

# 電子材料與元件技術特論

## SPECIAL TOPICS IN ELECTRONIC MATERIALS AND DEVICES

### *Electrical Properties-Part 2*

# Outline

- Carrier Concentration and Mass Action Law
- The Fermi Energy
- Conductivity of Extrinsic Si
- Direct and Indirect Recombination
- Carrier Injection

# Conductivity of a Semiconductor

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- **$n$**  and  **$p$**  are concentrations of electrons and holes in a semiconductor crystal
- Electrons and holes have drift mobility, so overall conductivity of the crystal can be given by:

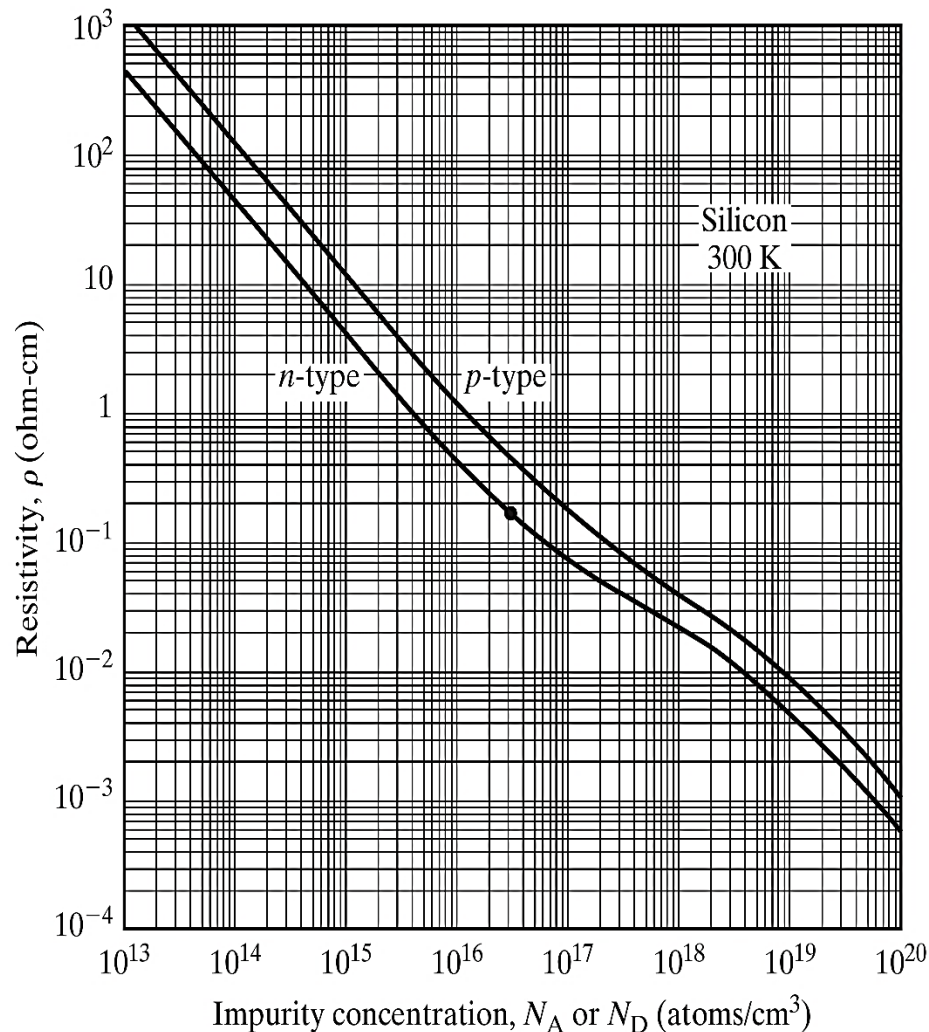
$$\sigma = en\mu_e + ep\mu_h$$

- **Drift Velocity and Net Force**

$$v_e = \frac{\mu_e}{e} F_{\text{net}} = \mu_e E \quad \mu_e = \frac{e\tau}{m_e^*} \quad v_h = \mu_h E \quad \mu_h = \frac{e\tau}{m_h^*}$$

$v_e$  = drift velocity of the electrons,  $F_{\text{net}}$  = net force,  $\tau$  = mean scattering time,  $m_e^*$  = effective mass of electron

# Resistivity vs. Doping



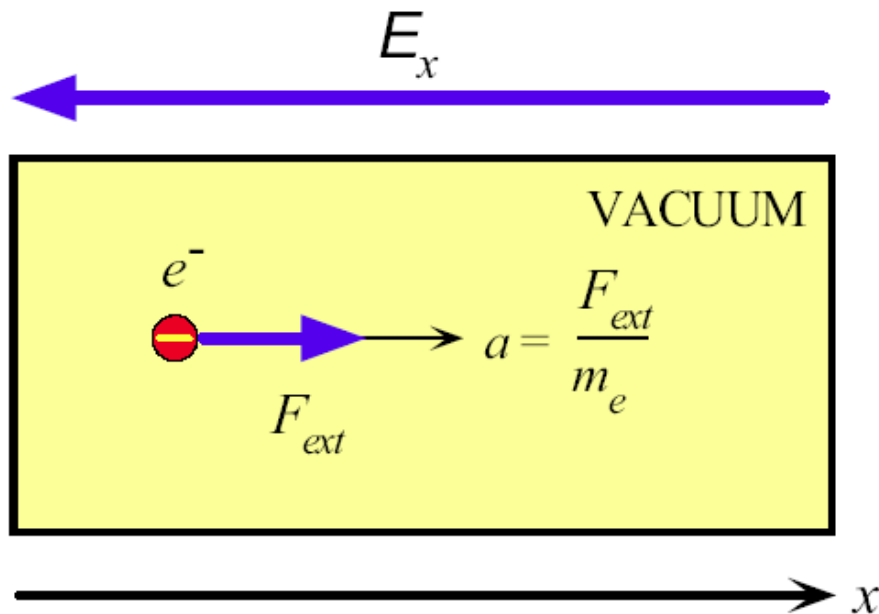
$$\rho = \sigma^{-1} = [q(\mu_n n + \mu_p p)]^{-1}$$

$$n\text{-type} : \rho \cong [q\mu_n (N_D)]^{-1}$$

$$p\text{-type} : \rho \cong [q\mu_p (N_A)]^{-1}$$

Room-temperature resistivity in *n*- and *p*-type silicon as a function of impurity concentration. (Note that these curves are valid for either donor or acceptor impurities but not for compensated material containing both types of impurities.)

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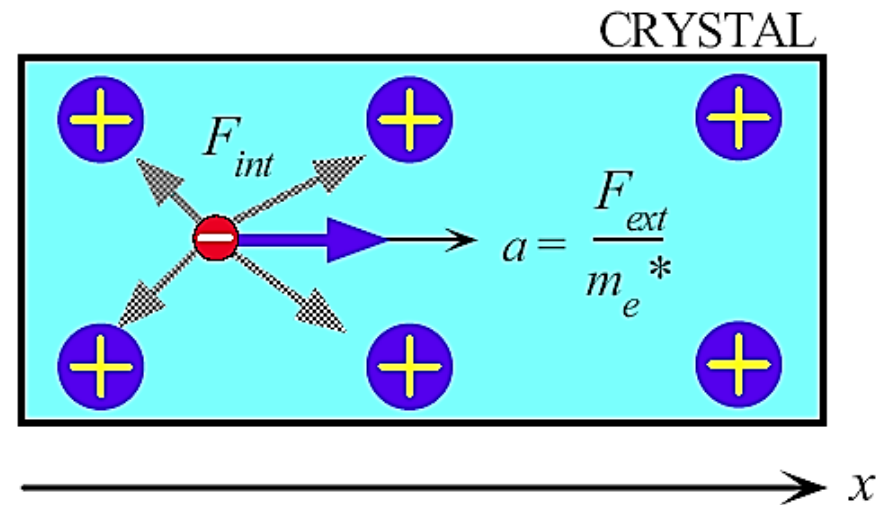


(a) An external force  $F_{ext}$  applied to an electron in vacuum results in an acceleration

$$\mathbf{a}_{vac} = \mathbf{F}_{ext} / m_e$$

$m_e$  = mass of an electron in free space

The effective mass is a quantum mechanical quantity that behaves in the same way as the inertial mass in classical mechanics

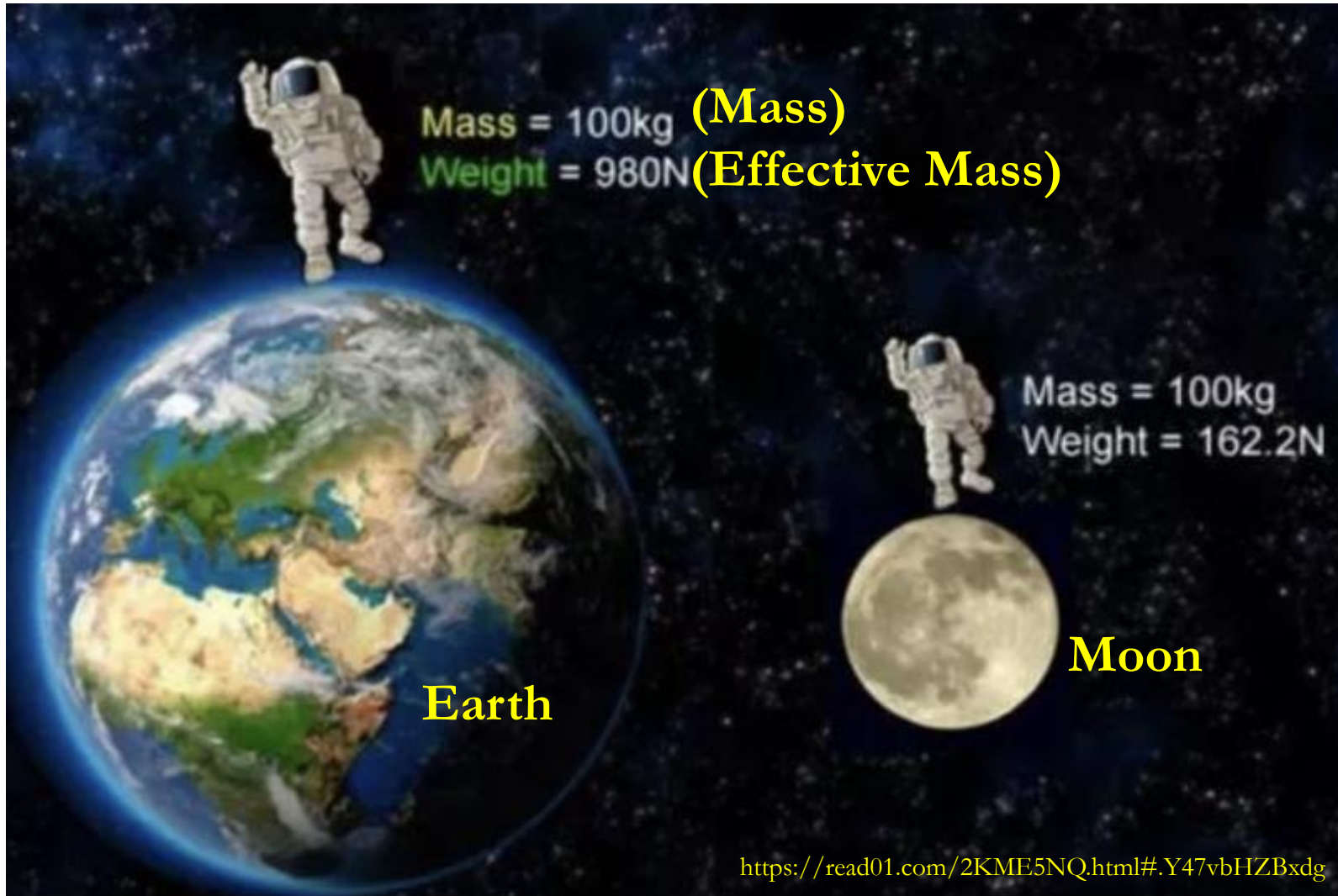


(b) An external force  $F_{ext}$  applied to an electron in a crystal results in an acceleration

$$\mathbf{a}_{cryst} = (\mathbf{F}_{ext} + \mathbf{F}_{int}) / m_e = \mathbf{F}_{ext} / m_e^*$$

$m_e^*$  = effective mass of electron

# Gravity of the Earth



# Statistical Mechanics

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- If you want to know the **number of charge carriers** and their temperature dependence, the question is: **how many energy levels** (density of states, DOS) do we have and what is the **chance that they are populated** independent of the temperature.
- While studying large number of particles, we are interested only in **statistical behavior** of the group as a whole rather than in the behavior of individual one.
- There are three distribution laws determining the **distribution of particles** among available energy states.

# Three Distribution Laws

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Gas molecular	Electrons	Phonons
classical	quantum	quantum
Boltzmann	Fermions	Bosons
Boltzmann-Maxwell	Fermi-Dirac	Bose-Einstein
w/o spin	Half-integer spin Pauli exclusion	Integer spin
distinguishable	indistinguishable	indistinguishable

Maxwell-Boltzmann  
(classical)

$$f(E) = \frac{1}{Ae^{E/kT}}$$

Fermi-Dirac  
(quantum)

$$f(E) = \frac{1}{Ae^{E/kT} + 1}$$

Bose-Einstein  
(quantum)

$$f(E) = \frac{1}{Ae^{E/kT} - 1}$$



# The Fermi-Dirac Function

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## □ The Fermi-Dirac Function:

The **probability** that an **energy level**,  $E$ , is **occupied by an electron**. (Value between 0 and 1)

$$f(E) = \frac{1}{1 + e^{(E-E_F)/KT}}$$

**The Fermi-Dirac  
distribution function**

## □ The Fermi-Dirac function at 0 K :

✓ At 0 K, when  $E < E_F$ , the  $f(E)=1$

$$f(E < E_F) = \frac{1}{1 + \exp(-\infty)} = 1$$

✓ At 0 K, when  $E > E_F$ , the  $f(E)=0$

$$f(E > E_F) = \frac{1}{1 + \exp(+\infty)} = 0$$

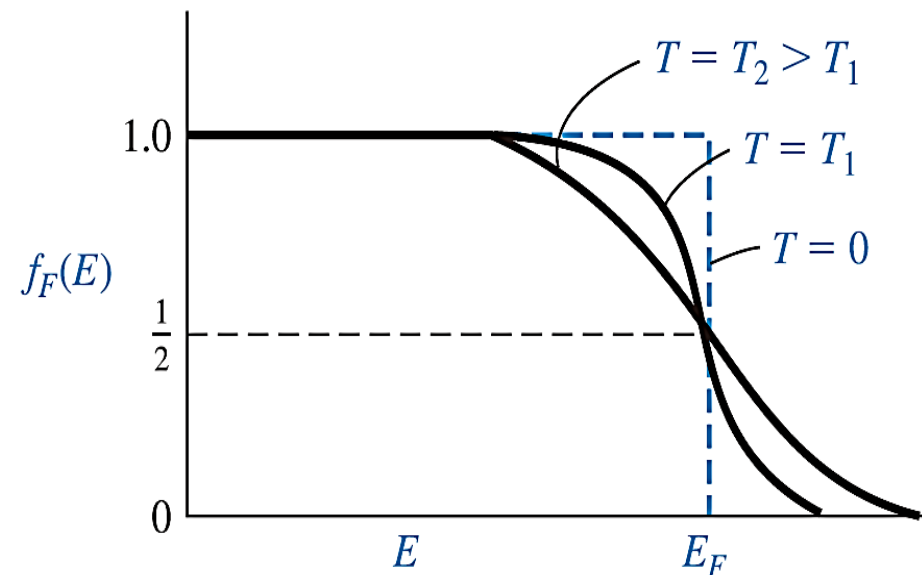
$$f(E_F) = 1/2 \text{ at } T > 0K$$

# The Fermi-Dirac Function

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- So **Fermi Energy ( $E_f$ )** is the energy level where the probability of finding an electron is  $\frac{1}{2}$ .

$$\mathbf{E = E_f} \quad f_E(E) = \frac{1}{1 + \exp\left(\frac{E - E_f}{kT}\right)} = \frac{1}{1 + \exp(0)} = \frac{1}{1 + 1} = \frac{1}{2}$$



- ◆ At  $T > 0K$ , there is non-zero probability that some states above  $E_f$  are occupied and some states below are empty.
- ◆ Some electrons jumping to higher energy levels with increasing thermal energy.

