates with volume flow rate I in laminar regime:

$$\mathbf{\Delta P_{12}} = \frac{\mathbf{8\eta L}}{\pi \mathbf{r^4}}\mathbf{I} \tag{4.3}$$

Replacing Equation 4.3inEquation 4.1results:

$$\rho \mathbf{g}(\mathbf{h} - \mathbf{2z}) = \frac{\mathbf{8\eta L}}{\pi \mathbf{r^4}}\mathbf{I} \tag{4.4}$$

Writing I in z function (Figure 4.6) and replacing:

$$\rho \mathbf{g}(\mathbf{h} - \mathbf{2z}) = \frac{\mathbf{8\eta L}}{\pi \mathbf{r^4}}[\mathbf{S}\frac{\mathbf{dz}}{\mathbf{dt}}] \tag{4.5}$$

Separating the variables dz and dt and integrating in the interval (0,z) and (0,t). Using the following initial condition: z = 0 at time t = 0. The resulting equation is:

$$\mathbf{ln}[\frac{(\mathbf{h} - \mathbf{2z})}{\mathbf{h}}] = -\mathbf{K}\frac{\rho}{\eta}\mathbf{t} \tag{4.6}$$

K is a characteristic geometrical constant of the device:

$$\mathbf{K} = \frac{\mathbf{gr^4}}{\mathbf{4LR^2}} \tag{4.7}$$

being R the radius of the circular section of the sryinge (S $=\pi R^2$).
So as to analyze the relationship between z and t, we will express time as a dependent variable and z as an independent variable, since z values are marked in the syringe, the Equation 4.6 it is obtained:

$$\mathbf{t} = -\frac{\eta}{\mathbf{K}\rho}\mathbf{ln}[\frac{(\mathbf{h} - \mathbf{2z})}{\mathbf{h}} \tag{4.8}$$

Changing variables as: y=t, x=$ln[\frac{(h-2z)}{h}]$ the equation Equation 4.8 can be written as the linear equation whose slope is: M=-$\frac{\eta}{K\rho}$ and the independent term N is nearly zero.

$$\mathbf{y} = \mathbf{Mx} + \mathbf{N} \tag{4.9}$$

Table 4.3: Measured data for calibrating with distilled water

z(mm)	time in chronometer	x = $ln[\frac{(h-2z)}{h}]$	y = t (s)
0			
5			
10			
15			
20			
25			
30			

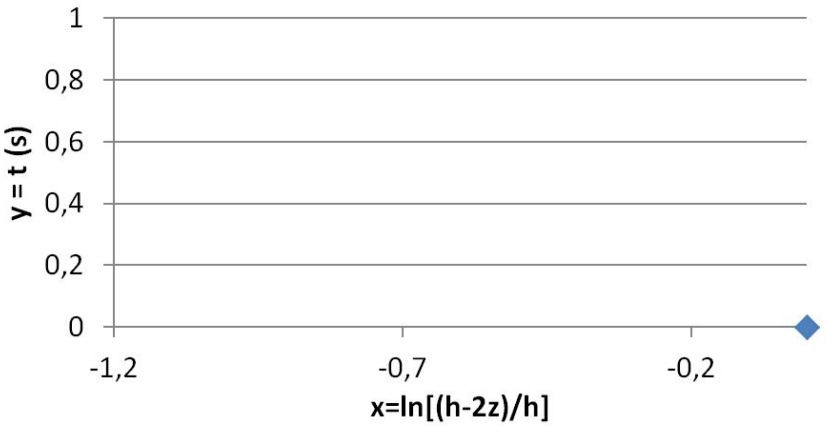

Figure 4.7: t(s) as a function of ln[(h-2z)/h(m)]

Table 4.4: Calibration table

Temperature	ρ (kg/m^3)	η (Pa·s)	M (s)	u(M)(s)

$$K = -\frac{\eta}{M\rho} =$$

Calibration. Measurement of k and u(k)

Distilled water is used for calibrating the measuring system. After taking readings about the time at certain heights, Table 4.3 has to be filled up, and then a graph in Figure 4.7 has to be represented. This Figure 4.7 depict the experimental data which are fitted to a linear equation. The steps to be followed, using the Excel spreadsheet, for finding out the characteristic parameters of the linear equation that defines the behaviour of distilled water in the measuring device and their uncertainties are:

1. Two columns are created for data: h_2(x)-F(y).
2. Ten empty boxes of the spreadsheet have to be highlighted (two columns and five rows)
3. Then it has to be written down "= estimación.lineal (y numbers: T; x numbers: R_2; verdadero; verdadero)".
4. Finally, click simultaneously: Ctrl-Mayus-Enter.

In boxes F1 and G1 are M and N, and in boxes F2 and G2 their uncertainties, respectively. In box F3 is the determination coefficient R^2.

It will be verified that the determination coefficient is $R^2 > 0,95$, and that the independent term is very low (N=0). The slope (M) is related with density and viscosity of the distilled water as well as with the characteristic parameter K of the measuring system. Both density and viscosity are tabulated as a function of the temperature. Therefore, the temperature is measured and written down along with the corresponding density and viscosity (Table 4.4).

Comparing Equation 4.9 to the Equation 4.8, it turns out that:

$$\mathbf{M} = -\frac{\eta}{\mathbf{K}\rho} \tag{4.10}$$

From Equation 4.10, K the geometric parameter of the measuring device is obtained.

$$K = -\frac{\eta}{M\rho} \tag{4.11}$$

This characteristic parameter K can be used in the same measuring device regardless the liquid is going to be measured. For this reason it will be necessary to determine its uncertainty. It is therefore important to determine its uncertainty, for which account is taken of the uncertainty of the coefficient M on which it depends. Actually the uncertainty of K should be afected also by the uncertainties of the density and the viscosity determined by a table whose input is the temperature measured in the laboratory, however their effect is difficult to assess indirectly.

