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Dynamic lot sizing model for retailers with multi suppliers, quantity discounts, and capacity constraints that consider advance demand informations

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Abstract:

The Dynamic lot sizing (DLS) model is widely used in production planning, to minimize inventory levels, which basically will minimize production costs. The DLS model tries to eliminate the assumption of a fixed demand level throughout the period used in the Economic Order Quantity (EOQ) model. The characteristics of business processes in online retailers are the basis of this research to develop DLS models. This research develops a DLS model for retailers with multi-supplier cases, quantity discounts, and capacity constraints that consider Advanced Demand Information (ADI). Based on the numerical test results, it appears that ADI has an effect on reducing total production costs in the DLS model. The results of the numerical tests also showed that the model was able to solve the problems of the production planning process for SMEs and retailers who considered ADI.

Key words:

Dynamic lot sizing, advance demand information (ADI), multi suppliers, quantity discounts, capacity constraints.

1. Introduction

Intensify competition between companies, shift from the seller's market towards the buyer's market, and allow customers to choose products freely. This competition requires business people to reduce operational costs while maintaining quality and service to consumers (Dombrowski, et al. 2017). This research is motivated by the problems faced by many micro, small, and medium enterprises (MSMEs) in Indonesia. Based on data from the Ministry of Cooperatives and Small and Medium Enterprises, MSMEs in Indonesia contribute 57% to the gross domestic product (GDP). Most of these businesses are engaged in online retailers, which have dynamic demand levels and need to plan inventory levels and demand to meet consumer demand with minimal production costs.

Optimization of inventory and demand levels is one way to control production costs so that companies continue to generate profits while still meeting quality standards according to customer demand (Randall et al., 2006). Purchasing goods, as one of the most strategic activities in supply chain management, can provide an opportunity to reduce production costs and increase profits for companies (Mazdeh, et al., 2015).

Based on the characteristics of most MSMEs in Indonesia as well as the system used as the object of research, this research developed a Dynamic Lot Sizing (DLS) model to optimize production planning and control for MSMEs. The problem faced by many MSMEs is limited production capacity and supply.

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Contributions made from this study include considering multi-supplier factors, quantity discounts, capacity limits, and Advanced Demand Information (ADI) for online retail cases in the DLS model. Thus, this research is expected to make a significant contribution to the understanding and application of dynamic lot sizing.

2. Literature review

This research focuses on three research scopes: production and inventory, suppliers, and online retail. The model of production-inventory integration was first proposed by Ford W. Harris in 1913, who developed a model now known as Economic Order Quantity (EOQ). Since then, many researchers have developed EOQ and Economic Production Quantity (EPQ) models with various characteristics and variables. The basic model of EOQ has the objective function of minimizing message costs and storage costs with a fixed or assumed constant number of requests throughout the period. The EOQ policy makes the costs incurred by the company for the production process optimal.

Wagner and Whitin (1958) explain that when the assumption of a fixed level of demand throughout the planning period is omitted and inventory costs vary from one period to another, the square root formula (which is applied to the average of overall demand and costs) no longer guarantees producing a solution with minimum costs. Wagner and Whitin saw the weakness of the EOQ model: that the amount of demand is fixed or assumed to be fixed, which often does not correspond to reality. To overcome the problem of varying demand and cost, Wagner and Whitin began to develop the Dynamic Lot Sizing (DLS) model, which is now being developed by many other researchers.

In fact, decisions in lot sizing problems have two main characteristics that are often overlooked in the development of a lot sizing model: stochastic demand characteristics and rolling-horizon-based planning that can influence lot sizing decisions (Bodt, et al., 1984). Dynamic lot sizing problems often arise as short-to medium-term production scheduling for single-item problems, which will essentially be a procedure for multi-items with capacity constraints (Lotfi & Yoon, 1994).

The classification of problem types in lot sizing can be based on several criteria, including the number of machines, the number of production levels, capacity limitations, the length of the production period, and so on. Broadly speaking, production system problems for lot sizing can be categorized into two types: big-time bucket and small-time bucket (Brahimi, et al., 2006). The two categories have differences in terms of the length of the planning period; the small-time bucket consists of planning with an hourly or daily period, while the big-time bucket plans with a weekly, monthly, or yearly period. Brahimi et al. (2017) classified the types of problems in the case of lot sizing using several parameters, including the degree of information, planning horizon, time scale, number of items, number of levels, associated costs, resource constraints, service policies, time-consuming activities, and goal functions.