# Example

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- Please find out the probability that an energy level  $3kT$  above  $E_f$  is occupied by an electron at  $T=300K$ .

$$f_F(E) = \frac{1}{1 + \exp\left(\frac{E - E_F}{kT}\right)} = \frac{1}{1 + \exp\left(\frac{3kT}{kT}\right)}$$

$$f_F(E) = \frac{1}{1 + 20.09} = 0.0474 = 4.74\%$$

- Most states at energies  **$3kT$  above  $E_F$**  are empty,  $f_F(E)=4.74\%$
- Thus the following approximation is valid:

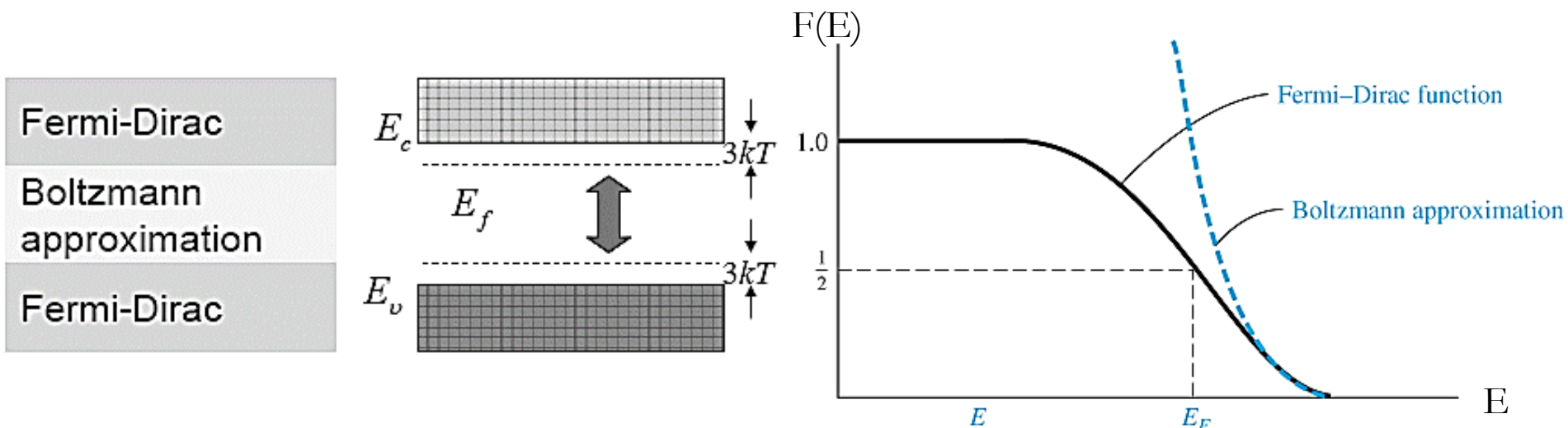
$$f(E) = \frac{1}{e^{(E-E_F)/KT}} \quad \text{Boltzmann Approximation}$$

# Boltzmann Distribution

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- The Fermi-Dirac probability for  $E - E_f > 3kT$  can be approximated by Boltzmann distribution.

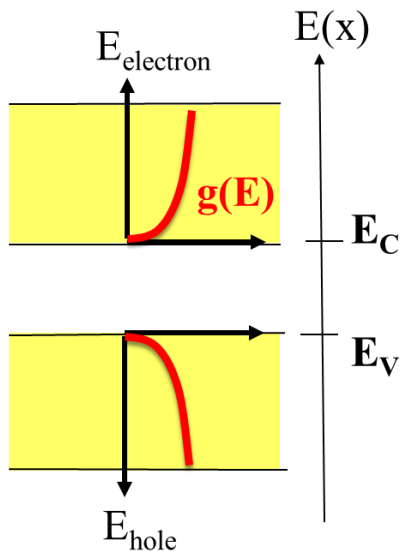
$$\left. \begin{array}{l} \text{n-type } E_c - E_f > 3kT \\ \text{p-type } E_f - E_v > 3kT \end{array} \right\} \Rightarrow \text{Boltzmann approximation holds}$$
  
 elsewhere  $\Rightarrow$  Fermi - Dirac distribution



# Density of States in 3D

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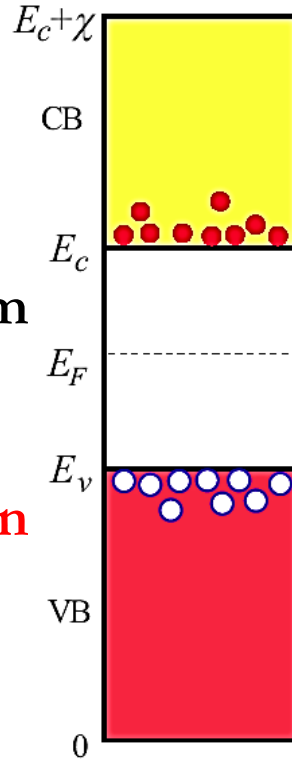
- **Density of States (DOS)**: the **number of available energy states** per unit volume per unit energy in a crystal.
- It is essential for deciding the **carrier concentrations** and energy distribution of carrier within a semiconductor.



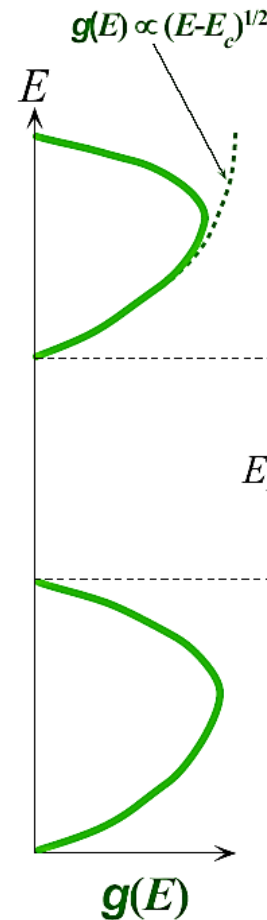
$$g_{cb}(E) = \frac{dN}{dE} = \frac{8\sqrt{2}\pi m_e^{*3/2}}{h^3} (E - E_c)^{1/2}$$

$$g_{vb}(E) = \frac{dN}{dE} = \frac{8\sqrt{2}\pi m_h^{*3/2}}{h^3} (E_v - E)^{1/2}$$

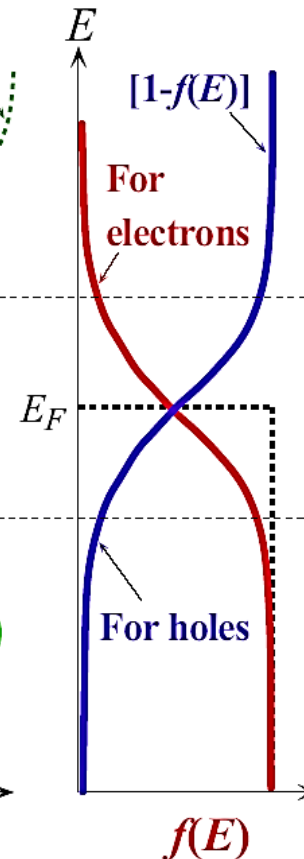
(a)



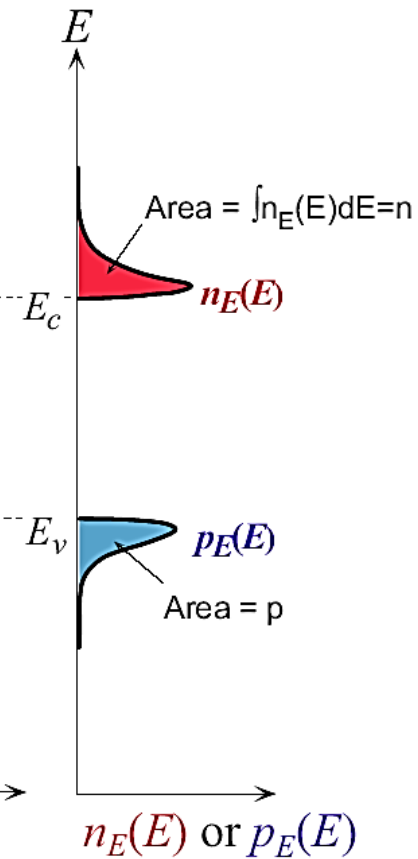
(b)



(c)



(d)



(d) The **product of  $g(E)$  and  $f(E)$**  is the energy density of electrons (number of electrons per unit energy per unit volume). The **area** under  $n_E(E)$  versus  $E$  is the **electron concentration**.

$$n = \int_{E_c}^{\infty} g(E)f(E)dE$$

# Electron Concentration

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## □ Electron Concentration in CB:

$$n = \int_{E_C}^{\infty} g(E)f(E)dE = 2 \left( \frac{2\pi m_e^* KT}{h^2} \right)^{\frac{3}{2}} e^{\frac{-(E_C - E_F)}{KT}} = N_c e^{\frac{-(E_C - E_F)}{KT}}$$

$$n = N_c \exp \left[ -\frac{(E_c - E_F)}{kT} \right]$$

The equation is valid for both intrinsic & extrinsic semiconductors.

## □ Effective Density of States at CB Edge:

$$N_c = 2 \left( \frac{2\pi m_e^* kT}{h^2} \right)^{3/2}$$

T=300K in Si  
 $N_c = 2.8 \times 10^{19} \text{ cm}^{-3}$

$E_c$  = conduction band edge,  $E_F$  = Fermi energy,  $k$  = Boltzmann constant,  $m_e^*$  = effective mass of the electron in the CB,  $h$  = Planck's constant

# Hole Concentration

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## □ Hole Concentration in VB:

$$p = \int_{-\infty}^{E_v} g(E)[1 - f(E)]dE = 2 \left( \frac{2\pi m_h^* KT}{h^2} \right)^{\frac{3}{2}} e^{\frac{-(E_F - E_v)}{KT}} = N_v e^{\frac{-(E_F - E_v)}{KT}}$$

$$p = N_v \exp \left[ -\frac{(E_F - E_v)}{kT} \right]$$

## □ Effective Density of States at VB Edge

$$N_v = 2 \left( \frac{2\pi m_h^* kT}{h^2} \right)^{3/2}$$

T=300K in Si

$N_v = 1.2 \times 10^{19} \text{ cm}^{-3}$

$N_v$  = effective density of states at the VB edge,  $m_h^*$  = effective mass of a hole in the VB,  $k$  = Boltzmann constant,  $T$  = temperature,  $h$  = Planck's constant



# Effective Density of States

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- $N_c$  &  $N_v$  are determined by the parameters of **effective masses** and **temperature** with power of **3/2**.

$$N_c = 2 \left( \frac{2\pi m_n^* kT}{h^2} \right)^{3/2} \quad N_v = 2 \left( \frac{2\pi m_p^* kT}{h^2} \right)^{3/2}$$

**Table 4.1** | Effective density of states function and effective mass values

	$N_c$ (cm <sup>-3</sup> )	$N_v$ (cm <sup>-3</sup> )	$m_n^*/m_0$	$m_p^*/m_0$
Silicon	$2.8 \times 10^{19}$	$1.04 \times 10^{19}$	1.08	0.56
Gallium arsenide	$4.7 \times 10^{17}$	$7.0 \times 10^{18}$	0.067	0.48
Germanium	$1.04 \times 10^{19}$	$6.0 \times 10^{18}$	0.55	0.37

T=300K in Si,  $N_c=2.8 \times 10^{19}$  cm<sup>-3</sup>;  $N_v=1.2 \times 10^{19}$  cm<sup>-3</sup>

# Example

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$$N_v(300K) = 1.04 \times 10^{19} \text{ cm}^{-3}$$

$E_f$  is 0.27 eV above  $E_v$

$\Rightarrow$  Find the hole concentration in VB at 400K

**Solution:**

$$\frac{N_v(400K)}{N_v(300K)} = \left(\frac{400}{300}\right)^{3/2} \Rightarrow N_v(400K) = (1.04 \times 10^{19}) \left(\frac{400}{300}\right)^{3/2} = 1.60 \times 10^{19} \text{ cm}^{-3}$$

$$p_o = N_v \exp \frac{-(E_f - E_v)}{kT}$$

$$= (1.60 \times 10^{19}) \exp\left(\frac{-0.27}{0.03453}\right) = 6.43 \times 10^{15} \text{ cm}^{-3}$$

# Mass Action Law

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$$np = n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$

$n_i$  = intrinsic concentration

The  $np$  product is a constant,  $n_i^2$ , that depends on the material properties  $N_c$ ,  $N_v$ ,  $E_g$ , and the temperature. If somehow  **$n$  is increased (e.g. by doping),  $p$  must decrease to keep  $np$  constant.**

**Mass action law applies**

**in thermal equilibrium**

**and**

**in the dark (no illumination)**

# Thermal Equilibrium

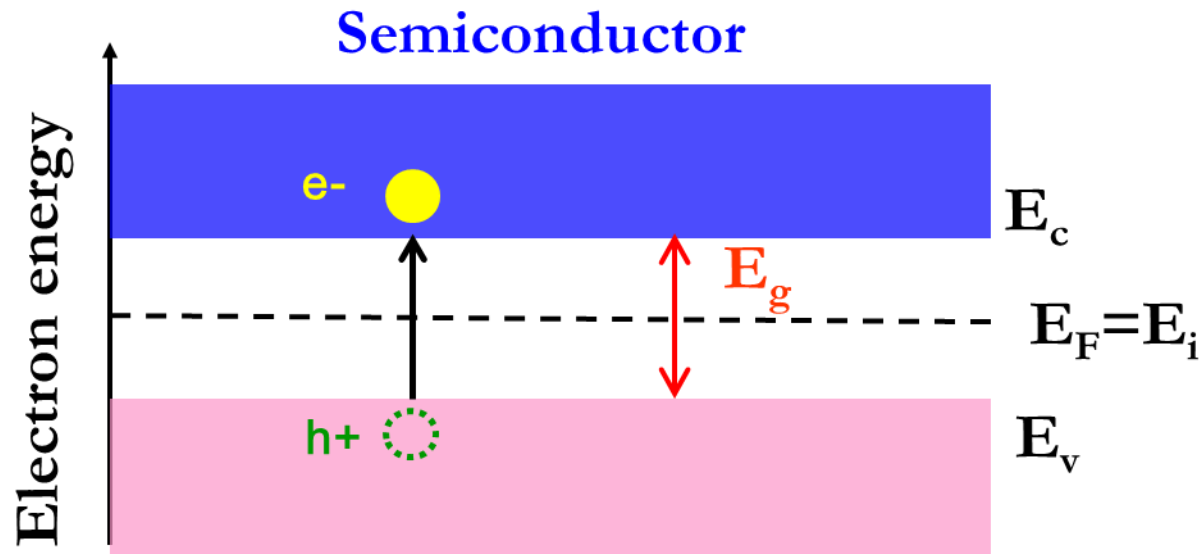
- **No external forces** such as voltages, **electric fields**, **magnetic fields**, or **temperature gradients** are acting on the semiconductor
- All properties of the semiconductor will be **independent of time** at equilibrium
- Equilibrium is the starting point for developing the physics of the semiconductor, and then explaining the characteristics when deviations from equilibrium occur.