Table 4.5: Data on K of different devices

Measurement	K_i	$(K_i - < K >)^2$
1st		
2nd		
3th		
4th		
5th		
6th		
7th		
8th		
9th		
10th		
11th		
12th		
$< K >=$		XXXXXXXXXXX
SUM=	XXXXXXX	
u(K)=		XXXXXXXXXXX

Table 4.6: Measured data for the liquid of unknown viscosity

z(mm)	time (chronometer)	x = $ln[\frac{(h-2z)}{h}]$	y = t (s)
0			
5			
10			
15			
20			
25			
30			

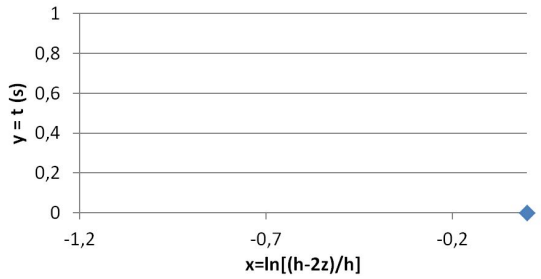

Figure 4.8: Graph of t(s)vs ln[(h-2z)/h(m)] for the liquid of unknown viscosity

4.4 RESULTS AND DISCUSSION

4.4.1 Direct measurement u(k)

Having considered all the calibrations done in N measuring devices in the laboratory, N=12 data on K (Table 4.5) can be used for calculating a direct uncertainty (Appendix A, página 76):

$$u\left(K\right) = \sqrt{\frac{\sum_{i=1}^{12}\left(\langle K\rangle - K_i\right)^2}{12\left(12-1\right)}} \tag{4.12}$$

We take K as the corresponding to our measuring device but we use the direct uncertainty that has just been calculated in the above paragraph.

4.4.2 Experimental measurements

The measuring procedure described in subsection 4.3.2 has to be followed for the liquid of unknown viscosity. Copy the Excel spreadsheet prepared for measuring and calibrating of distilled water is recomended, but in this case the K±u(K) determined in the calibration has to be fixed because it is a known characteristic parameter of the measuring system.

Following the steps of page 49 for the determination of characteristic parameters of a linear equation, the slope (M) will be obtained for the liquid of unknown viscosity. The knowledge of parameters such as: K (obtained from water) and density of the liquid of unknown (measured in the previous laboratory) (ρ) allows for computing the viscosity of the measured liquid.

The density of the liquid has to be known for the viscosity determination. Assuming the same experimental temperature, and being the same liquid, the density and the uncertainty will be taken from the previous chapter (these data have to be written down in Table 4.7).

Table 4.7: Previous measurements and their uncertainties

	Measurement	Uncertainty
K ($m^2 \cdot s^{-2}$) (water)		
M (-)		
ρ ($kg \cdot m^{-3}$)		

$$u(\eta) = \sqrt{(u_K(\eta))^2 + (u_M(\eta))^2 + (u_\rho(\eta))^2} \tag{4.13}$$

$$u_K(\eta) = \frac{\partial \eta}{\partial K} u(K) =$$

$$u_M(\eta) = \frac{\partial \eta}{\partial M} u(M) =$$

$$u_\rho(\eta) = \frac{\partial \eta}{\partial \rho} u(\rho) =$$

Table 4.8: Indirect measurement with partial uncertainties

η ($Pa \cdot s$)	$u_K(\eta)$	$u_M(\eta)$	$u_\rho(\eta)$	$u(\eta)$

Table 4.9: Measurement of the viscosity of a liquid

$\eta \pm u(\eta)$ ($Pa \cdot s$)	$u(\eta)/\eta$ (%)

Known M, K and ρ of the liquid is posible to calculate η from the Equation 4.10, according to:

$$\eta = -KM\rho \tag{4.14}$$

4.4.3 Calculation of the uncertainties

1. Uncertainty of K is calculated after the calibration, having considered every value of K obtained by the N measuring devices of the lab.
2. The uncertainty of the slope M is given from the "estimacion.lineal" developed in the Excel spreadsheet.
3. The uncertainty of the density is given by the direct uncertainty calculated for the set of N=12 measurements taken in the previous lab (Table 3.11 in page 38).

Uncertainty of the viscosity ($u(\eta)$)

As an indirect measure, the uncertainty of density is calculated from the Equation 4.13 (Appendix A, page 77).

The uncertainty of η is the result of the errors made when measuring parameter K (calibration), measuring the slope M for the liquid whose viscosity is unknown and measuring its density (page 38).

To assess the contribution of each of these variables in the uncertainty of η, the uncertainties of η with respect to each of these variables have to be computed by the product of the partial derivative of η with regard of each variable and the uncertainty of the corresponding variable.

Solve the partial derivatives and calculate the partial uncertainties (Table 4.8).

Finally, calculate the uncertainty of the viscosity (Equation 4.13) and its relative uncertainty. Complete Table 4.9 using the normalized writting rules.

Chapter 5

ULTRASOUND WAVES CHARACTERIZATION

5.1 ABSTRACT

Ultrasounds are sound waves of higher frequency than detectable by the human ear, which is set to 20 kHz. Ultrasound has many applications in the biotechnology area. On one hand low power ultrasounds are used for measuring: sonography, speed of fluids, etc., on the other hand high power ultrasounds are used for accelerating the dissolution of products, breaking cells, etc.. The oscilloscope is an electronic device used to represent signals (electrical potential difference) as a function of time, and to characterize its amplitude and its frequency if they are periodic, and its phase difference if they have the same frequency. In this lab, generated and received ultrasonic waves with frequency of 40 kHz will be characterized by the digital oscilloscope and the propagation velocity of sound in the air will be measured.

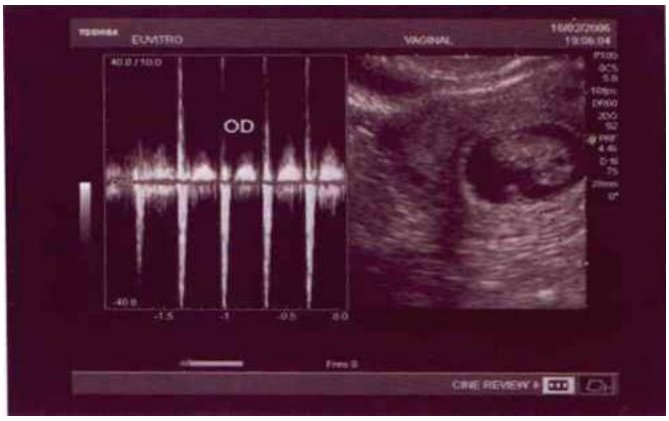

Figure 5.1: Ecography.

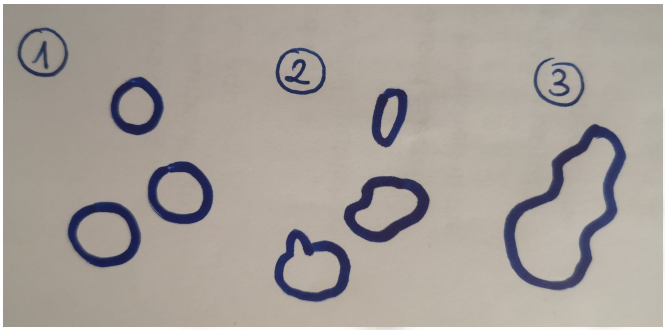

Figure 5.2: Cavitation of oily tissue particles.