Table 1. Lot Sizing Problem Classification (source: Brahimi et al., 2017).

Parameter	Classification
Degree Information	Deterministic, Stochastic
Horison	Limited, Unlimited
Time Scale	Discrete (small period, large period), Continuous
Number of Items	Single Item, Multi Item
Number of Levels	Single Level, Multi Level (Serial, Tree, etc)
Related Costs	Setup Related (startup, ordering), Inventory Related (Saving Cost, Backlogging, Lost Sale), Capacity Related (Regular Hours, Overtime, Subcontracting)
Resource Limits	Sum (Single, Multi), Type (Constant, Varies)
Service Policy	No Delays, Backorders, Lost Sales, Subcontracts
Time-consuming Activities	Setup Time (Setup Minor, Setup Major), Processing Time (Zero, Con-stant, Varies), Lead Time, Transport Time
Purpose	Cost Minimization, Service Level Maximization, Smooth Production Load, Profit Maximization

Xu, et al. (2017) saw a change in the direction of the retail business towards an online sales system. For this reason, the study tried to develop a dynamic lot-sizing decision-making model for sales through online channels by utilizing ADI. The research focuses on how information obtained through online sales can be used to determine the optimal lot size. Ghaniabadi & Mazinani, (2017) Ghaniabadi and Mazinani (2017) developed a DLS model by considering multi-supplier, quantity discount, and backlogging in a dynamic lot sizing model. The research saw that the current model did not include these three factors, so to solve it, a model was made that could include all three factors. The resulting model has an objective function to minimize total inventory quantities and purchase costs.

Boctor (2022) presents two heuristic models for minimizing setup and inventory holding costs in single-machine capacitated lot-sizing and scheduling problems with delivery dates and quantities. It aims to minimize setup costs and inventory holding costs, and it proposes two solution heuristics. Another piece of research on the DLS model presented by Gati and Banyai (2023) investigates the effectiveness of the Wagner-Whitin and Silver-Meal algorithms in solving the problem of dynamic lot sizing and demonstrates that significant cost savings can be achieved using both algorithms.

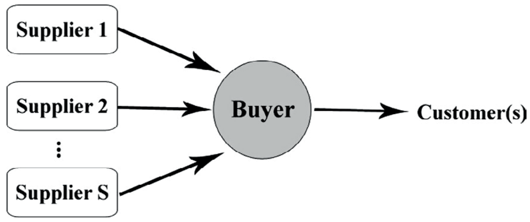

Figure 1. Multi Supplier Model Illustration (source: Mazdeh et al., 2015).

Mazdeh et al., (2015) stated that when the supply capacity of suppliers is unlimited, the decision to buy from more than one different supplier in the same period is not optimal. Jaruphongsa et al., (2005) develop an algorithm for dynamic lot sizing problems by considering multiple suppliers without quantity discounts. The variable to consider is when retailers have a choice of suppliers and modes of transportation, taking into consideration the capacity and cost of each mode of transportation. Zhao & Klabjan (2012) developed a dynamic lot sizing model by considering the selection of several suppliers simultaneously who have fixed costs and variable costs. The DLS model, with consideration of multiple suppliers and the presence of quantity discounts, was developed by Ghaniabadi and Mazinani (2017). The research developed a DLS model by combining multi-supplier, quantity discounts, and backlogging simultaneously.

2.1. Supplier selection

Today, a manufacturer is free to choose several suppliers as an option to supply the raw materials it needs. The selection of several existing suppliers is one of the other considerations that need to be considered so that the production process can run well and optimally. Most literature studies consider only a single supplier. The existence of a single supplier is the simplest assumption and is used in basic research for dynamic lot sizing problems. The existence of several suppliers in a production process becomes a natural thing and can be a valid assumption in reality (Bai & Xu, 2010). Some studies related to supplier selection include Bai & Xu (2010), Lee et al., (2013), and Mazdeh et al., (2015). The selection of suppliers gives companies a number of choices to determine the origin of the supply of raw materials needed. Figure 1 shows a model in which a company can place orders with multiple suppliers. Each supplier has a different product pricing scheme with different rebate schemes.