# Mass Action Law

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$$np = n_i^2 = N_c N_v \exp\left(-\frac{E_g}{kT}\right)$$

$$n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right) = n_0 \exp\left(-\frac{E_g}{2kT}\right)$$



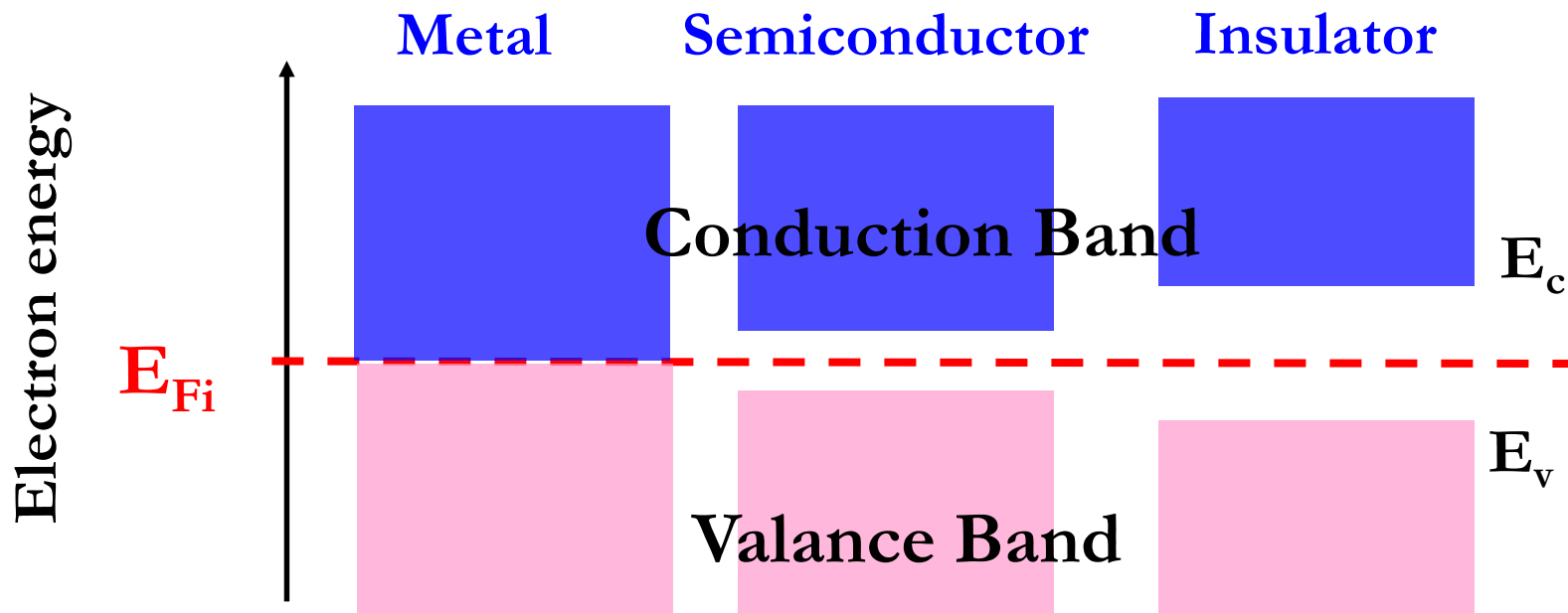
# Outline

- Carrier Concentration and Mass Action Law
- **The Fermi Energy**
- Conductivity of Extrinsic Si
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# The Fermi Energy (Level)

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- At 0 K, all electron states below **Fermi energy** ( $E_F$ ) are filled, and all electron states above  $E_F$  are vacant.
- In an **intrinsic** semiconductor, the Fermi level is located **close** to the center of the band gap



# Fermi Energy in Intrinsic Semi.

$$p = N_v \exp\left[-\frac{(E_{Fi} - E_v)}{kT}\right]$$

$$p = n_i = \sqrt{N_c N_v} \exp\left(-\frac{E_g}{2kT}\right)$$

$$E_{Fi} = E_v + \frac{1}{2} E_g - \frac{1}{2} kT \ln\left(\frac{N_c}{N_v}\right) = \boxed{E_v + \frac{1}{2} E_g} - \frac{3}{4} kT \ln\left(\frac{m_e^*}{m_h^*}\right)$$

$E_{\text{midgap}}$

The intrinsic Fermi level,  $E_{Fi}$ , is located **near** midgap.

$E_{Fi}$  = Fermi energy in the intrinsic semiconductor,  $E_v$  = valence band edge,  $E_g = E_c - E_v$  is the bandgap energy,  $k$  = Boltzmann constant,  $T$  = temperature,  $N_c$  = effective density of states at the CB edge,  $N_v$  = effective density of states at the VB edge

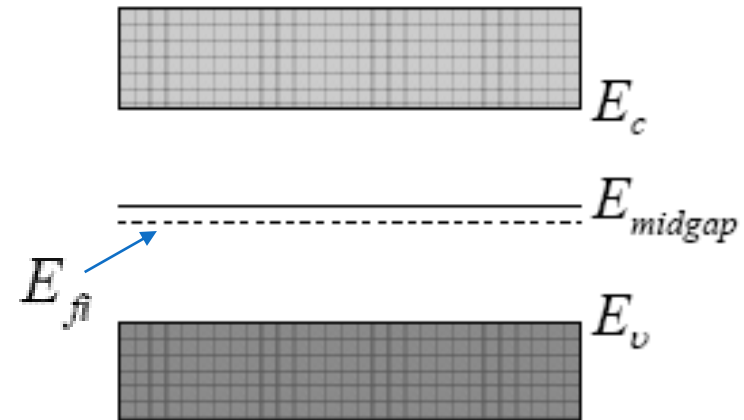


# Intrinsic Fermi-level Position

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- Even in intrinsic semiconductor, Fermi level is **not** exactly **at center** between the CB and VB.

$$E_{Fi} = E_{midgap} - \frac{3}{4}KT \ln\left(\frac{m_e^*}{m_h^*}\right)$$



**Table 4.1** | Effective density of states function and effective mass values

	$N_c$ (cm <sup>-3</sup> )	$N_v$ (cm <sup>-3</sup> )	$m_n^*/m_0$	$m_p^*/m_0$
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# Properties of Si, Ge, and GaAs

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**Table 5.1** Selected typical properties of Ge, Si, and GaAs at 300 K

	$E_g$ (eV)	$\chi$ (eV)	$N_c$ (cm <sup>-3</sup> )	$N_v$ (cm <sup>-3</sup> )	$n_i$ (cm <sup>-3</sup> )	$\mu_e$ (cm <sup>2</sup> V <sup>-1</sup> s <sup>-1</sup> )	$\mu_h$ (cm <sup>2</sup> V <sup>-1</sup> s <sup>-1</sup> )	$m_e^*/m_e$	$m_h^*/m_e$	$\epsilon_r$
Ge	0.66	4.13	$1.04 \times 10^{19}$	$6.0 \times 10^{18}$	$2.3 \times 10^{13}$	3900	1900	0.12a 0.56b	0.23a 0.40b	16
Si	1.10	4.01	$2.8 \times 10^{19}$	$1.2 \times 10^{19}$	$1.0 \times 10^{10}$	1350	450	0.26a 1.08b	0.38a 0.60b	11.9
GaAs	1.42	4.07	$4.7 \times 10^{17}$	$7 \times 10^{18}$	$2.1 \times 10^6$	8500	400	0.067a,b	0.40a 0.50b	13.1

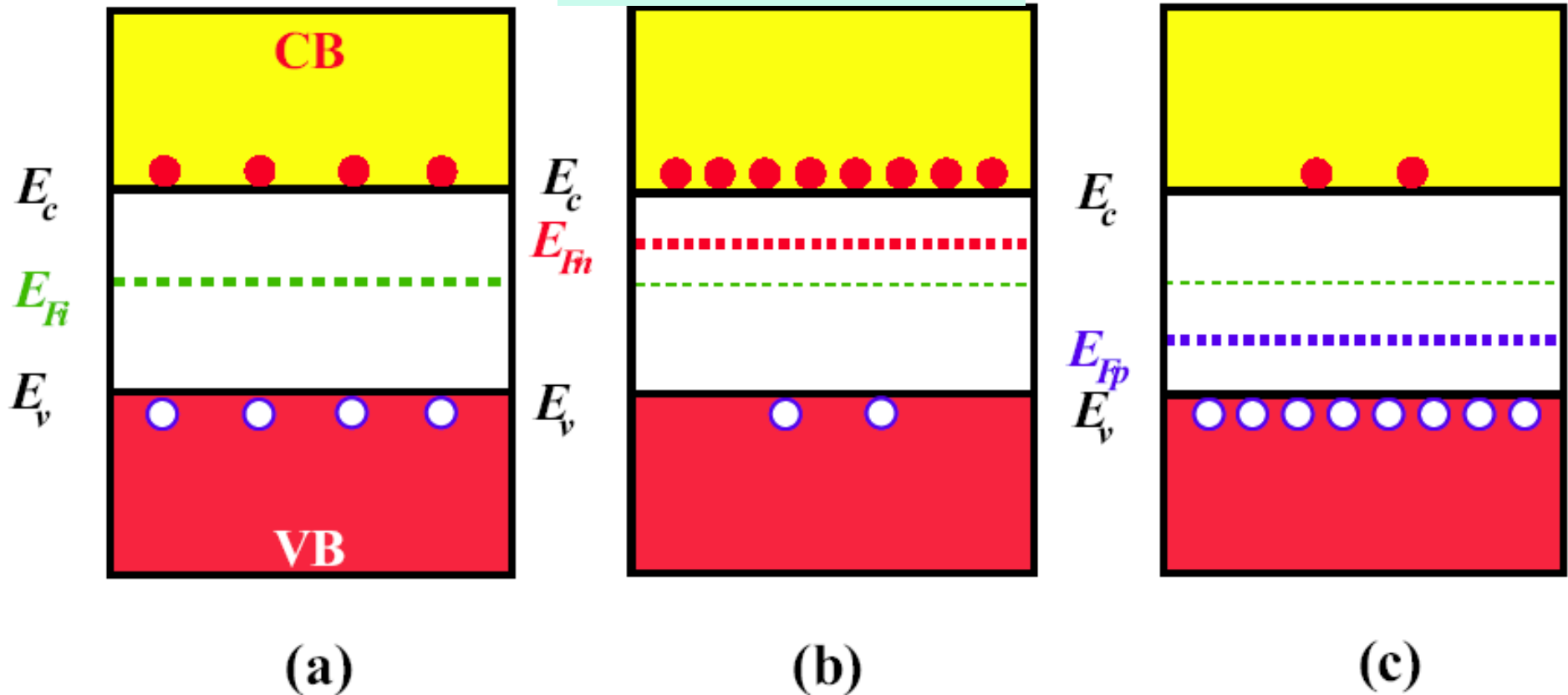
NOTE: Effective mass related to conductivity (labeled a) is different than that for density of states (labeled b). In numerous textbooks,  $n_i$  is taken as  $1.45 \times 10^{10}$  cm<sup>-3</sup> and is therefore the most widely used value of  $n_i$  for Si, though the correct value is actually  $1.0 \times 10^{10}$  cm<sup>-3</sup>. (M. A. Green, *J. Appl. Phys.*, **67**, 2944, 1990.)

# Position of Fermi Level

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$$n = N_c \exp\left[-\frac{(E_c - E_F)}{kT}\right]$$

$$p = N_v \exp\left[-\frac{(E_F - E_v)}{kT}\right]$$



Energy band diagrams for

In all cases,  $np = n_i^2$

(a) Intrinsic

(b) n-type semiconductor

(d) p-type semiconductor

# Average Electron Energy in CB

$$\overline{E}_{\text{CB}} = E_c + \frac{3}{2}kT$$

$\overline{E}_{\text{edge}}$  = average energy of electrons in the CB,  $E_c$  = conduction band edge

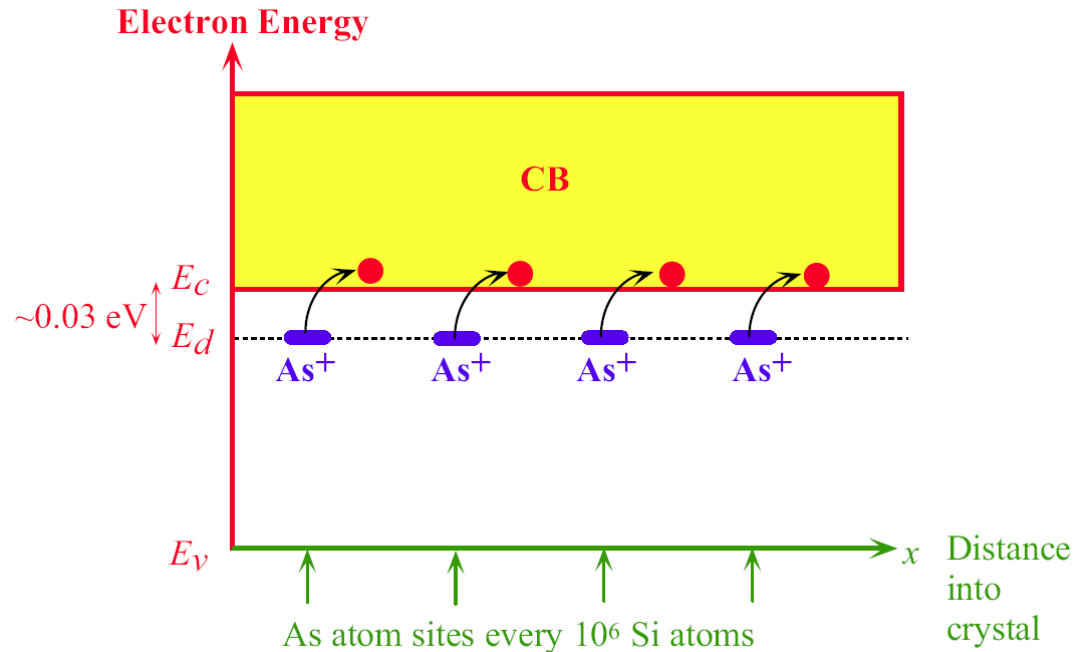
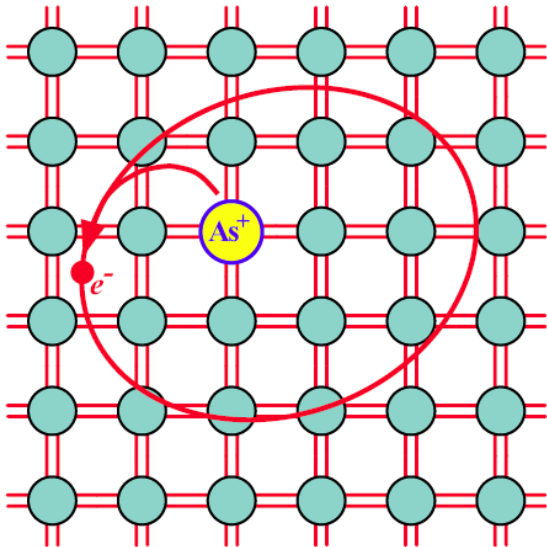
$(3/2)kT$  is also the average kinetic energy per atom in a monatomic gas (kinetic molecular theory) in which the gas atoms move around freely and randomly inside a container.