Figure 5.3: Homogenization of an emulsion and degassing. Source: www.hielscher.com

5.2 INTRODUCTION

Ultrasounds are sound waves of frequency between 20 kHz to 200 MHz, this frequency is higher than that detectable by the human ear, which is set within the range [20 Hz - 20 kHz]. Infrasound frequency sounds are less than 20 kHz, which is below the audible limit of humans. Ultrasounds are generated and detected by piezoelectric crystals (eg. quartz). Piezoelectrics deform when an electrical voltage is applied to them and produce a voltage when they deform. If a sinusoidal alternating voltage is applied in a piezoelectric (emitter), it changes its shape and regains the original shape in a harmonic motion of the same frequency as the voltage. This crystal is a vibrating source that produces a harmonic simple motion of air molecules and therefore leads to variations in the density and pressure of air, these things cause the ultrasound wave. The receiver detects this pressure wave and it produces an electrical voltage of the same frequency.

There are two types of ultrasonic applications. Low power waves (less than 1 mW) are used to measure properties of bodies using the laws of propagation and to make images of the layers they pass through (sonography of a baby, Figure 5.1). For example, the position of a body, its size, its speed (pulsations, Figure 5.1) or ultrasonic absorption of waves, in different parts of a body, can also be measured with this technique. High-powered waves have mechanical effects such as cavitation in liquids, which can be used to break cell membranes (Figure 5.2), to dissolve homogeneously (Figure 5.3), to speed up chemical reactions, to clean up solids inside liquids, welding, etc.

The oscilloscope is an electronic device that allows you to observe and measure the electrical potential difference as a function of time, with high sample rates up to the order of gigahertz. Oscilloscopes can usually measure two signals in two different channels. The oscilloscope screen has a horizontal axis which represents time and a vertical axis which represents the signal amplitude in volts. It can also represent a signal on the screen on the vertical axis vs. the other signal on the horizontal axis.

OBJETIVES

The digital oscilloscope is used in the first part of this lab to characterize two electrical sinusoidal signals that are delayed in time. In the second part of the lab low power waves are generated and received. The main properties of these waves are observed and measured. With the help of the oscilloscope, the main parameters of emitted and reflected waves are characterized: amplitude, frequency and phase difference. Finally, the velocity of propagation of sound in air will be measured.

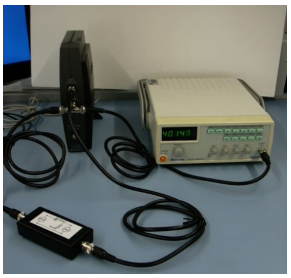

Figure 5.4: Mesuring device.

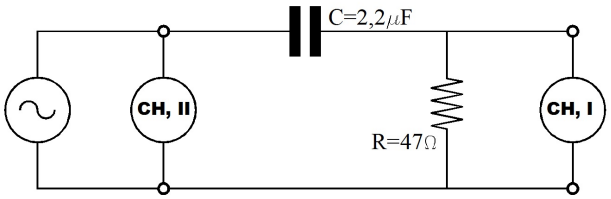

Figure 5.5: Wiring diagram.

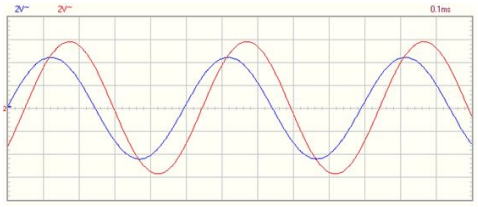

Figure 5.6: Graph of Voltage (V) as a function of time (s) in the oscilloscope.

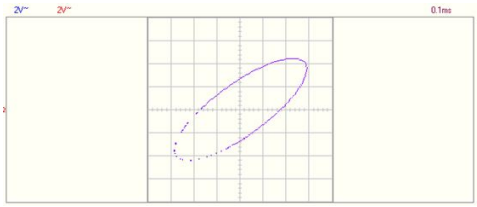

Figure 5.7: Graph of ChI vs ChII.

5.3 MATERIALS AND METHODS

5.3.1 Materials needed for the study of electrical signals in the oscilloscope

In Figure 5.4 the experimental device is shown. It consist of a generator of variable frequency signals, R-C circuit and coaxial wires and a two-channel oscilloscope. Circuit of Figure 5.5 has to be assemble. One signal is measured in Channel I (Ch I) and the generator signal is measured in Channel II (Ch II).

Experimental method

1. The generator has to be turned on with the following options: sinusoidal signal, key 10k, button 0.2, 2kHz and maximum amplitude) and the oscilloscope has to be switched on from the computer by program: PC-Lab200se/OK.
2. Press RUN and ON of Channel I (only **Ch I**, switch off the Ch II). A single signal appears, blue line in Figure 5.6. Press AUTOSCALE button so as to fix the wave and this channel will be taken as the reference wave.
3. Choose Time/DIV as 50 μs, and display only 1-2 waves.
4. Tune by Vol/DIV the maximum scale in the vertical axis for ChI that will be 1V.
5. Since the signal has to be symmetrical with respect to the horizontal axis, its position can be changed by using GND button and moving the cursor until the signal is over the horizontal axis.
6. Come back to the original signal by pressing AC key (todas teclas del Ch I.
7. Move the horizontal cursor until the crest is on the graduated vertical axis and then adjust the amplitude of the signal generator so as the peak reaches 3.8 V in the graduated scale of the oscilloscope.
8. Move the horizontal cursor until the wave starts in t=0.(Zero trigger level)
9. Press ON of the **Ch II**. Two overlapped waves appeared like Figure 5.6. Ch II is the red one. You do **not** press **Autoset** again.
10. Center ChII with GD key of ChII.
11. Chose the graduated scale Volt/Div just for seeing the highest amplitude of the signal.
12. Captur image (FILE/Save Image), save as Figure 5.8.