2.2. Discount quantity

Quantity rebates are a fundamental pricing strategy in the retail industry. The policy of offering rebates to customers has become a very important area of research in supply chain management (Zhou, 2007). A quantity price discount is a policy given by product sellers to consumers for purchases of a certain quantity. The greater the quantity of product purchases by consumers, the greater the discount given by sellers to consumers. It is normal to consider rebates in the matter of lot sizing. In the case of quantity-based discounts, when the buyer places a large order, the supplier will reduce the purchase price based on a predetermined price scheme (Mazdeh, et al., 2015).

The quantity discount issue determines the number of orders in a dynamic environment, i.e., the number of demand levels changes over time, orders are made periodically, and rebates are available for purchases of a certain amount (Chung, et al., 1987). The research resulted in an efficient algorithm to produce optimal solutions when the all-unit quantity discount scheme was applied. Federgruen & Lee (1990) developed algorithms for dynamic lot sizing by considering two types of discount schemes: all-unit quantity discount and incremental discount.

An all-unit quantity discount provides a discount if the quantity purchased is included in the quantity level provided by the supplier, and the discount is given starting from the first unit. An incremental quantity discount provides a discount for a certain number of units that are included in one price lag and different prices for units that are included in other price breaks (Lee, et al., 2013).

2.3. Advanced demand information

The cost structure of both retail store and warehouse e-fulfillment strategies will differ signifcantly, and that the addition of new processes to retailers daily operations such as the cost of picking and the cost of delivery brings a great challenge for online sales cost management (Rodriguez-Garcia et al., 2023). One piece of information that helps to improve performance related to customer needs in the future especially in online retailer is known as advance demand information (Tabar & Sahin, 2015). In real systems, ADI refers to a customer who orders a product before the desired or expected deadline, so that a customer places an order early so that the company has information on the number of future requests. The time between the order and the goods that must be received by the customer is called the demand lead time.

Xu et al. (2017) developed a DLS model by utilizing ADI for retail cases with online sales channels. The research produced a model that can solve DLS problems for retail cases with online sales channels. Online sales are simplified with conditions that are homogeneous customer characteristics with a number of lead times L. This scenario only categorizes customers into one type, or the absence of customers with preferential treatment. Retailers in this category are usually small and medium-sized businesses that generalize all their customers so that retailers treat all customers equally without privileges.

Orders with online channels come during the t period, then are delivered to consumers with a lead time during L; the tolerable delay is during G. The company has the option to deliver orders to consumers during the t + L period or during the t + L + G period. Orders that are successfully fulfilled during the t + L period do not result in additional costs for the company, while if the product is fulfilled during the t + L + G period, there are a number of late costs that must be incurred by the company. An illustration of the ordering process with the concept of an online channel is illustrated in Figure 2.

The initial inventory amount is assumed to be zero ($I_o = 0$). The demand for a number of products that come in period t is denoted as dt. A number of requests are notated as v_t^i, i = 0, …, L+G-1 Defined as a number of requests that come in the period before period t that have not been fulfilled in period t. As example v_t^0 represents the number of unfulfilled requests in period t that are part of dt, while v_t^1 represents the number of requests that have not been fulfilled in the period t-1. All requests that cannot be fulfilled in the current period will be transferred as advance demands at the beginning of the next period.

Figure 3 illustrates an illustration of the addition process by considering Advance Demand Information (ADI) in a homogeneous customer scenario. Number

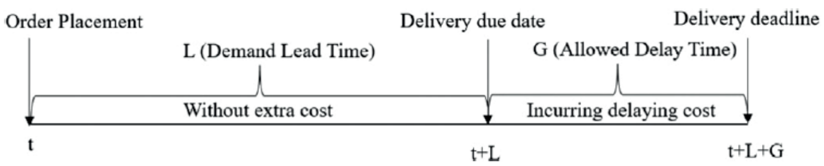

Figure 2. Online Order Process Delivery (source: Xu, et al., 2017).