The electron in CB behaves as if it were “free” with a mean kinetic energy that is  $(3/2)kT$  and an effective mass  $m_e^*$ .

# Outline

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# $n$ -type Si

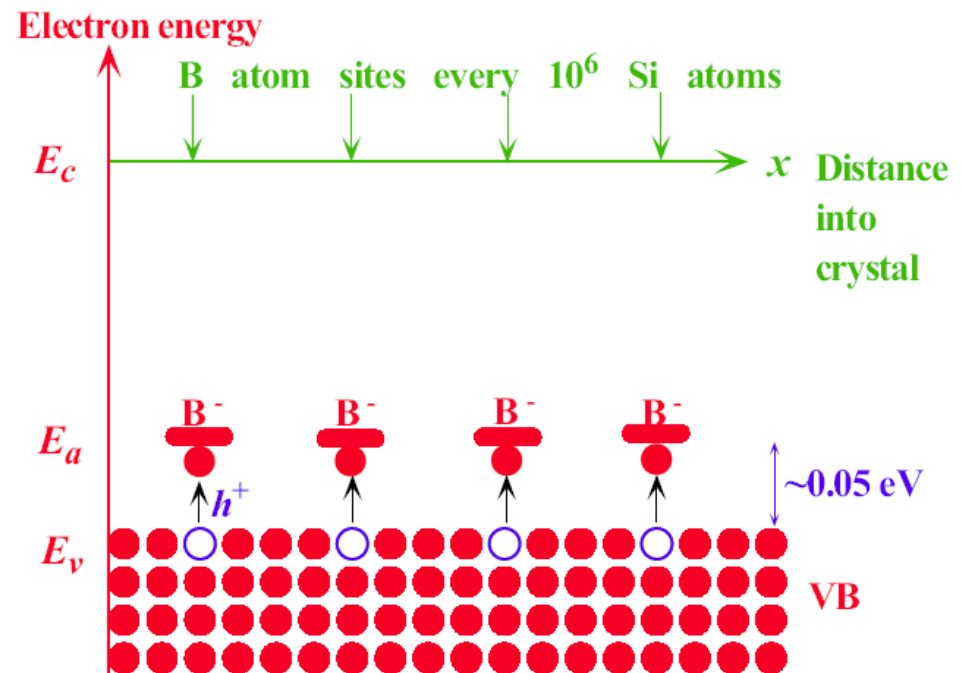
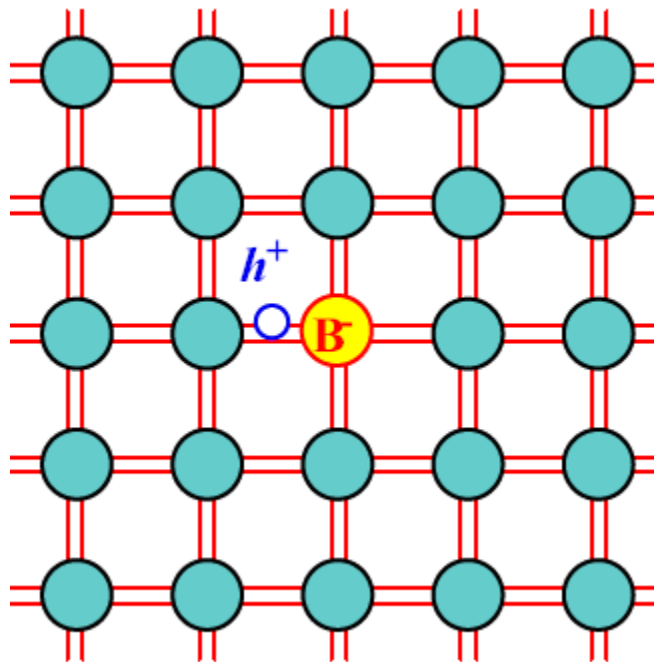


The fifth electron of As is left orbiting the As site. The energy required to release the free fifth electron into the CB is very small.

**Energy band diagram** for an  $n$ -type Si doped with 1 ppm As. There are **donor energy levels ( $E_d$ )** just below  $E_c$  around  $As^+$  sites.

# $p$ -type Si

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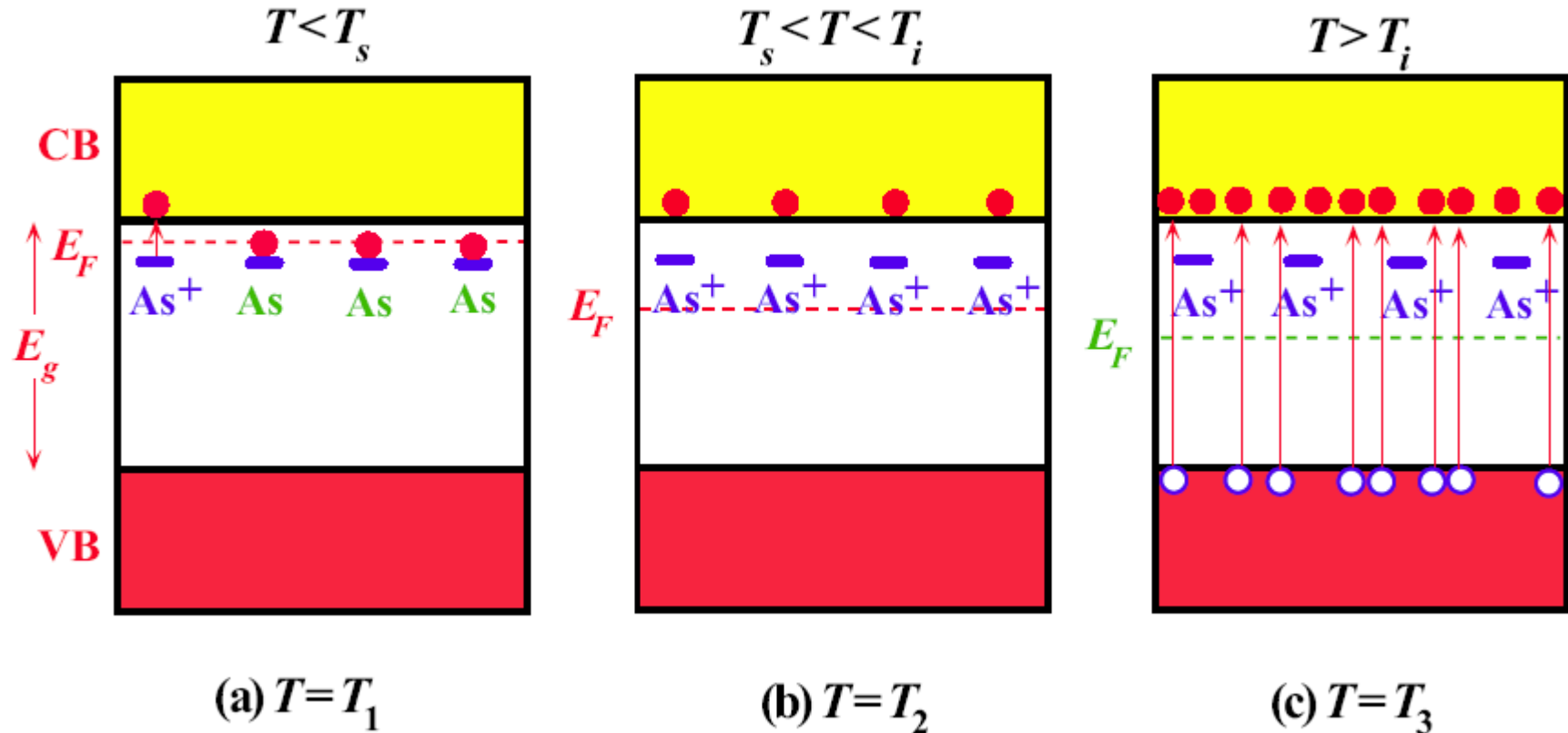
- **Energy band diagram** for a  $p$ -type Si doped with 1 ppm B.
- There are **acceptor energy levels**  $E_a$  just above  $E_v$  around  $B^-$  sites. These acceptor levels accept electrons from the VB and therefore create holes in the VB.

# Ionization Energy

**Table 5.2** Examples of donor and acceptor ionization energies (eV) in Si

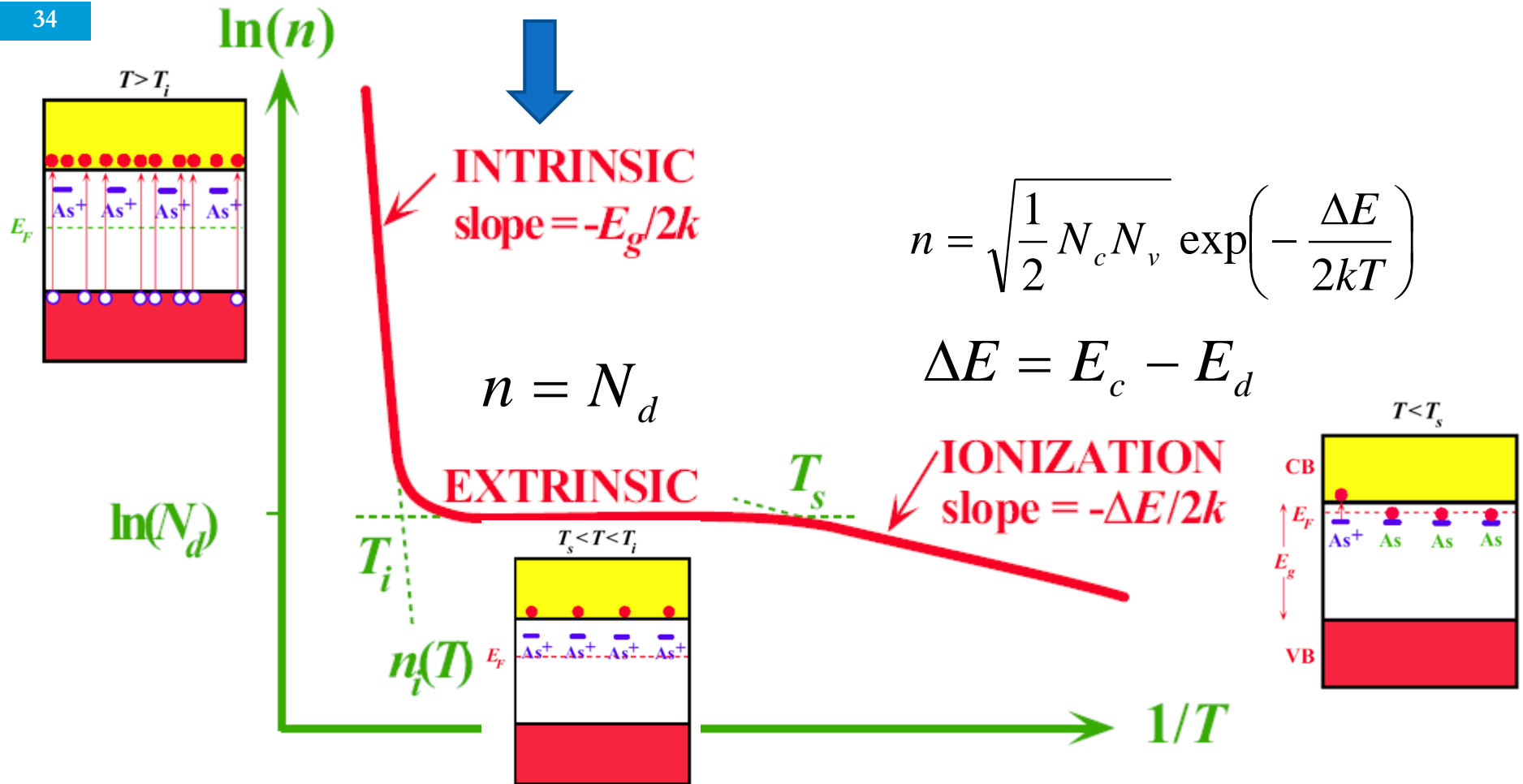
Donors			Acceptors		
P	As	Sb	B	Al	Ga
0.045	0.054	0.039	0.045	0.057	0.072





- (a) Below  $T_s$ , the electron concentration is controlled by the ionization of the donors.
- (b) Between  $T_s$  and  $T_i$ , the electron concentration is equal to the concentration of donors since they would all have ionized.
- (c) At high temperatures, thermally generated electrons from the VB exceed the number of electrons from ionized donors and the semiconductor behaves as if intrinsic.