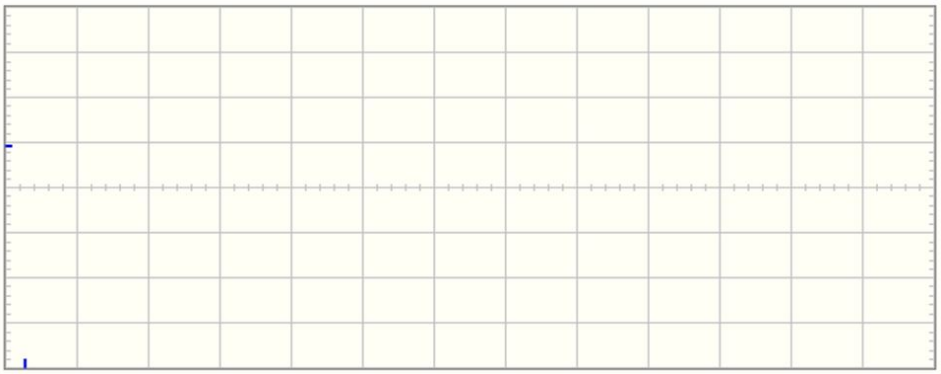

Figure 5.8: Graph of the Voltage (Volts) as a function of time for both channels

Table 5.1: Table of data

Medida	Y_i (div)	Y_i (Volts)	t_0 (div)	t_0 (s)	f_0 (Hz)	ω (rad/s)	φ (rad)
Ch I							
Ch II							

Is Ch II delayed or advanced with respect to Ch I?

td (s) = φ_d (rad)= $\omega \cdot t_d$ =

φ_{adv}=

φ_{del}=

φ_d=$\varphi_{adv} - \varphi_{del}$= φ_{II} (rad)=

5.3.2 Physical Model

Signal characterization over time

Print and stick the image of Figure 5.8 in page 60. Fill in Table 5.1 measuring all parameters in Figure 5.8.
The characteristic parameters are: Amplitude (YI, in Volts), angular frequency (ω in rad/s) and initial phase (φ en rad). Equation 5.1 and Equation 5.2.

$$\mathbf{y_I} = \mathbf{Y_I} \cdot \mathbf{sen}(\omega \mathbf{t} + \varphi_\mathbf{I}) \tag{5.1}$$

$$\mathbf{y_{II}} = \mathbf{Y_{II}} \cdot \mathbf{sen}(\omega \mathbf{t} + \varphi_\mathbf{II}) \tag{5.2}$$

The amplitude is measured in (DIV) between the horizontal axis (time) and the peak, then it is changed to Volts when it is multiplied by its scale Vol/DIV. You can move the wave to the left of the screen to use subdivisions more comfortably.

The period ($\mathbf{t_0}$)(in seconds) is measured by counting the number divisions along the horizontal axis between two times in which the wave appears once, and multiplied by the value of the TIME/DIV of the selected scale. The frequency is the inverse of the period $\mathbf{f} = \mathbf{t_0^{-1}}$. The angular frequency is calculated by multiplying frequency with 2π, so: $\omega = \mathbf{f} \cdot \mathbf{2}\pi$. The initial phase ($\varphi_\mathbf{I}$) of Ch I is zero, since signal at t=0 is zero.

The same calculation has to be repeated for Ch II.

The phase difference is the positive difference between the advanced phase and the delayed phase: $\varphi_\mathbf{d} = \varphi_\mathbf{adv}$ - $\varphi_\mathbf{del}$. The advanced phase is related to the wave which is above when time is zero, and the delayed phase corresponds to the wave that is below.

The $\varphi_\mathbf{d}$ has to be calculated by the following steps: first count the divisions between two successive intersections of the two waves with the time axis (when they have the same tendency), second calculate characteristic time (td) multiplying the counted divisions by the TIME/DIV scale, finally compute the $\varphi_\mathbf{d}$ by multiplying the td by ω.

For instance in Figure 5.6 ChI signal is above ChII signal when t=0, therefore $\varphi_\mathbf{adv}=\varphi_\mathbf{I}=0$ and $\varphi_\mathbf{del}=\varphi_\mathbf{II}$. When $\varphi_\mathbf{d}$ is calculated, the value of $\varphi_\mathbf{II}$ is easily obtained as $-\varphi_\mathbf{d}$, the minus sign shows the delay.

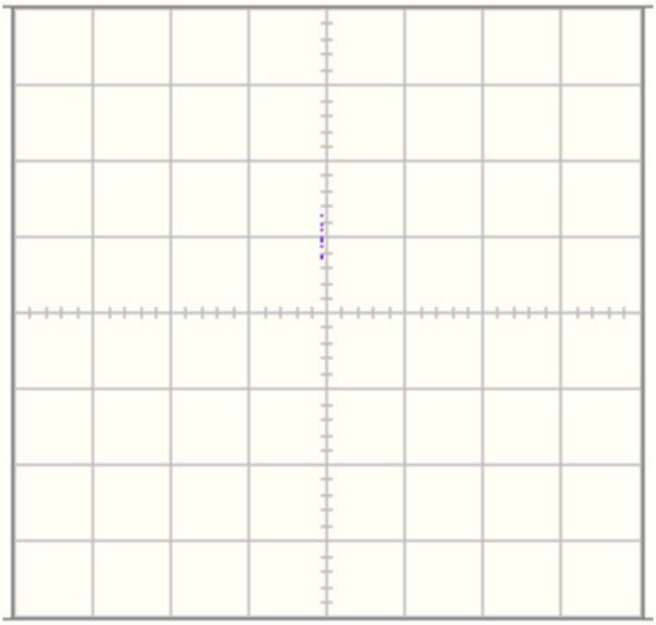

Figure 5.9: Graph ChI (Volts) vs ChII (Volts)

Table 5.2: Table of data

$\mathbf{y_{II}(t=0, y_I=0)}$ (DIV)	$\mathbf{Y_{II}}$ (DIV)	$\mathbf{sen\varphi_d = \frac{y_{II}(y_I=0)}{Y_{II}}}$	$\mathbf{\varphi_d(rad)}$

Lissajauss Elipse

In order to see another way of signal representation, the following steps must be followed:

1. Press Math/XY key. In this option the oscilloscope plots el osciloscopio Ch I in the vertical axis and Ch II in the horizontal axis. The result is an ellipse that looks like the one there is in Figure 5.7.
2. Center the elipse using GD comands of each channel. Press GND and change position of Ch I signal by moving the cursor until it is on the horizonal axis of the screen, then came back to AC. Press GND and change position of Ch II by moving the cursor until the signal is on the vertical axis of the screen, and return to AC.
3. Use PERSIST in case you want to enlarge the width of the line.
4. Save image (FILE/Save Image) for Figure 5.9
5. Print and paste the graph: Figure 5.9 in page 62.

The XY option plot Ch I vs Ch II. If the main axis of the ellipse has positive slope, Ch II is delayed, if not Ch II is advanced.