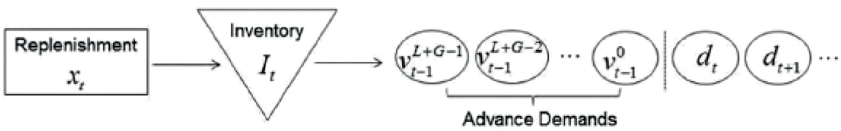

Figure 3. Inventory Model with ADI dan Homogen Customer (source: Xu, et al., 2017).

Table 2. Research Position.

Name	Sales Channel		Supplier Selection		Price Scheme			
	Conventional	Online	Single	Multi	Incremental Discount	All Unit Quantity Discount	ADI	Capacity Constraints
Florian M. (1971)	✓		✓					✓
Federgruen A. (1990)	✓		✓		✓	✓		
Chyr (1999)	✓		✓			✓		
Hu J. (2002)	✓			✓	✓			
Moqri M. (2001)	✓			✓				
Zhao Y. (2012)	✓			✓				
Lee A. (2013)	✓			✓	✓			
Choudhary D. (2014)	✓			✓	✓	✓		
Brahimi N. (2015	✓							✓
Mazdeh (2015)	✓			✓	✓	✓		
Xu H. (2017)		✓	✓				✓	
Ghaniabadi (2017)	✓			✓	✓	✓		
Gati (2023)	✓	✓	✓					✓
Ferdian R. (present)		✓		✓		✓	✓	✓

of additions in the period t notated with xt, While the amount of inventory in the period t notated with It. all requests are accommodated on a First Come First Served (FCFS) basis, which means that first-come, first-served requests will be served first.

Nakade & Seino (2021) discuss two types of imperfect advance demand information, where the actual arrival time of demand is stochastic and may not occur with a given probability. This research formulates the models as a Markov decision process. The optimal ordering policy is derived to minimize the total expected cost. The experimental results show that the total expected cost under the optimal policy in the case of the known lead time is smaller than that in the unknown lead time case, but the difference becomes small when the holding cost is small or the fraction of urgent demand is large.

The characteristics considered in this research included the sales channels used by retailers, supplier selection, pricing schemes, ADI, and capacity constraints used in each research model. The position of this research among other studies in the field of existing DLS models, is shown by Table 2. Based on the literature review, there is no model that considers the characteristics of online sales, with consideration of multi-supplier, all unit quantity discount scheme, ADI, and capacity limits at the same time, so in this study try to combine some of these criteria to see how it affects the function of the goal of minimizing total costs.

3. Model development

Simple single-item lot sizing problems (SILSP) and their multiple developments often result in definitive solution methods such as dynamic programming, polyhedral approaches, branch-and-cut, and branch-and-bound algorithms. Some research focuses on the development on calculation time at a certain level of complexity. The basic SILSP model for the problem without capacity is a relatively easy-to-solve problem that can be solved in general in O(TlogT) and O(T) with some assumptions. The dynamic programming algorithm of dynamic lot sizing was first created by Wagner and Whitin (1958). The algorithm is an algorithm that can be solved in $O(T^2)$. Several subsequent studies tried to develop the WW algorithm to produce algorithms that were more efficient in real-world implementation, but some of these developments did not improve the complexity of the WW algorithm (Brahimi, et al., 2017).

This research considers how the ordering process must be determined by the company for suppliers to replenish inventory. The focus of this research is on companies with the concept of selling products through online sales channels. The model developed is in the form of a decision-making model for ordering optimal raw materials by considering several suppliers, quantity discounts, and sales through online channels.

3.1. Notation

Some of the mathematical notations consisting of indexes, variables, parameters and decision variables used in this thesis research model are as follows:

Indices	
s	supplier index (1, …, S)
t	period index (1, …, T)
k	discount rate index (1, …, K)
Parameters	
P_{st}	fixed message costs to suppliers s in period t
C_{kst}	product prices for orders for discount rate *k* to suppliers *s* in period *t*
Q_{kst}	upper limit of the amount of the k price level for supplier *s* in period *t*
z	number of unfulfilled requests in period 0
h_t	saving costs in period *t*
d_t	number of requests in period *t*
G	maximum allowable delay
L	*lead time* promised to consumers
b	delay *cost* per period
j	production capacity of the enterprise per period
Decision Variables	
R_{st}	ordering decision, value 1 if the company placed an order with supplier *s* in period *t*, value 0 if vice versa
X_{kst}	order amount for discount rate *k* to supplier *s* in period *t*.
Y_{kst}	order decision, worth 1 if the company ordered raw materials at price level *k* to supplier s in period t, is worth 0 if vice versa
v_t^i	number of requests that came in previous period that were still not fulfilled at the end of period t, with a value of i = 0, …, L+G-1
I_t	amount of inventory in period *t*