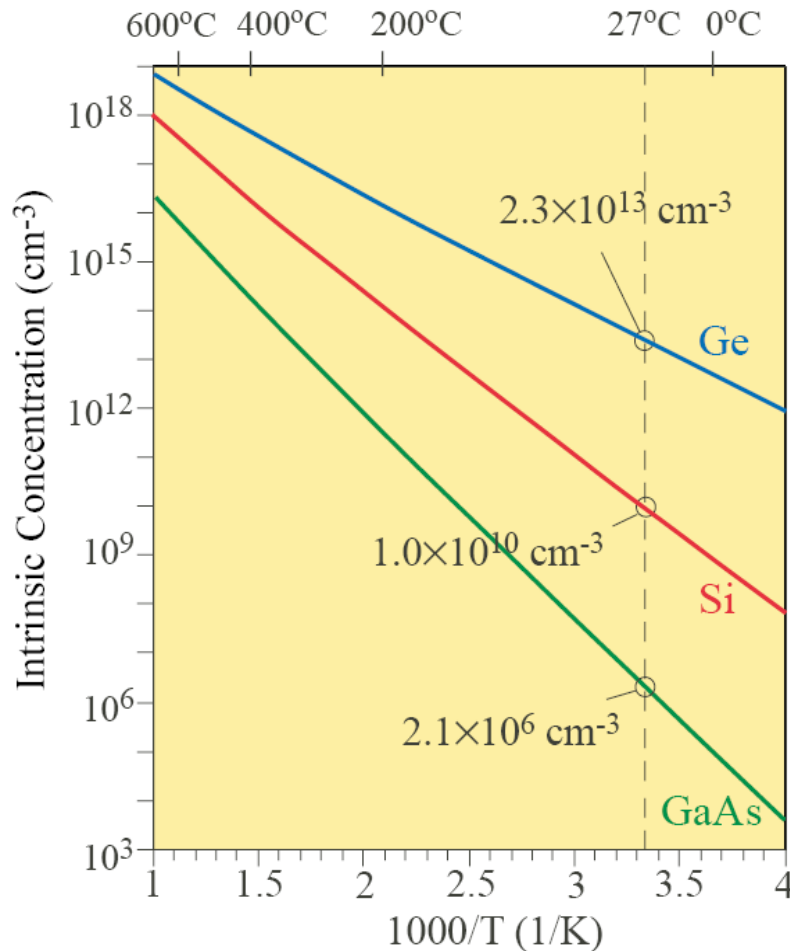
34



The temperature dependence of the electron concentration in an  $n$ -type semiconductor.

# Intrinsic Carrier Concentration

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The temperature and bandgap dependence of the intrinsic concentration.

Material	Band Gap (eV)	Electrical Conductivity $[(\Omega\text{-m})^{-1}]$	Electron Mobility ( $\text{m}^2/\text{V}\cdot\text{s}$ )	Hole Mobility ( $\text{m}^2/\text{V}\cdot\text{s}$ )
<b>Elemental</b>				
Si	1.11	$4 \times 10^{-4}$	0.14	0.05
Ge	0.67	2.2	0.38	0.18
<b>III-V Compounds</b>				
GaP	2.25	—	0.03	0.015
GaAs	1.42	$10^{-6}$	0.85	0.04
InSb	0.17	$2 \times 10^4$	7.7	0.07
<b>II-VI Compounds</b>				
CdS	2.40	—	0.03	—
ZnTe	2.26	—	0.03	0.01

$$n_i^2 = N_c N_v \exp \left[ \frac{-E_g}{kT} \right]$$

- In **thermal equilibrium**, the semiconductor crystal is **electrically neutral**.
- The charge-neutrality condition is used to determine the thermal equilibrium electron and hole concentration as a function of impurity doping concentration.
- **Compensated semiconductor:**
  - ✓ A semiconductor contains both **donor** and **acceptor** impurity atoms **in the same region**.
  - ✓ **N-type compensated semiconductor** ( $N_d > N_a$ )
  - ✓ **P-type compensated semiconductor** ( $N_a > N_d$ )
  - ✓ **Completely compensated semiconductor** ( $N_a = N_d$ )

# Compensation Doping

## More donors than acceptors

$$N_d - N_a \gg n_i$$

$$n = N_d - N_a$$

$$p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d - N_a}$$

- P-type semiconductor doped with  $N_a$  acceptors can be converted to n-type by simply having more  $N_d$  than  $N_a$ . The effect of donors compensates for the effect of acceptors and vice versa.
- The e- concentration is then given by  $N_d - N_a$  when  $N_a$  is larger than  $n_i$ .
- When both acceptors and donors are present, e- will **recombine** with holes, so **the mass action law** is obeyed.

## More acceptors than donors

$$N_a - N_d \gg n_i$$

$$p = N_a - N_d$$

$$n = \frac{n_i^2}{p} = \frac{n_i^2}{N_a - N_d}$$

# *n*-type Conductivity

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$$n = N_d - N_a = N_d \qquad p = \frac{n_i^2}{n} = \frac{n_i^2}{N_d}$$

$$\sigma = eN_d\mu_e + e\left(\frac{n_i^2}{N_d}\right)\mu_h \approx eN_d\mu_e$$

$\sigma$  = electrical conductivity

$e$  = electronic charge

$N_d$  = donor atom concentration in the crystal

$\mu_e$  = electron drift mobility,  $n_i$  = intrinsic concentration,

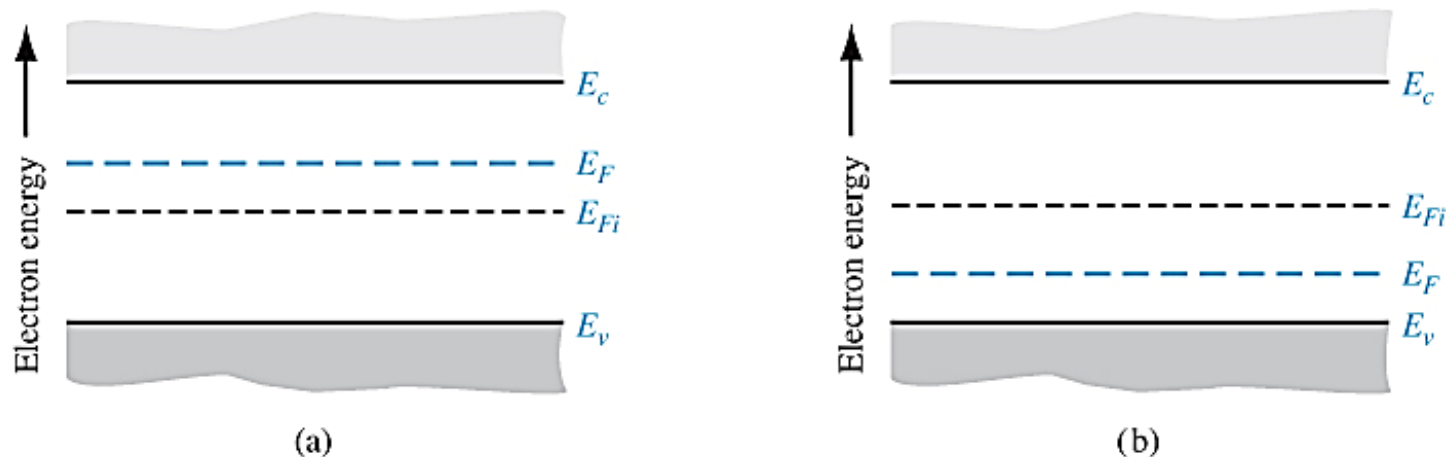
$\mu_h$  = hole drift mobility

# Degenerate and Nondegenerate

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## Nondegenerate semiconductor:

- If the doping is low, the impurity atoms are spread far enough apart so that there is **no interaction** between **donor electrons** (or acceptor holes), the impurities introduced **discrete, noninteracting energy states**.



**Figure 4.17** | Position of Fermi level for an (a) n-type ( $N_d > N_a$ ) and (b) p-type ( $N_a > N_d$ ) semiconductor.

# Degenerate and Nondegenerate

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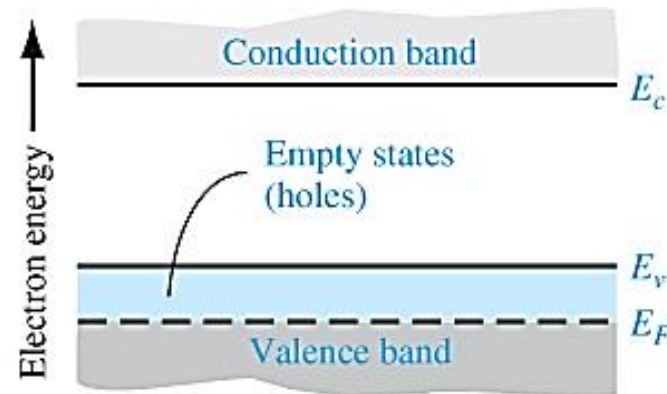
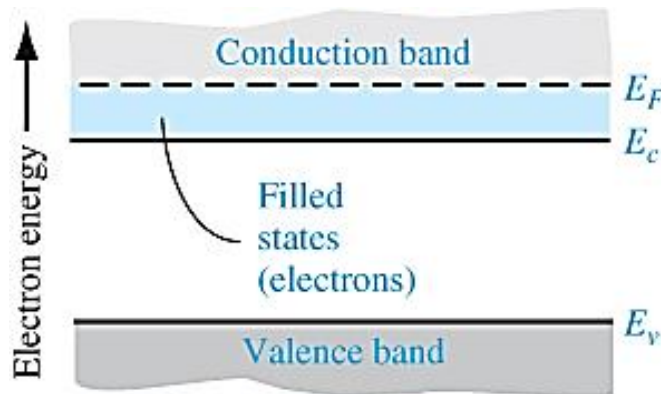
## Degenerate semiconductor:

- If the **doping is high**, donor electrons interact with each other. The single discrete donor energy will **split into a band**.
- The band may overlap the conduction band
- If the **concentration** exceeds **effective density of states ( $N_c$ )**,  $E_F$  lies within the CB for n-type semiconductor.

**Fermi level in the CB:**  
**Metallic conduction**

$$n = N_c \exp\left[-\frac{(E_c - E_F)}{kT}\right]$$

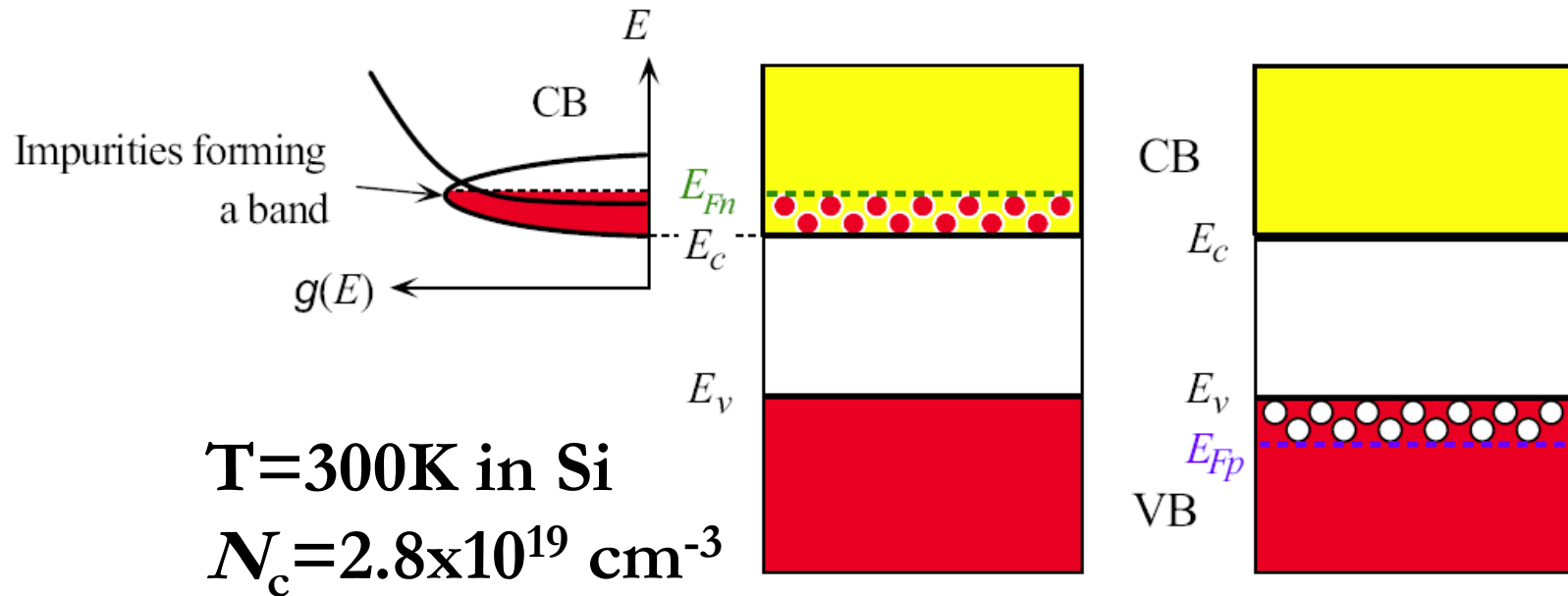
$T=300\text{K}$  in Si  
 $N_c=2.8 \times 10^{19} \text{ cm}^{-3}$





# Degenerate Semiconductors

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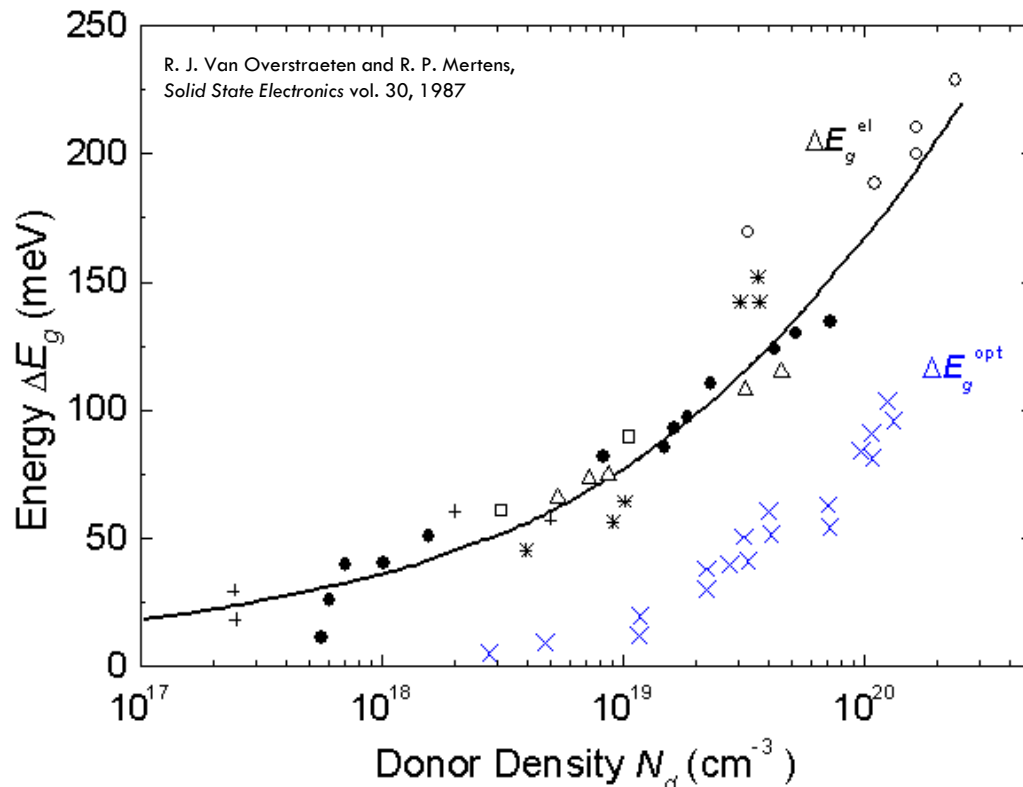
Excessively doped to  $10^{19} - 10^{20} \text{ cm}^{-3}$  (a)

(b)

- (a) Degenerate  $n$ -type semiconductor. Large number of donors form a band that overlaps the CB.
- (b) Degenerate  $p$ -type semiconductor.