When t = 0 the signal of ChI is zero: $y_I = 0$ and the signal of ChII is equal to: $y_{II}(t=0) = Y_{II} sen\varphi_{II}$, then $sen\varphi_{II} = y_{II}(t=0)/Y_{II}$. The denominator is the amplitude of Ch II that is half the base of the rectangle which inserts the elipse, and the numerator is the value of the ellipse when $y_I = 0$ that is the point where the ellipse intersects with the horizontal axis.

Fill in Table 5.2 and obtain the phase difference following this graph Figure 5.9.

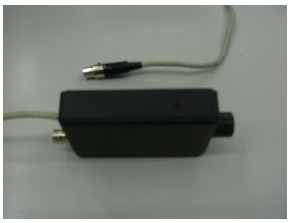

Figure 5.10: Ultrasound emitter

Figure 5.11: Ultrasound receiver

Figure 5.12: Power source of 30V

Figure 5.13: Signal generator

Figure 5.14: Two-channel oscilloscope

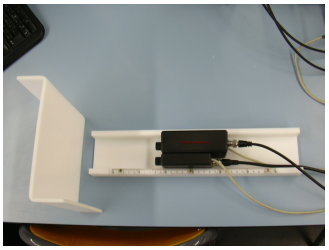

Figure 5.15: Placement of transmitter and receiver in the measurement

5.3.3 Materials for measuring the speed of ultrasounds

The device to produce, detect and characterize the ultrasounds is made of the following:

1. Ultrasound emitter (Figure 5.10). It has two connectors: one of them is a BNC type to plug in the signal generator which is variable with frequency and the other is composed by three conductor wires to plug in a power source.

2. Ultrasound receiver (Figure 5.11). It has a BNC connector to plug in ChII of the oscilloscope and so as to measure the electrical signal that ultrasound produces, and another of three wires to plug in the signal generator.

3. Power source supplies 30 V (Figure 5.12) so as to provide the piezoelectric with symmetrical signal (-15 V, 0 V, +15 V) for receiving and emitting ultrasounds.

4. Signal generator with variable frequency (Figure 5.13). Sinusoidal wave has to be selected and the frequency has to be chosen from the 100 kHz range and it has to be turned on at a maximum frequency of 40 kHz. The amplitude is $\frac{3}{4}$ of the maximum. In the output, it has two BNC connectors which are connected, one to the emitter and another to the ChI of the oscilloscope.

5. Two-channel osciloscope (Figure 5.14). Emitting signal is measured in ChI and receiving signal is measured in ChII. The oscilloscope is an electronic instrument used to represent electrical signals on the screen (electric potential difference) in time function, as well as compositions of signals, ie: a signal in function of another.

6. Plastic guide graduated in mm to slide the emitter-receiver set. They face the plastic screen which reflects the ultrasound waves (Figure 5.15). The first steps are made in a fixed position emitter-receiver. To measure the velocity of ultrasound the reflective screen must be approached.

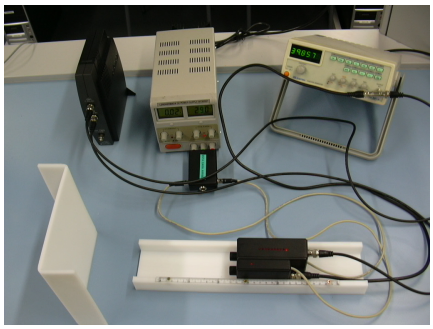

Figure 5.16: Ultrasound measuring device.

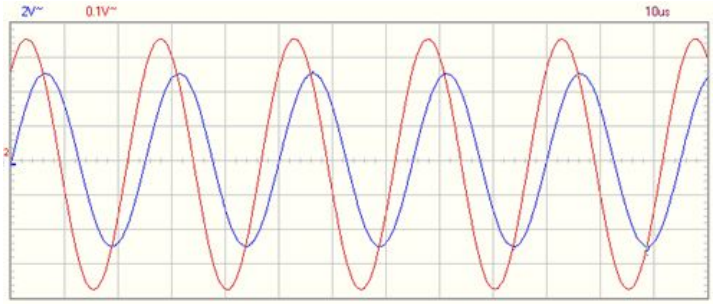

Figure 5.17: Electric signals as a function of time

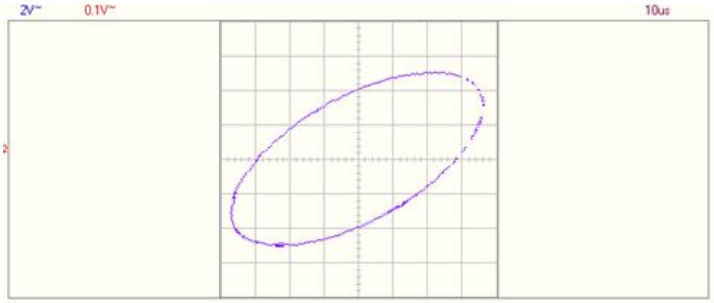

Figure 5.18: Graph ChI vs ChII

Experimental method for ultrasound wave characterization

First the circuit is connected as shown in Figure 5.16. Grey wires of emitter and receiver are plugged to the power source. The coaxial wire is connected between the emitter and the signal generator, just where there is a T-shaped connector. The other end of the T-shaped connector is attached to another coaxial wire that goes to the ChI of the oscilloscope. Turn on the power source (sinusoidal signal, key 100 K, button 40,000 kHz, amplitude to $\frac{3}{4}$ of the largest), power source (30 V) and oscilloscope (computer/PC-Lab2000se/OK). In order to define Equation 5.1 the oscilloscope signals have to be analyzed, the steps to follow are similar to those described in page 59.

Observation of ChI

First of all adjust Ch I (using ChI commands). Press buttons: RUN and AUTOSET while is only activated Ch I (ON). One signal is shown in blue, which is Ch I Figure 5.17. Only one or two cycles must appear by tunning Time/DIV to 5 μs. The greatest size of the signal has to be shown, so Vol/DIV control of Ch I has to be adjusted to 2V. Wave of Ch I has to be symmetric with respect to the horizontal axis, therefore using GND, the position command and the cursor the wave has be centered. Press SINGLE just as to freeze the image.

Observation of ChII

Press the button of Ch II ON . Place the emitter-receiver set at a certain distance from the reflective screen, until the signal of ChII (receiver) is advanced with respect to the signal of ChI (emitter) Figure 5.17). Both signals Figure 5.17. The signal of Ch II is in red. Center Ch II (analogous to Ch I, with its own commands). Choose the Vol/DIV scale so that the signal is well visible. Press SINGLE to freeze the image and save it as Figure 5.19.