The assumptions used in this model include

- Type of customer is homogeneous, in the absence of priority customers.
- This type of retailer is a retailer who sells goods only through online channels or has an ADI on the number of requests.
- Discount scheme provided by suppliers is the All-Unit Quantity Discount scheme.
- Types of products on the company's production system are single item and single level.
- Planning horizon is limited, with discrete planning time.
- Production process is uninterrupted and sufficient to meet the maximum amount of capacity.
- Messaging fees, storage fees, and late fees for each product are constant and known for each period.
- Quality of production results is always in accordance with customer requests without any defective products.
- Initial number of I_0 and end I_T setups amounts to zero.
- Lead time is known and certain.
- Process of delivering finished products to consumers does not experience interruptions and delays.
- Raw materials at suppliers are always available according to needs.
- Production capacity is the same in each period.

3.2. Model formulation

The objective function of the research model is to minimize total costs consisting of 4 main cost components: fixed costs, ordering costs per unit, storage costs, and delay costs. Total costs are calculated based on the sum of all major cost components throughout the period.

Objective Function:

$$min\,Z = \sum_{t=1}^{T}\left(\sum_{s=1}^{S} p_{st}R_{st} + \sum_{k=1}^{K}\sum_{s=1}^{S} C_{kst}X_{kst} + h_t I_t + \sum_{m=1}^{G-1} b_t v_t^{L+m}\right) \quad (1)$$

Constraints:

$$I_t + d_t - \sum_{k=1}^{K}\sum_{s=1}^{S} X_{kst} = \sum_{m=0}^{L+G-1} v_t^m - \sum_{m=0}^{L+G} z_0^m \; ; t = 1 \quad (2)$$

$$I_t - I_{t-1} + d_t - \sum_{k=1}^{K}\sum_{s=1}^{S} X_{kst} = \sum_{m=0}^{L+G-1} v_t^m - \sum_{m=0}^{L+G-1} v_{t-1}^m \; ; t = 2, \ldots, T \quad (3)$$

$$\sum_{k=1}^{K}\sum_{s=1}^{S} X_{kst} - z_n^{L+G-1} \geq 0 \; ; t = 1 \quad (4)$$

$$I_{t-1} - v_{t-1}^{L+G-1} + \sum_{k=1}^{K}\sum_{s=1}^{S} X_{kst} \geq 0 \quad ; t = 2, \ldots, T \quad (5)$$

$$\sum_{m=1}^{L+G-1} v_t^m \geq \sum_{m=0}^{L+G-1} z_n^m - \sum_{k=1}^{K}\sum_{s=1}^{S} X_{kst} \quad ; t = 1 \quad (6)$$

$$\sum_{m=1}^{L+G-1} v_t^m \geq \sum_{m=0}^{L+G-1} z_n^m - \sum_{k=1}^{K}\sum_{s=1}^{S} X_{kst} + I_{t-1} \; ; t = 2, \ldots T \quad (7)$$

$$\sum_{m=0}^{L+G-1} v_t^m \geq \sum_{m=0}^{L+G-1} z_n^m - \sum_{k=1}^{K}\sum_{s=1}^{S} X_{kst} + d_t \ ; t = 1 \quad (8)$$

$$\sum_{m=0}^{L+G-1} v_t^m \geq \sum_{m=0}^{L+G-1} v_{t-1}^m - \sum_{k=1}^{K}\sum_{s=1}^{S} X_{kst} - I_{t-1} + d_t \ ; t = 2,\ldots T \quad (9)$$

$$v_t^m \leq z_n^m ; t = 1 \ ; \forall n, m \quad (10)$$

$$v_t^i \leq v_{t-1}^{i-1}, v_t^0 \leq d_t ; t = 1,\ldots T \ ; i = 1,\ldots L + G - 1 \quad (11)$$