# Band Gap Narrowing

- If the dopant concentration is a significant fraction of the silicon atomic density, the energy-band structure is perturbed → the band gap is reduced by  $\Delta E_G$ :



$$\Delta E_G \cong 3.5 \times 10^{-8} N^{1/3} \sqrt{\frac{300}{T}}$$

$$N = 10^{18} \text{ cm}^{-3}: \Delta E_G = 35 \text{ meV}$$

$$N = 10^{19} \text{ cm}^{-3}: \Delta E_G = 75 \text{ meV}$$

**RESISTIVITY OF INTRINSIC AND DOPED Si** Find the resistance of a  $1 \text{ cm}^3$  pure silicon crystal. What is the resistance when the crystal is doped with arsenic if the doping is 1 in  $10^9$ , that is, 1 part per billion (ppb) (note that this doping corresponds to one foreigner living in China)? Given data: Atomic concentration in silicon is  $5 \times 10^{22} \text{ cm}^{-3}$ ,  $n_i = 1.0 \times 10^{10} \text{ cm}^{-3}$ ,  $\mu_e = 1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ , and  $\mu_h = 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ .

For the intrinsic case, we apply

$$\sigma = en\mu_e + ep\mu_h = en(\mu_e + \mu_h)$$

so 
$$\begin{aligned}\sigma &= (1.6 \times 10^{-19} \text{ C})(1.0 \times 10^{10} \text{ cm}^{-3})(1350 + 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) \\ &= 2.88 \times 10^{-6} \Omega^{-1} \text{ cm}^{-1}\end{aligned}$$

Since  $L = 1 \text{ cm}$  and  $A = 1 \text{ cm}^2$ , the resistance is

$$R = \frac{L}{\sigma A} = \frac{1}{\sigma} = 3.47 \times 10^5 \Omega \quad \text{or} \quad 347 \text{ k}\Omega$$

When the crystal is doped with 1 in  $10^9$ , then

$$N_d = \frac{N_{\text{Si}}}{10^9} = \frac{5 \times 10^{22}}{10^9} = 5 \times 10^{13} \text{ cm}^{-3}$$

At room temperature all the donors are ionized, so

$$n = N_d = 5 \times 10^{13} \text{ cm}^{-3}$$

The hole concentration is

$$p = \frac{n_i^2}{N_d} = \frac{(1.0 \times 10^{10})^2}{(5 \times 10^{13})} = 2.0 \times 10^6 \text{ cm}^{-3} \ll n_i$$

Find the resistance of a  $1 \text{ cm}^3$  pure silicon crystal. What is the resistance when the crystal is doped with arsenic if the doping is 1 in  $10^9$ , that is, 1 part per billion (ppb) (note that this doping corresponds to one foreigner living in China)? Given data: Atomic concentration in silicon is  $5 \times 10^{22} \text{ cm}^{-3}$ ,  $n_i = 1.0 \times 10^{10} \text{ cm}^{-3}$ ,  $\mu_e = 1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ , and  $\mu_h = 450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}$ .

Therefore,

$$\begin{aligned}\sigma &= en\mu_e = (1.6 \times 10^{-19} \text{ C})(5 \times 10^{13} \text{ cm}^{-3})(1350 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) \\ &= 1.08 \times 10^{-2} \Omega^{-1} \text{ cm}^{-1}\end{aligned}$$

Further,

$$R = \frac{L}{\sigma A} = \frac{1}{\sigma} = 92.6 \Omega$$

Notice the drastic fall in the resistance when the crystal is doped with only 1 in  $10^9$  atoms.

Doping the silicon crystal with boron instead of arsenic, but still in amounts of 1 in  $10^9$ , means that  $N_a = 5 \times 10^{13} \text{ cm}^{-3}$ , which results in a conductivity of

$$\begin{aligned}\sigma &= ep\mu_h = (1.6 \times 10^{-19} \text{ C})(5 \times 10^{13} \text{ cm}^{-3})(450 \text{ cm}^2 \text{ V}^{-1} \text{ s}^{-1}) \\ &= 3.6 \times 10^{-3} \Omega^{-1} \text{ cm}^{-1}\end{aligned}$$

Therefore,

$$R = \frac{L}{\sigma A} = \frac{1}{\sigma} = 278 \Omega$$

The reason for a higher resistance with  $p$ -type doping compared with the same amount of  $n$ -type doping is that  $\mu_h < \mu_e$ .

# Example 5.4

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**COMPENSATION DOPING** An  $n$ -type Si semiconductor containing  $10^{16}$  phosphorus (donor) atoms  $\text{cm}^{-3}$  has been doped with  $10^{17}$  boron (acceptor) atoms  $\text{cm}^{-3}$ . Calculate the electron and hole concentrations in this semiconductor.

## EXAMPLE 5.4

### SOLUTION

This semiconductor has been compensation doped with excess acceptors over donors, so

$$N_a - N_d = 10^{17} - 10^{16} = 9 \times 10^{16} \text{ cm}^{-3}$$

This is much larger than the intrinsic concentration  $n_i = 1.0 \times 10^{10} \text{ cm}^{-3}$  at room temperature, so

$$p = N_a - N_d = 9 \times 10^{16} \text{ cm}^{-3}$$

The electron concentration

$$n = \frac{n_i^2}{p} = \frac{(1.0 \times 10^{10} \text{ cm}^{-3})^2}{(9 \times 10^{16} \text{ cm}^{-3})} = 1.1 \times 10^3 \text{ cm}^{-3}$$

Clearly, the electron concentration and hence its contribution to electrical conduction is completely negligible compared with the hole concentration. Thus, by excessive boron doping, the  $n$ -type semiconductor has been converted to a  $p$ -type semiconductor.

**EXAMPLE 5.5**

**THE FERMI LEVEL IN *n*- AND *p*-TYPE Si** An *n*-type Si wafer has been doped uniformly with  $10^{16}$  antimony (Sb) atoms  $\text{cm}^{-3}$ . Calculate the position of the Fermi energy with respect to the Fermi energy  $E_{Fi}$  in intrinsic Si. The above *n*-type Si sample is further doped with  $2 \times 10^{17}$  boron atoms  $\text{cm}^{-3}$ . Calculate the position of the Fermi energy with respect to the Fermi energy  $E_{Fi}$  in intrinsic Si. (Assume that  $T = 300$  K, and  $kT = 0.0259$  eV.)

**SOLUTION**

Sb gives *n*-type doping with  $N_d = 10^{16} \text{ cm}^{-3}$ , and since  $N_d \gg n_i (= 1.0 \times 10^{10} \text{ cm}^{-3})$ , we have

$$n = N_d = 10^{16} \text{ cm}^{-3}$$

For intrinsic Si,

$$n_i = N_c \exp\left[-\frac{(E_c - E_{Fi})}{kT}\right]$$

whereas for doped Si,

$$n = N_c \exp\left[-\frac{(E_c - E_{Fn})}{kT}\right] = N_d$$

where  $E_{Fi}$  and  $E_{Fn}$  are the Fermi energies in the intrinsic and *n*-type Si. Dividing the two expressions,

$$\frac{N_d}{n_i} = \exp\left[\frac{(E_{Fn} - E_{Fi})}{kT}\right]$$

so that

$$E_{Fn} - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right) = (0.0259 \text{ eV}) \ln\left(\frac{10^{16}}{1.0 \times 10^{10}}\right) = 0.36 \text{ eV}$$

When the wafer is further doped with boron, the acceptor concentration is

$$N_a = 2 \times 10^{17} \text{ cm}^{-3} > N_d = 10^{16} \text{ cm}^{-3}$$



# Example 5.5

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The semiconductor is compensation doped and compensation converts the semiconductor to  $p$ -type Si. Thus

$$p = N_a - N_d = (2 \times 10^{17} - 10^{16}) = 1.9 \times 10^{17} \text{ cm}^{-3}$$

For intrinsic Si,

$$\underline{n_i = N_v \exp\left[-\frac{(E_{Fi} - E_v)}{kT}\right]}$$

whereas for doped Si,

$$\underline{p = N_v \exp\left[-\frac{(E_{Fp} - E_v)}{kT}\right]} = N_a - N_d$$

where  $E_{Fi}$  and  $E_{Fp}$  are the Fermi energies in the intrinsic and  $p$ -type Si, respectively. Dividing the two expressions,

$$\frac{p}{n_i} = \exp\left[-\frac{(E_{Fp} - E_{Fi})}{kT}\right]$$

so that

$$\begin{aligned} E_{Fp} - E_{Fi} &= -kT \ln\left(\frac{p}{n_i}\right) = -(0.0259 \text{ eV}) \ln\left(\frac{1.9 \times 10^{17}}{1.0 \times 10^{10}}\right) \\ &= -0.43 \text{ eV} \end{aligned}$$

# Outline

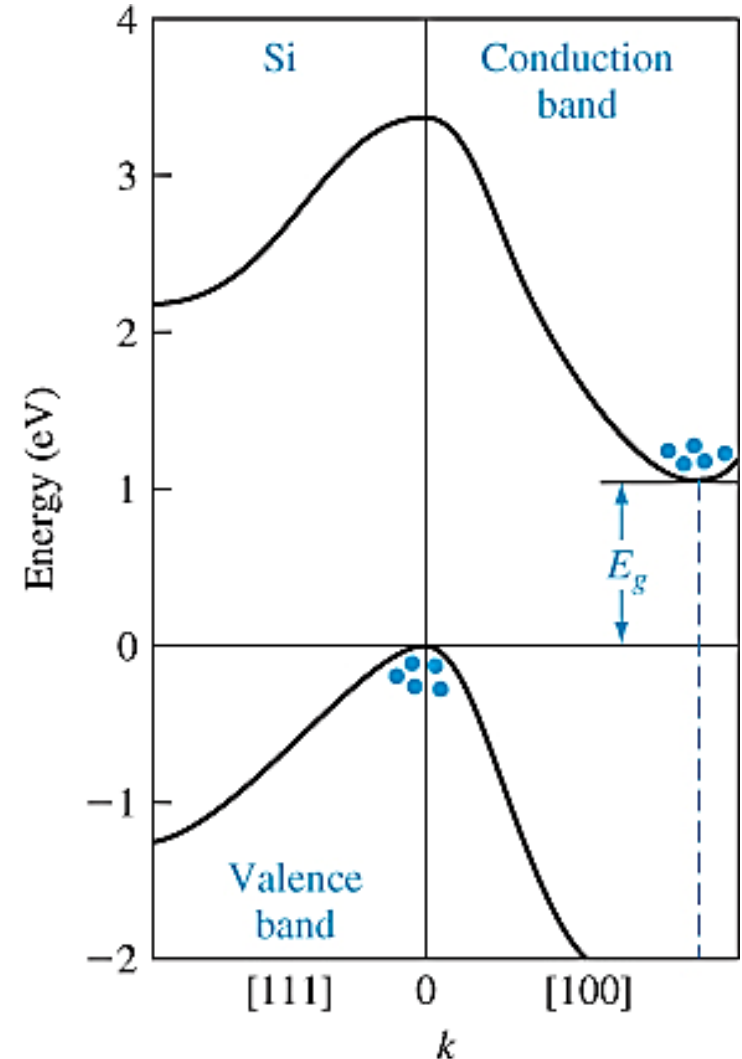
- Carrier Concentration and Mass Action Law
- The Fermi Energy
- Conductivity of Extrinsic Si
- **Direct and Indirect Recombination**
- Carrier Injection



# Extending Energy Band Theory to 3D

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- The **E-k diagram** is symmetric in  $k$  so that there is no new information in the negative axis.
- It is practice to plot E-k with [100] direction along + $k$  axis, and [111] direction along  $-k$  axis.

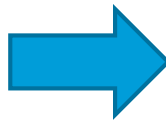


# E-k Diagrams of Si and GaAs

50

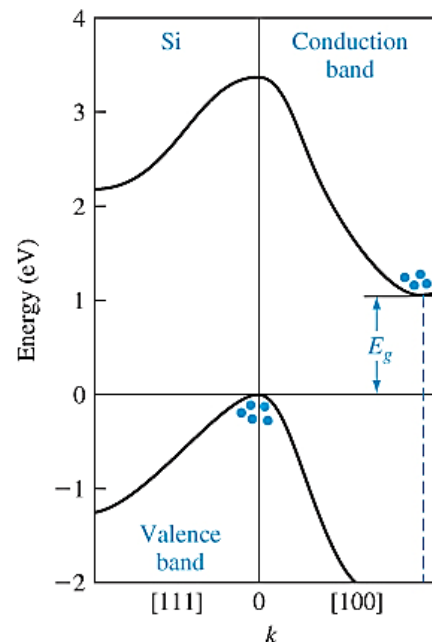
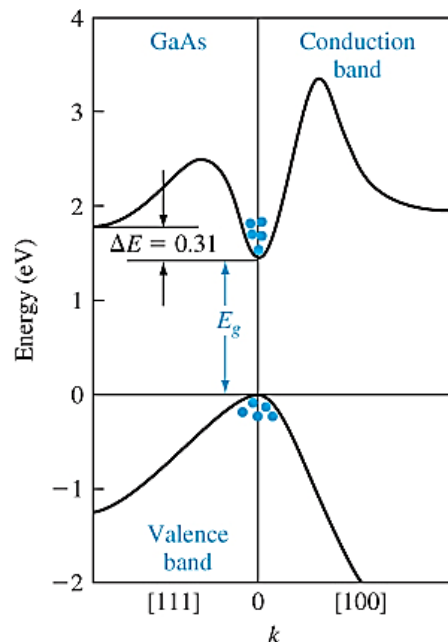
- Effective mass can be different along 3 different k-vectors. Therefore, we consider average effective mass.

$$\text{mass: } \frac{1}{m^*} = \frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k^2}$$



$\frac{d^2 E}{dk^2}$  is larger in GaAs than in silicon

So  $m^*$  in GaAs <  $m^*$  in silicon



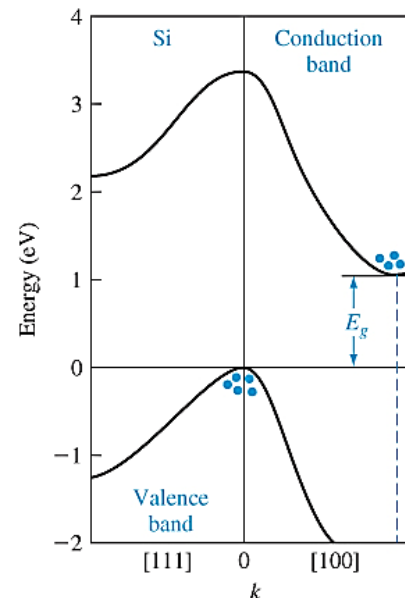
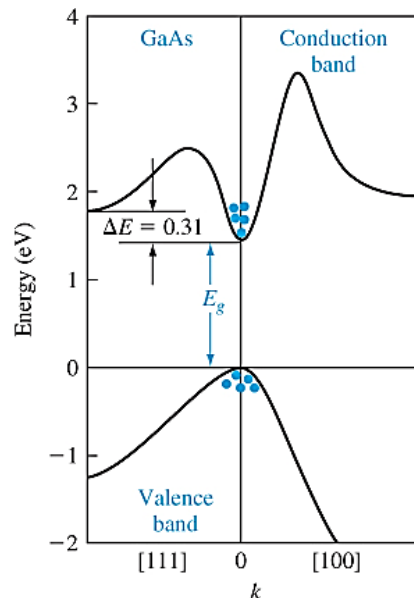
P.25

# Direct & Indirect Bandgap

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- **Direct bandgap**: minimum CB energy and maximum VB energy occur at same  $k$  value.
- **Indirect bandgap**: the minimum CB is not at  $k=0$  so that the transitions for electron between CB & VB includes **an interaction with the crystal**. Then, **crystal momentum is conserved**.
- Direct bandgap semiconductors are used for making optical devices.