Observation of ChI vs ChII

Select Math/XY plot, the graph shown in the oscillocope represents Ch I in the vertical axis and Ch II in the horizontal axis. It is an ellipse like that of the Figure 5.18. In order to center the ellipse the GND of Ch I will be press, then the Position will be adjust until the line is on the horizontal axis, then return to the ellipse by AC. Repeat the process but now with the ChII comands of the oscillocope. Finally press SINGLE to freeze the image and save it as Figure 5.20.
Undo the ellipse by pressing XY plot button and check that the saved image
in Figure 5.19 does not change, so that both images represent the same transmitter-receiver position.

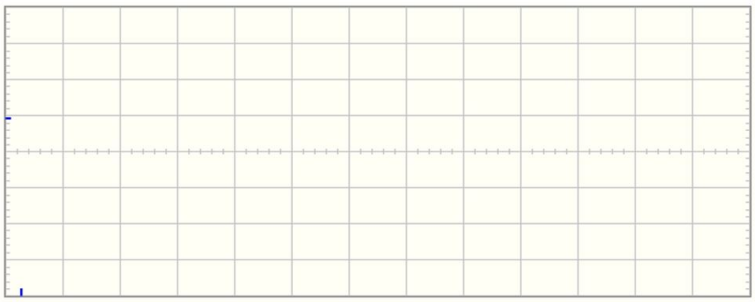

Figure 5.19: Voltage (Volts) vs time(s) for ChI and ChII that are connected to the emitter and receiver of ultrasounds

Table 5.3: Data table Figure 5.19

Medida	Y_i (div)	Y_i (Volts)	t_0 (div)	t_0 (s)	f_0 (Hz)	ω (rad/s)	φ (rad)
Ch I							
Ch II							

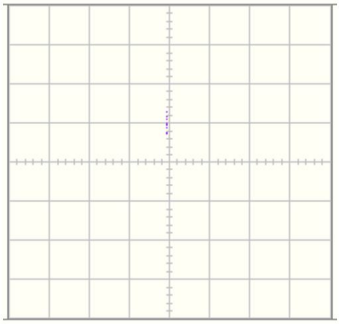

Figure 5.20: Representación ChI (Volts) vs ChII (Volts)

Table 5.4: Data from Figure 5.20

$y_{II}(t=0, y_I=0)$ (div)	Y_{II} (div)	$sen\varphi_d = \frac{y_{II}(y_I=0)}{Y_{II}}$	$\varphi_d(rad)$

5.4 RESULTS AND DISCUSSION

5.4.1 Characterization of the electrical signals being in the piezoelectrics

Print and paste the saved images for Figure 5.19 and Figure 5.20.

Fill out Table 5.3 and Table 5.4 the same way as Table 5.1 and Table 5.2 were completed, according to the instructions of pages 61 y 63.

5.4.2 Changes in the shape of Lissajous curve

The phase difference between two waves is dependent on the distance between the emitter-receiver and the screen. If we move the emitter and receiver together towards the screen, we will see that the shape of the ellipse changes due to the variation of the phase difference.
The emitted wave travels from the emitter to the plastic screen, when the wave reaches the screen it is reflected traveling in the opposite direction until reaching the receiver. Depending on the distance traveled by the wave, the received signal arrives with a different phase from the emitted signal. Sometimes the distance is such that both signals are in phase, and the shape of the Lissajaus curve appears as a positive straight line instead. In Figure 5.21 different shapes of the Lissajouss curve can be seen as regard of the phase difference between both signals.

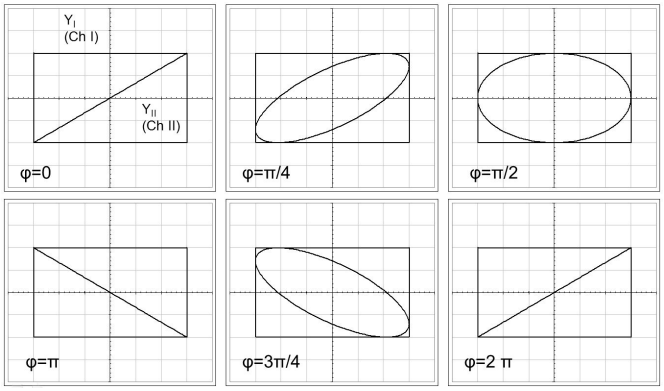

Figure 5.21: ChI vs ChII when they are connected to the emitter and receiver at different distances (x) from the screen

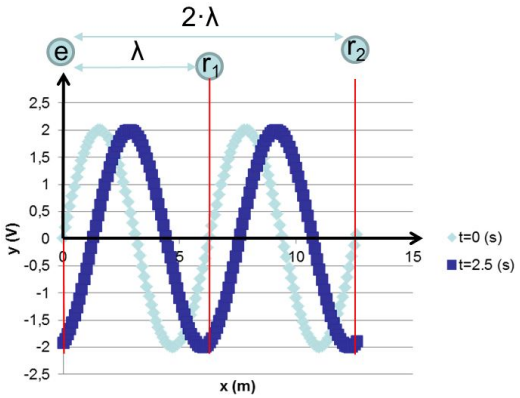

Figure 5.22: Voltage (Volts) vs distance x (m) at two different times

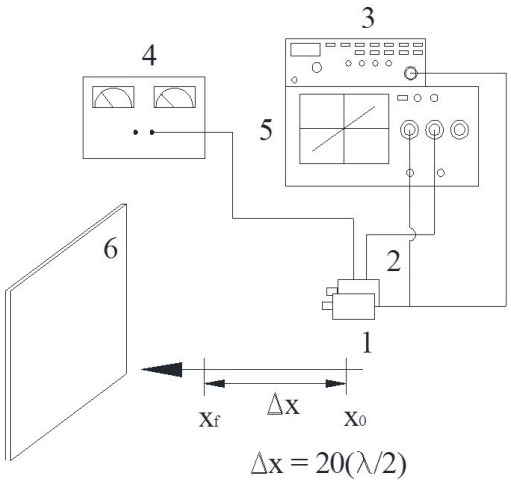

Figure 5.23: Assembly diagram emitter (1) and receiver (2) which approach towards the reflecting screen from a position x_0 when the signals are in phase. (3) signal generator, (4) power source, (5) oscilloscope, (6) reflective screen.