$$\sum_{k=1}^{K} X_{kst} \leq M R_{st} ; s = 1,\ldots, S;\ t = 1,\ldots, T \quad (12)$$

$$\sum_{s=1}^{S} R_{st} \leq 1 ; s = 1,\ldots, S;\ t = 1,\ldots, T \quad (13)$$

$$I_T = 0, \quad \sum_{i=0}^{L+G-1} v_T^i = 0 \quad (14)$$

$$\sum_{k=1}^{K} Y_{kst} \leq 1 ;\ t = 1,\ldots, T \quad (15)$$

$$X_{kst} - Y_{kst} \geq 0 ; t = 1 \ ; \forall k, s \quad (16)$$

$$X_{kst} - Y_{kst}\left(1 + Q_{(k-1)st}\right) \geq 0; t = 2,\ldots, T; \forall k, s \quad (17)$$

$$X_{kst} - Y_{kst} Q_{kst} \leq 0; \forall k, s, t \quad (18)$$

$$\sum_{k=1}^{K}\sum_{s=1}^{S} X_{kst} \leq j ;\ t = 1,\ldots, T \quad (19)$$

$$v_t^i \geq 0, x_{kst} \geq 0, I_t \geq 0, \ ; i = 0,\ldots, L + G - 1 \ ; t = 1,\ldots, T \quad (20)$$

$$Y_{kst} = 0,1; \forall k, s, t \quad (21)$$

$$R_{st} = 0,1 ; \forall s, t \quad (22)$$

The amount of inventory in the warehouse needs to be limited so that the amount is balanced (2) and (3). Requests that have not been met with maximum delay must be guaranteed to be fulfilled in period t. This equation ensures that the number of requests that have not been met until the period can be fulfilled in period (t-1)t so that the goods arrive at the customer before the maximum expected time of (L+G) (4) and (5). The number of requests in the period is carried over to period (t-1)t for requests that have not been fulfilled in that period (6) (7) (8) (9) (10) (11).

The value of M which is a very large positive number to ensure that the number of bookings is sufficient to be able to meet the number of requests in period t. The amount value of M can be assumed to be greater than or equal to the total sum of requests in period 1 to period T (12). In each order period, it must be ensured that the company only places an order against one supplier (13). The amount of inventory at the end of the T period was 0 so that at the end of the planning period there was no inventory available in the warehouse. The final value of inventory can be set according to company policy to provide a number of products for safety stock or not, so this barrier is dynamic and can change according to company policy (14).

Quantity of orders placed by the Company shall be between the lower and upper limits of the k price level given by the supplier for each supplier and in each period (15) (16) (17) (18). Within the constraints of the production capacity of the production system, it must be ensured that the total number of orders in each period for all suppliers and the price level is less than the production capacity (19). The type of each variable, the values v, X, and I need to be ensured to be positive numbers, while the values of variables Y and R are binary numbers (20) (21) (22).

3.3. Numerical experiment

In this study, model validation was performed using numerical experiments data from observed in a small and medium enterprise (SME) that sells shoe products. The company markets and sells its products through online channels; they had several choices of suppliers to order leather raw materials. Each supplier has a different quantity discount, so the company needs to determine which supplier can provide the most optimal price for its production process. The data used is the number of customer requests per week shown in Table 3.

The length of time it takes to make a shoe is only about one week, but due to limited staff, the company promises consumers that every shoe purchase can be completed and delivered within three weeks. The

Table 3. Demand.

Period	0	1	2	3	4	5	6
Demand	9	6	5	3	7	1	2

Period	7	8	9	10	11	12
Demand	13	8	7	9	3	7

delay tolerated by the company is for one week. A summary of the parameters used in the company's production process is shown in Table 4.

Table 4. Production Parameter.

Selling Price	Rp	700,000
Delay Cost (bt)	Rp	35,000
Order Cost (pt)	Rp	200,000
Inventory Cost (It)	Rp	10,000
Lead Time (L)		3
Max Delay Time (G)		1

There are 2 suppliers of leather raw materials, each supplier has a discount scheme as shown in Table 5.

Table 5. Discount Scheme.