*Direct Eg  
Transition  
Ex. GaAs*

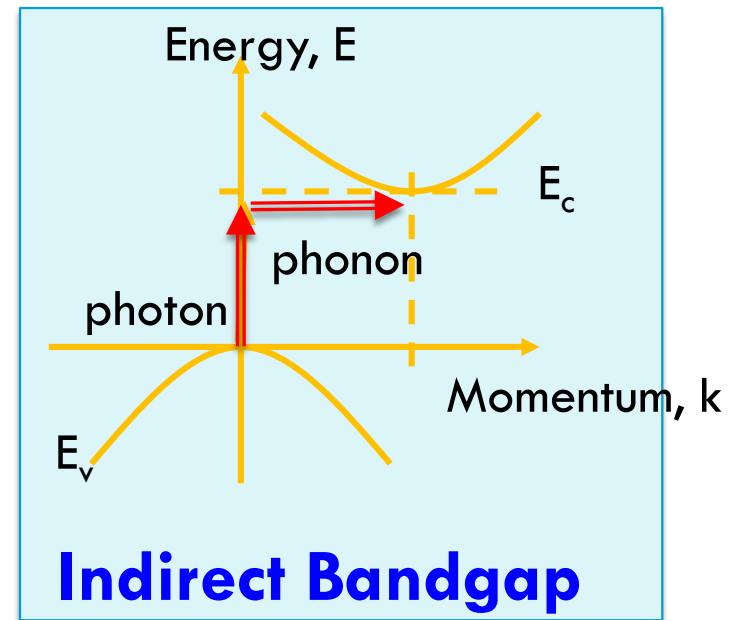
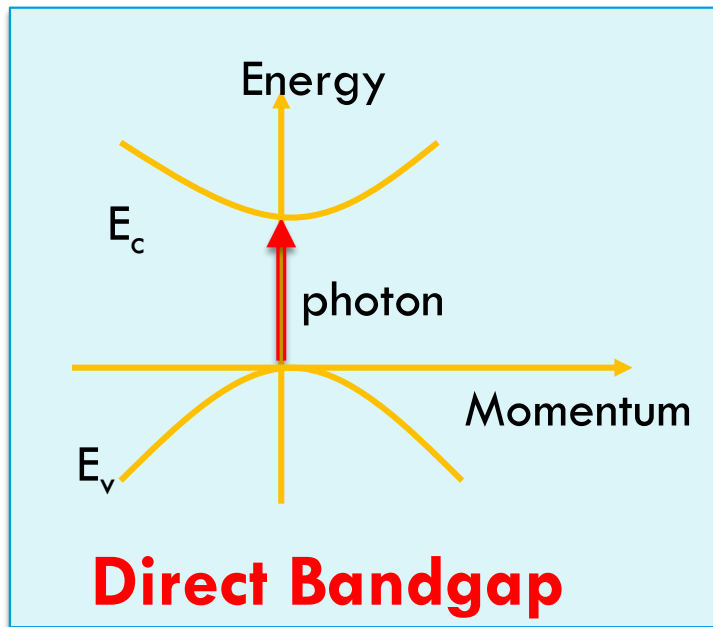


*Indirect Eg  
Transition  
Ex. Si*

# Direct & Indirect Bandgap

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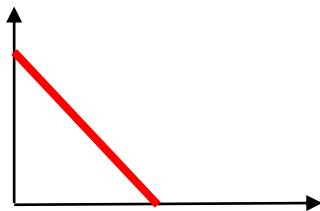
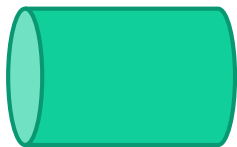
- The direct and indirect band gap behavior is reflected in **absorption coefficients** in solar cell.



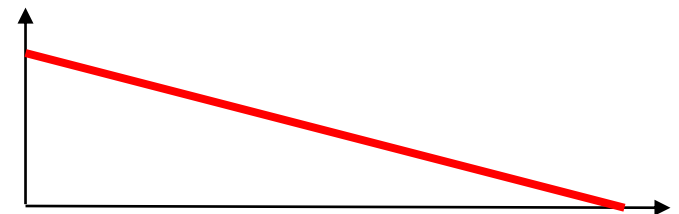
- **High** absorption probability
- **Thinner** material is required

- **Low** absorption probability
- **Thicker** material is required

absorption



without absorption



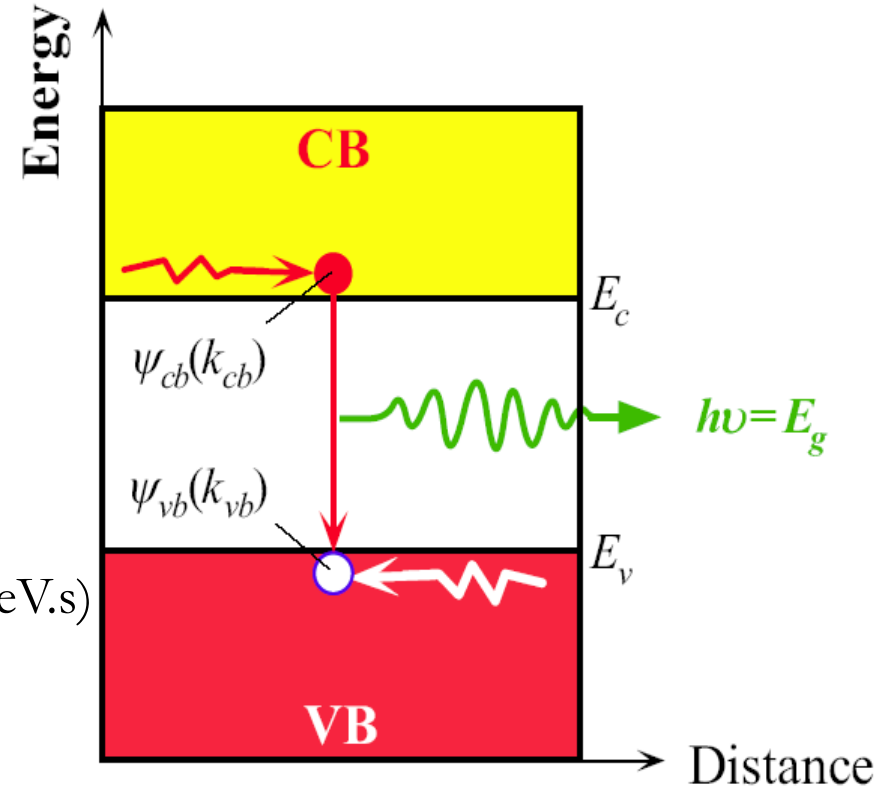
# Direct Recombination

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$$E = h\nu = h \frac{c}{\lambda}$$

$$E(eV) = \frac{1.2398}{\lambda (\mu m)}$$

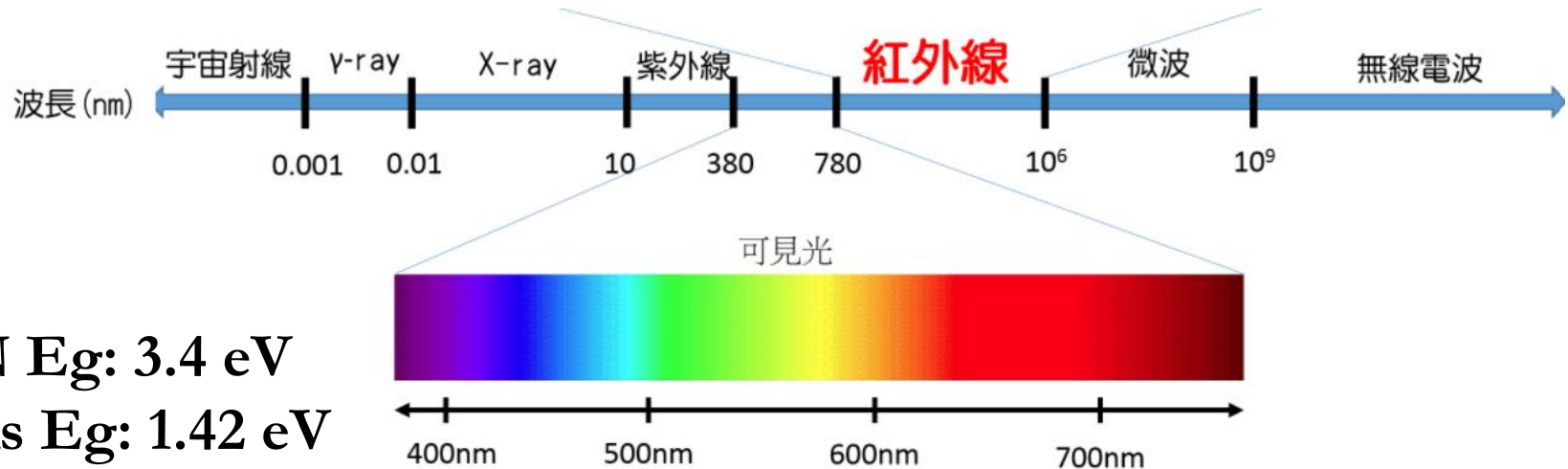
h: Planck constant ( $6.626 \times 10^{-34}$  J.s =  $4.13 \times 10^{-15}$  eV.s)  
 c is the speed of light in vacuum ( $3 \times 10^{14}$   $\mu m/s$ )  
 $\lambda$  is the photon's wavelength



**Direct recombination in GaAs and InSb**

**Wavevector  $k_{cb} = k_{vb}$  so that momentum conservation is satisfied.**

# Wavelength of the Light

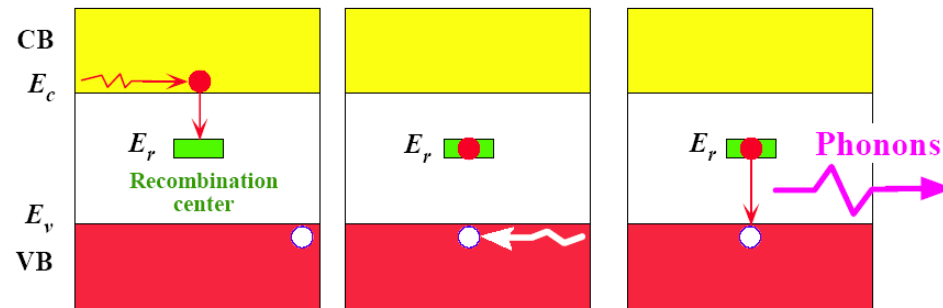


GaN  $E_g$ : 3.4 eV  
GaAs  $E_g$ : 1.42 eV

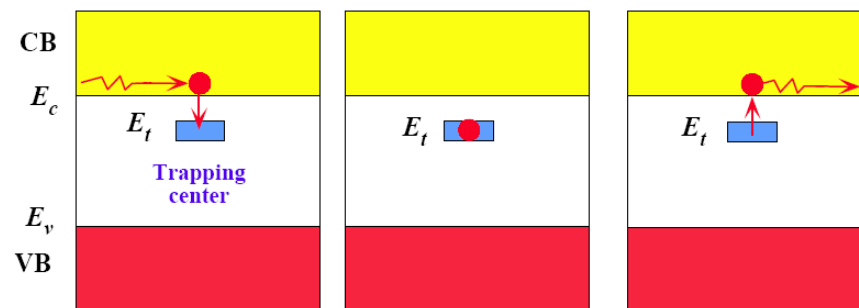
# Indirect Recombination

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- (a) **Recombination** in Si via a recombination center which has a localized energy level at  $E_r$  near the middle of  $E_g$ .
- (b) **Trapping** and detrapping of electrons by trapping centers. A trapping center has a localized energy level near  $E_c$ .



(a) Recombination



(b) Trapping



# Indirect Recombination

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- In real semiconductors, there are some crystal defects and these defects create discrete electronic energy states within the forbidden energy band.
- Recombination through the defect (trap) states is called indirect recombination.

- Shockley-Read-Hall recombination:

Assumes that a single trap center exists at an energy  $E_t$  within the bandgap.

# Outline

- Carrier Concentration and Mass Action Law
- The Fermi Energy
- Conductivity of Extrinsic Si
- Direct and Indirect Recombination
- **Carrier Injection**

# Low-level Injection

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## □ Injection :

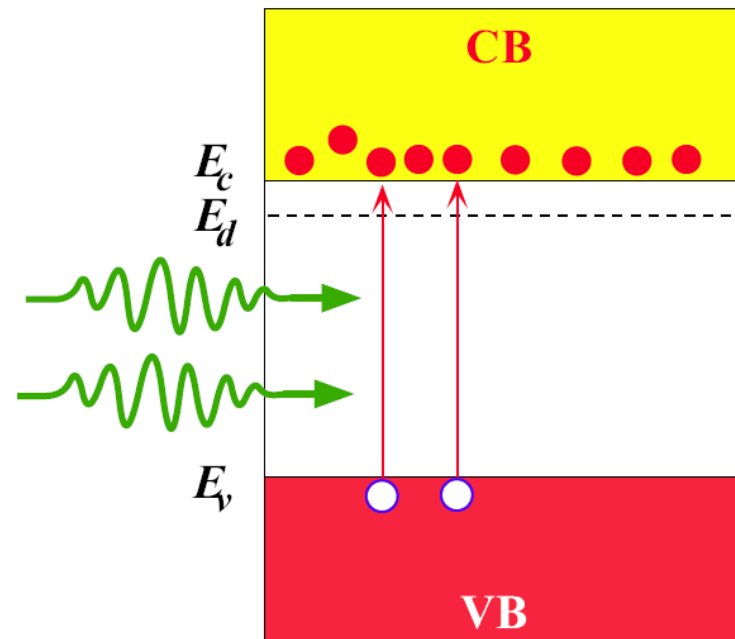
A process of introducing excess carriers in the semiconductor.