Table 5.5: Measuring data and results

Δx (m)	λ (m)	v (m/s)

If the distance travelled by the ultrasound wave from the emitter to the receiver is an integer number of wavelength, (λ) the amplitude of the wave in the emitter is the same as the amplitude found in the receivers r_1 y r_2, as it can be seen in Figure 5.22. As a result the received signal is in phase with the emitted signal, in that case the XY-plot will appear as a positive straight line. For any other intermediate distance in which there are a receiver, the received signal will be out of phase with respect to the emitted and the XY-plot will show an ellipse or a negative straight line.

5.4.3 Wavelength measurement

As we have seen, if we move the emitter-receiver set towards the screen, the Lissajous curve changes. When a positive straight line is plot in the oscilloscope, the emitted and received wave are in phase. x_0=0 will be assigned just at this point, and after that the emitter-receiver set will continue aproaching to the screen. When the positive straight line comes back, the waves are in phase yet again. This means that the wave covers one wavelength (λ) less than before (half wavelength to go and half wavelenght to return), so the emitter-receiver set has cover half of the wavelength.

If the total displacement is Δx, after seeing 20 times the positive straight line. It can be said that Δx =20·(λ/2), therefore the wavelength could be calculated by λ=Δx/10.

$$\lambda = \frac{\mathbf{\Delta x}}{\mathbf{10}} \tag{5.3}$$

5.4.4 Speed of sound in air

We already know the frequency of the ultrasonic wave f_0 (that is the same as the frequency of the signal generator) and the wave length λ from the last section.

The wavelength is the travelled distance by a wave in a propagation medium at the characteristic speed of the medium, during one period of time (t_0).

$$\lambda = \mathbf{v} \cdot \mathbf{t_0} \tag{5.4}$$

Since the period is related to the frequency as: t_0=f_0^{-1} and substituting its value in Equation 5.4, the speed is ease to compute by:

$$\mathbf{v} = \lambda \cdot \mathbf{f_0}$$

Appendix A

MEASUREMENTS AND UNCERTAINTIES

A.1 INTRODUCTION

Measurement of physical quantities and their treatment are an essential part of Physics. Measurement is the comparison of a physical quantity to another quantity called the basic unit. In Physics, there are basic units and others derived from the basic units. For example, the International System uses the meter (m), kilogram (kg), second (s) Ampere (A) and Kelvin (K) as basic units. Direct comparison between quantity and unit can only be carried out in certain cases, for example if we measure the length of a pencil we have measuring devices that allow us to know its length (ruler, Vernier Caliper ...) but we do not always have measurement equipment, if we want to measure less than one micrometer or more than one megameter we have to use physical laws that indirectly allow us to measure these quantities.

Direct measurements can be obtained with measuring instruments. Direct measurements are used in equations to obtain other measurements related to them, these results are indirect measures.

All measurements have errors. Errors can be human if their source is the person performing the measurement or instrumental if their cause is due to the measuring apparatus itself. Moreover these errors can be classified as systematic or accidental.

Systematic errors are repeated in all actions and you can remove them by calibrating the measuring system (person-instrument). Accidental errors can not be removed because they are not predictable. Accidental errors are random and they can not be corrected, we can only indicate the magnitude of these by a tolerance band around the result of measurement, the width of this band defines the uncertainty. The standard uncertainty (u) indicates the symmetrical range of a measure (m) so that the true value (measurand) is in to the interval with a certain probability.

m ± u(m) which it means [m-u(m), m+u(m)].

It has the same units as the measurement and is indicative of the quality of the measure, the smaller the uncertainty the better the quality of the measurement. It includes only random errors.

A.2 DIRECT MEASUREMENT AND UNCERTAINTY

A.2.1 Direct measurement from a single reading

MEASUREMENT BY SIMPLE COMPARATION (ruler, Vernier scale, balance with graduated weights)

There will be a single measurement when the measurand lies between two extreme values (X_1,X_2) between which the probability distribution is uniform (rectangular distribution). For example measurements with a ruler, a vernier or a balance with calibrated weights.

The measuring system certainly gives the interval in which the measurand is included, the range of extremes[X_1,X_2]. Measure is given as the mean interval (average) (Equation A.1)centered in the symmetric interval whose semi-interval of error is Equation A.2.

$$< \mathbf{X} >= \frac{\mathbf{X_1} + \mathbf{X_2}}{\mathbf{2}} \tag{A.1}$$

$$\mathbf{\Delta X} = \frac{\mathbf{X_1} - \mathbf{X_2}}{\mathbf{2}} \tag{A.2}$$

If the measurement appears as $X \pm \Delta X$, the true value of the measurement is in this interval with 100% of likelihood. So as to turn the semi-interval error into a standard deviation, we must bear in mind that the probability distribution of the measurand is

uniform throughout the interval [X_1,X_2](rectangular distribution). In order to get a 58% likelihood, the type B standard uncertainty is defined as:

$$\mathbf{u_B(X)} = \frac{\mathbf{\Delta X}}{\sqrt{\mathbf{3}}} \tag{A.3}$$

If a measurement is shown as X $\pm$ u_B(X) we can say with a 58% of probability that the true value of the measurement is in this interval ($1/\sqrt{3}$=0,58). This 58% de probabilidad el verdadero valor de la medida se encuentra en este intervalo. Nevertheless, this likelihood is equivalent to 68% of probability corresponding to the interval in which data have normal distribution.

MEASURING WITH CALIBRATED EQUIPMENT BY THE MANUFACTURER

When the measurement is performed by a complex electronic device, we can read the measurement on the device's screen and it is contained in an interval due to accidental errors. In this case the semi-interval error Δ X is determined after a calibration process, and how to calculate it depends on the type of device:

- Pointer instruments:
$$\Delta X = \frac{C}{100} \cdot FE + g \cdot (D_{min})$$
Being: C, class number; FE, maximum value of the measuring range; g, reading factor (0,$\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$,1); D_{min}: minimum division.

- Digital measuring instruments:
  - $$\Delta X = \frac{\delta_1}{100} \cdot X + C \cdot (FE)$$
  Being: δ_1,accurancy; X, measurement reading; C, class number; FE, maximum value of the measuring range.
  - $$\Delta X = \frac{\delta_1}{100} \cdot X + N \cdot R$$
  Being: δ_1, accurancy; X, measurement reading; N, number of digits (1,2,3..); R, unit of the last digit.

In this case the measurement is with high probability (near to 100%) in the range of X $\pm\Delta$(X).