Supplier 1

Purchase Quantity (unit)	Price
1-5	Rp 58.000
6-10	Rp 55.000
11-dst	Rp 52.000

Supplier 2

Purchase Quantity (unit)	Price
1-3	Rp 56.000
4-6	Rp 55.000
7-9	Rp 54.000
10-12	Rp 53.000

The results of model testing using more complete company data can be seen in Table 6.

Table 6. Model Result.

Period	0	1	2	3	4	5	6
Demand	0	6	5	3	7	1	2
v0	7	3	5	0	7	1	2
v1	2	0	3	0	0	7	1
v2	0	0	0	0	0	0	7
v3	0	0	0	0	0	0	0
Order (R1t)	0	12	0	11	0	0	0
Order (R2t)	0	0	0	0	0	0	0
Inventory (I)	0	0	0	0	0	0	0

Period	7	8	9	10	11	12
Demand	13	8	7	9	3	7
v0	8	8	7	9	1	0
v1	0	8	8	7	7	0
v2	0	0	8	1	0	0
v3	0	0	0	0	0	0
Order (R1t)	15	0	0	15	12	15
Order (R2t)	0	0	0	0	0	0
Inventory (I)	0	0	0	0	0	0

Testing company data without online sales, so there is no lead time (L=0, G=0) resulted in a total cost of Rp. 5,780,000. The use of a number of lead times and late fees resulted in a 7.27% improvement in total costs.

3.4. Model testing to see the effect of L and G values

Based on the research of Xu et al., (2017), lead time (L) and maximum delay (G) will result in lower costs. In this study, testing was carried out on this by testing data from Onderhoud to be used in models for different L and G values. As shown in Figure 4 that the greater the value of L, the lower the total cost, while the greater the value of G decreases but does not have a significant impact.

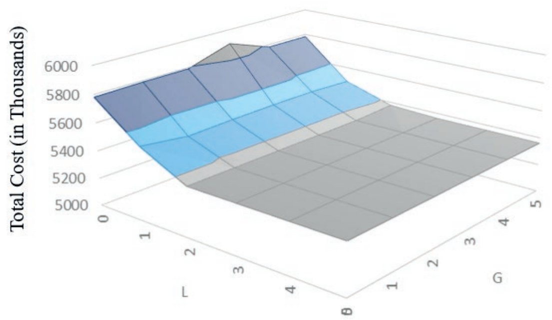

Figure 4. Cost Comparison with the Difference in L and G values.

The results of testing using company data show that when the L value is greater than 2, the decrease in total costs becomes insignificant and even tends to be stable. These conditions can be a consideration for companies to determine the value of lead time and maximum delay that need to be applied to the sales system. This condition is in line with research by Xu et al., (2017), which states that increasing demand lead time for consumers will have a significant impact on total costs compared to decreasing lead time for suppliers.

3.5. The effect of the ratio of late costs to saving costs

Testing by comparing the amount of late costs with storage costs was carried out to see how much influence the two cost variables had on the total cost. The ratio used is the late cost divided by the storage cost (w/t). The data used in the test is company data obtained by changing the ratio between late fees and storage costs. The summation of late fees and storage costs (w+h) is kept constant throughout the test by then varying the value of each cost. The test results are shown in Figure 5, i.e., at a ratio greater than 0, continue to increase the total cost until the ratio is close to 1.

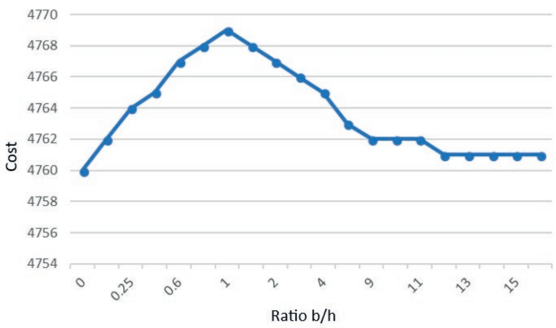

Figure 5. The effect of the ratio of late costs to storage costs.

When the ratio is greater than 1, the total cost decreases further until it is close to the original total cost when the ratio is 0. This condition can be a consideration for the company, namely by making a comparison ratio between late costs and storage costs to be made away from one. The ratio of comparison between late costs and storage costs that is close to the value of one has the potential to increase the total production costs incurred.

3.6. Model testing with period and supplier addition

The next test is to determine the effect of the number of suppliers and periods on the number of iterations and computational time required by the model. Tests are carried out with changes in the number of planning periods and the number of suppliers owned by the company. The limit used in the test is to see how many periods and alternative numbers of suppliers the model can complete in a maximum time span of 2 hours. The time span of 2 hours was chosen as a reference because when the optimal solution is produced by the model for more than 2 hours, the model can be said to be less good. The computational results in the form of the number of iterations and time required to produce the optimal solution are shown in Table 7.

It can be seen in the test results that when the company has three alternative suppliers with 50 planning periods, the model produces an optimal solution with a compute time of 2 hours, 4 minutes, 56 seconds, and 25 780 445 iterations. This condition is no longer feasible for the model to produce an optimal solution. The test results in Table 6 show the conclusion that when the production planning period is more than 50 periods, the model is not good to use because it takes a long time.

Table 7. Comparative computational results of the number of periods and suppliers.

Period	Supplier	Number of Iterations	Time
15	2	3439	00:00:01
	3	9908	00:00:04
	4	28652	00:00:11
	5	35342	00:00:13
20	2	6325	00:00:02
	3	46062	00:00:10
	4	37877	00:00:14
	5	67266	00:00:20
30	2	12389	00:00:03
	3	136580	00:00:40
	4	131095	00:00:30
	5	196742	00:00:57
40	2	576524	00:02:47
	3	1329573	00:06:26
	4	4544213	00:22:01
	5	1574544	00:42:26
50	2	13128448	01:03:37
	3	25780445	02:04:56
	4	31974933	02:34:57
	5	46157978	03:43:41

After testing the model by changing some of the parameters used, some characteristics of the model can be known. The model can already solve the problem of dynamic lot sizing with multiple suppliers, quantity discounts, and capacity limits for sales with ADI. The model class that this model generates is integer linear programming (ILP).

The results of testing using data from previous studies have shown that the model can still produce optimal solutions to simple dynamic lot sizing problems. The advantages that this model can provide are that it can consider supplier selection factors, discounts, and capacity limits on dynamic lot sizing problems with the use of ADI. The addition of both the number of periods and the number of suppliers can be accommodated by the model and result in a minimal solution. Testing was carried out by increasing the number of planning periods to 30 periods, with four suppliers still able to produce an optimal solution from the model.

The DLS model in this study experienced problems when facing capacity limitations that were too small. Too small a capacity limitation cannot produce a solution. This condition can also be seen in several existing studies that reveal that capacity limits can increase the complexity of NP-hard problems. By using this model, the problem of the real system of ordering raw materials for companies with multiple suppliers and quantity discounts with ADI can be solved.

4. Result and discussion

Dynamic Lot Size model that considers ADI by combining several characteristics, including multi-supplier, quantity discounts, and capacity constraints proposed to solve DLS problem. In general, the development model in this study can solve problems in retailers that have ADI, and companies have a choice of several suppliers with a discount scheme from suppliers to minimize total production or order costs, so that the results of the model development in this study can already solve cases in SMEs that are the object of real case study research. The effect of ADI on retailers can reduce production costs, because retailers can plan more definitely because of the certainty of the number of requests in a period. The results show that the optimal demand lead time for Onderhoud is 3 periods, so increasing the demand lead time more than 3 periods will not increase profits for the company and will reduce the level of service to consumers. In the case of production planning with more than 60 production periods and more than 5 suppliers, the development model is less effective because it requires a long computational time. The ratio of the comparison of storage costs and late costs must be made away from number 1, so as to produce the minimum total costs.

The proposed model can be used as a decision tool for production planner or anyone who need production plan with several characetristic. The model can minimize total production cost. From the perspective of a production planner, this model is expected to speed up the decision-making process related to production plans for the upcoming period.

The model developed in this study still has shortcomings and limitations, so it is expected that there are still some developments and improvements that can be made to produce a better model in future research. Development of dynamic lot sizing models for the process of selling products through brick and click. In brick-and-click sales, retailers sell products conventionally (directly) as well as online. Brick-and-click sales conditions are now starting to be widely used by retailers to sell their products. The dynamic lot sizing model needs to consider the economic aspects from the supplier side.

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