It also supports a uniform distribution of probabilities and the standard uncertainty of type B is also: $u_B(X) = \frac{\Delta X}{\sqrt{3}}$.

This measure is written as X $\pm$ u_B(X) and it determines a symmetric interval around the measured X in which the measurand lies with a 58% likelihood.

A.2.2 Direct measurement N repetitions

N measurements will be made, when reading the device, each measurement is different. There will also be several measurements when the measurement process includes human error far greater than the uncertainty of the measuring apparatus, for example, measuring a time interval by a timer managed by one person.

In general if the measurement process includes random factors affecting the measurement, in addition to the measurement device itself. Let's assume that the N measurements obtained follow a normal distribution.

If the frequency distribution is a normal frequency distribution, the graph looks like a bell, symmetric around the core of the values obtain most often, as we can see at the figure:

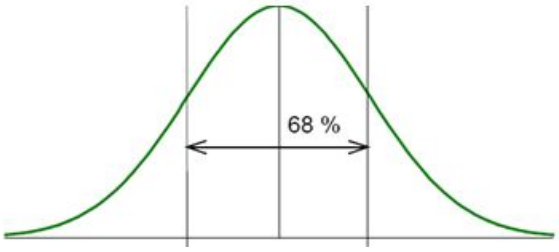

Figure A.1: Frequency distribution

The average of the measurement is calculated as:

$$< \mathbf{X} >= \frac{\mathbf{1}}{\mathbf{N}} \cdot \sum_{\mathbf{i=1}}^{\mathbf{N}} \mathbf{X_i} \tag{A.4}$$

The standard uncertainty, associated with a direct measurement with N repetitions is determined by the dispersion of values of the mean value, it is called standard uncertainty of type A, and is calculated as:

$$\mathbf{u_A(X)} = \sqrt{\frac{\sum_{\mathbf{i=1}}^{\mathbf{N}} \left(\langle \mathbf{X} \rangle - \mathbf{X_i}\right)^{\mathbf{2}}}{\mathbf{N\,(N-1)}}} \tag{A.5}$$

Symmetric interval determined the measurerement and its uncertainty X $\pm$ u_A(X) has a 68% chance of containing the measurand..

If you make a single measurement, uncertainty of the measurement is of type B. If you make N repetitions of the same size and uncertainty of the measuring apparatus is much less than the uncertainty associated with the process of measurement, uncertainty is considered as type A. If the uncertainties of the measurement apparatus and process are the same order you calculate the combined standard uncertainty:

$$\mathbf{u_C} = \sqrt{\mathbf{u_A^2} + \mathbf{u_B^2}} \tag{A.6}$$

The interval determined by the measurement <X> $\pm$ u_C(X) and its combined uncertainty will have a 68% likelihood of containing the measurand. Hereafter the standard uncertainty accompanying the measure is of type A, B or combined. It will be written as u(X) where X is the measurement.

A.3 INDIRECT MEASUREMENT AND UNCERTAINTY

The measure is called indirect measurement Y if it is calculated through a function that depends on other variables measured directly X_i, and their combined standard uncertainty are known.

$$Y(X_i) = (X_1, X_2, ..X_N)$$

Uncertainty is calculated considering direct measures are independent and their uncertainties are small, so they may be considered as differentials:

$$\mathbf{u}(\mathbf{Y}) = \sqrt{\sum_{\mathbf{i=1}}^{\mathbf{N}} (\frac{\partial \mathbf{Y}}{\partial \mathbf{X_i}} \mathbf{u}(\mathbf{X_i}))^{\mathbf{2}}} \tag{A.7}$$

A.4 STANDARD DEVIATION FOR NON EXACT NUMBERS

The error range for a non-exact number depends on the number of decimal places that are taken for calculation. For example, if we take number π with n=2 decimal places the semi-interval of error will be equal to: $\Delta X = 5 \cdot 10^{-(n+1)}$

Taking into consideration that it has a uniform probability distribution, the standard uncertainty will be: $u_B(X) = \frac{\Delta X}{\sqrt{3}}$.

In many situations the uncertainty of number π is not taken into account because the use of a lot of decimals (n>10) in the calculator or worksheets has a negligible effect on the uncertainty of the indirect measurement.

A.5 EXPANDED UNCERTAINTY AND RELATIVE UNCERTAINTY

The expanded uncertainty is defined as the product of combined standard uncertainty and the coverage factor k ($U_k = k \cdot u_X$) This is done to raise the likelihood of finding the measurand in the interval X $\pm$ U_k(X)

- If k=1 the true value of the quantity lies in the interval X $\pm$ U_1(X) with 68% likelihood.
- If k=2 the true value of the quantity lies in the interval X $\pm$ U_2(X) with 95% likelihood.
- If k=3 the true value of the quantity lies in the interval X $\pm$ U_3(X) with 99% likelihood.

It is useful to express expanded uncertainty as a relative uncertainty in percentage as: (u(X)/X)·100. This value allows us to analyze the measurement quality. If the relative uncertainty is less than 1% of the measurement it is a very good measurement. Between 1 and 5% are considered acceptable values. This are the values commonly found in labs.

A.6 WRITING RULES

A.6.1 Uncertainty

- If the first significant number of the uncertainty is higher than 2, the uncertainty will be written as an integer of one digit multiplied by base ten to the corresponding power. So after calculating the uncertainty, its significant number has to be rounded up, to leave the number correctly written with an integer (from 3 to 9), multiplied by base ten to the corresponding power. Example 1: u=0,005820 (kg) will be written as $6{\cdot}10^{-3}$ (kg).
- If the first significant number of the uncertainty is lower than 3, the uncertainty will be written as an integer of two digits multiplied by base ten to the corresponding power. As always, it has to be rounded up, to leave the number correctly written with an integer (from 10 to 29), multiplied by base ten

to the corresponding power. Example 2: u=0,00245 (m) will be written as $25 \cdot 10^{-4}$(kg).

A.6.2 Measurement

- Measurements must also be written with a scientific notation with the same power as the uncertainty and they must be written as a whole number (without decimals).
- Therefore so the last digit that appears in the units has to be rounded up.
- Following the Example 1: m=32,0289 (kg) will be

  m $\pm$ u(m) = [32029$\pm$6] $\cdot 10^{-3}$ (kg)

  It can be written as well as: 32,029 $\pm$ 0,006 (kg).

- Following the Example 2: d=0,05458 (m) will be

  m $\pm$ u(d) = [546$\pm$25] $\cdot 10^{-4}$ (m)

  It can be written as well as: 0,0546 $\pm$ 0,0025 (m).