□ Low-level photoinjection into an  $n$ -type semiconductor in which  $n_n < n_{n0}$

□  $n_n = n_{n0} + \Delta n_n$

□  $p_n = p_{n0} + \Delta p_n$

□  $\Delta n_n = \Delta p_n$  ( $e^-h^+$  pair)



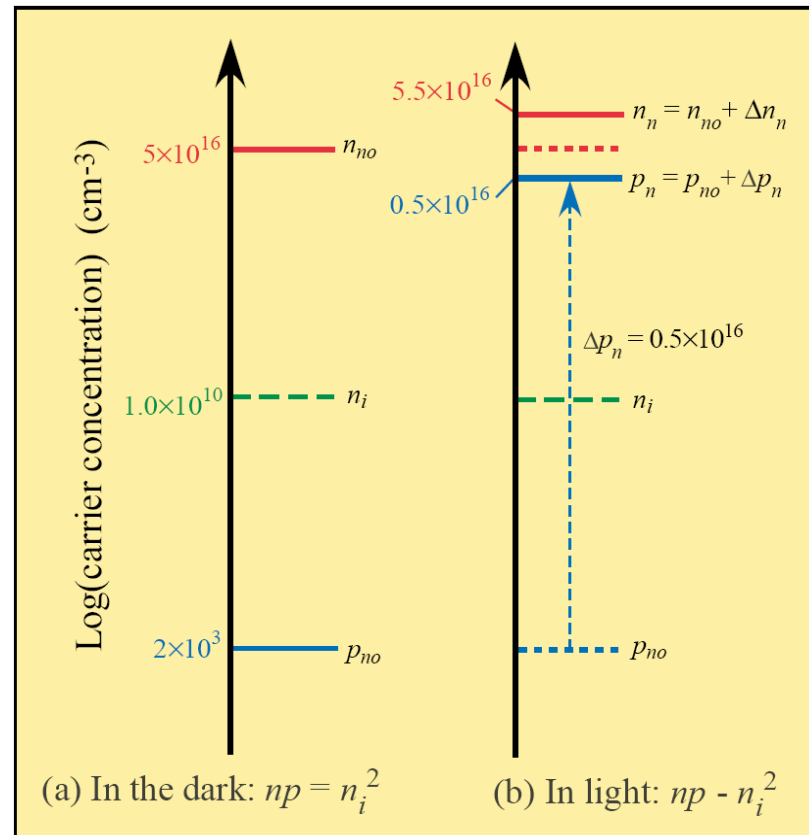
# Low-level Injection

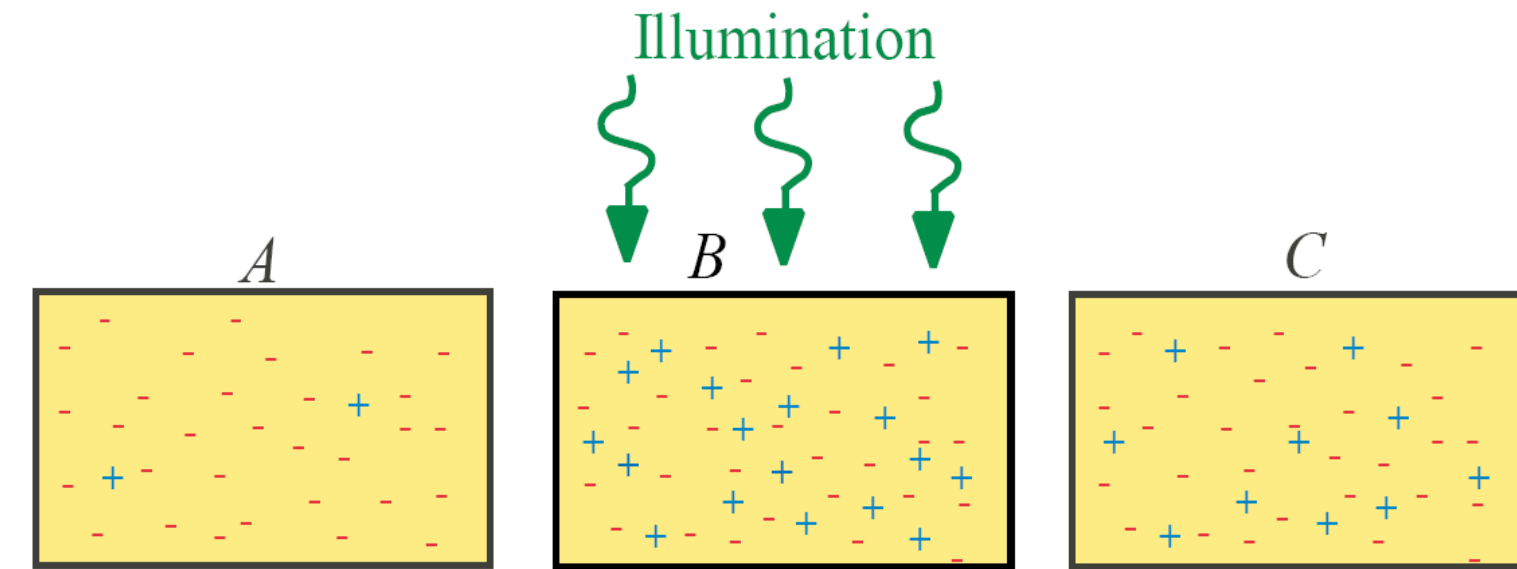
60

- Low-level injection in an  $n$ -type semiconductor does not significantly affect  $n_n$  but drastically affects the minority carrier concentration  $p_n$ .

$$\Delta n_n = 0.1 n_{n0} = 0.5 \times 10^{16} \text{ cm}^{-3}$$

$$\Delta p_n = \Delta n_n = 0.5 \times 10^{16} \text{ cm}^{-3}$$





*n*-type semiconductor in the dark.

$$p_n = p_{no} \ll n_{no}$$

Illumination with  $h\nu > E_g$  creates excess holes:

$$p_n = p_{no} + \Delta p_n = \Delta n_n$$

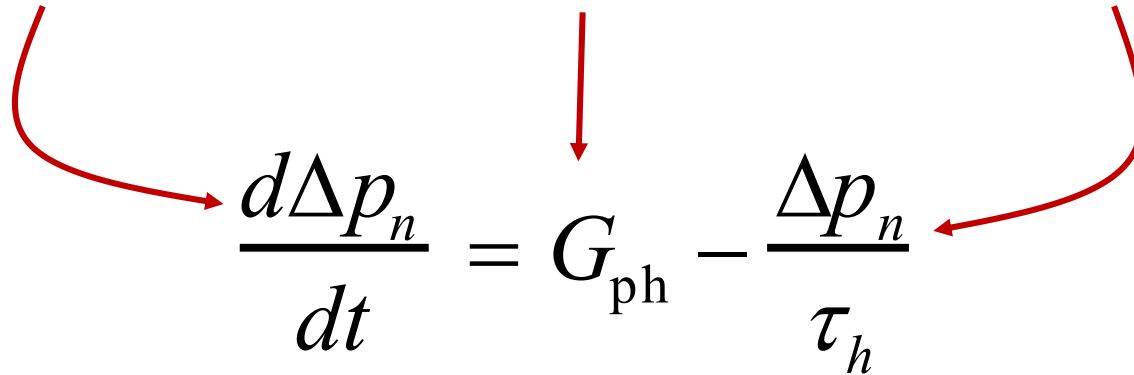
In dark after illumination. Excess holes are disappearing by recombination.

Light is switched on (B), then off again (C)

Illumination of an *n*-type semiconductor results in excess electron and hole concentrations.

After the illumination, the **recombination process** restores equilibrium; **the excess electrons and holes simply recombine.**

Rate of increase in excess  $h^+$  = Rate of photogeneration - Rate of recombination of excess  $h^+$

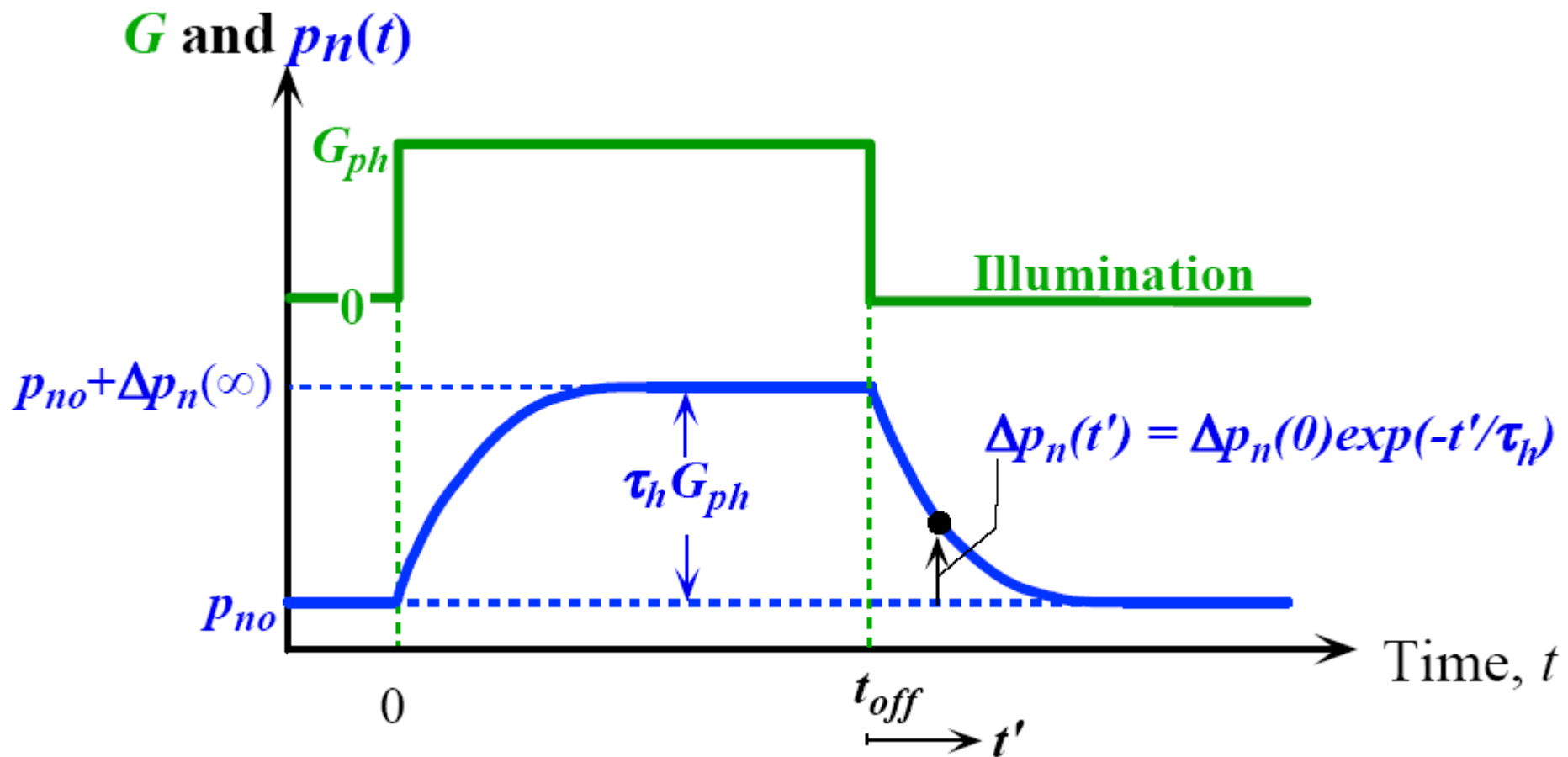

$$\frac{d\Delta p_n}{dt} = G_{ph} - \frac{\Delta p_n}{\tau_h}$$

$$\Delta p_n(t') = \Delta p_n(0) e^{-\left(\frac{t'}{\tau_h}\right)}$$

$\Delta p_n$  = excess hole (minority carrier) concentration in  $n$ -type

$G_{ph}$  = rate of photogeneration

$\tau_h$  = **minority carrier lifetime** (mean recombination time)



- ❑ Illumination is switched on at time  $t = 0$  and then off at  $t = t_{off}$ .
- ❑ The excess minority carrier concentration  $p_n(t)$  rises exponentially to its steady-state value with a time constant  $\tau_h$ . From  $t_{off}$ , the excess minority carrier concentration decays exponentially to its equilibrium value.

**PHOTORESPONSE TIME** Sketch the hole concentration when a step illumination is applied to an  $n$ -type semiconductor at time  $t = 0$  and switched off at time  $t = t_{\text{off}} (\gg \tau_h)$ .

**EXAMPLE 5.10**

**SOLUTION**

We use Equation 5.27 with  $G_{\text{ph}} = \text{constant}$  in  $0 \leq t \leq t_{\text{off}}$ . Since Equation 5.27 is a first-order differential equation, integrating it we simply find

$$\ln \left[ G_{\text{ph}} - \left( \frac{\Delta p_n}{\tau_h} \right) \right] = -\frac{t}{\tau_h} + C_1$$

where  $C_1$  is the integration constant. At  $t = 0$ ,  $\Delta p_n = 0$ , so  $C_1 = \ln G_{\text{ph}}$ . Therefore the solution is

$$\Delta p_n(t) = \tau_h G_{\text{ph}} \left[ 1 - \exp \left( -\frac{t}{\tau_h} \right) \right] \quad 0 \leq t < t_{\text{off}} \quad [5.28]$$

We see that as soon as the illumination is turned on, the minority carrier concentration rises exponentially toward its steady-state value  $\Delta p_n(\infty) = \tau_h G_{\text{ph}}$ . This is reached after a time  $t > \tau_h$ .

At the instant the illumination is switched off, we assume that  $t_{\text{off}} \gg \tau_h$  so that from Equation 5.28,

$$\Delta p_n(t_{\text{off}}) = \tau_h G_{\text{ph}}$$

We can define  $t'$  to be the time measured from  $t = t_{\text{off}}$ , that is,  $t' = t - t_{\text{off}}$ . Then

$$\Delta p_n(t' = 0) = \tau_h G_{\text{ph}}$$

Solving Equation 5.27 with  $G_{\text{ph}} = 0$  in  $t > t_{\text{off}}$  or  $t' > 0$ , we get

$$\Delta p_n(t') = \Delta p_n(0) \exp \left( -\frac{t'}{\tau_h} \right)$$

where  $\Delta p_n(0)$  is actually an integration constant that is equivalent to the boundary condition on  $\Delta p_n$  at  $t' = 0$ . Putting  $t' = 0$  and  $\Delta p_n = \tau_h G_{\text{ph}}$  gives

$$\Delta p_n(t') = \tau_h G_{\text{ph}} \exp \left( -\frac{t'}{\tau_h} \right) \quad [5.29]$$

We see that the excess minority carrier concentration decays exponentially from the instant the light is switched off with a time constant equal to the minority carrier recombination time. The time evolution of the minority carrier concentration is sketched in Figure 5.27.



# Wafer Electrical Testing

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## Micromanipulator Prober (Parametric Testing)

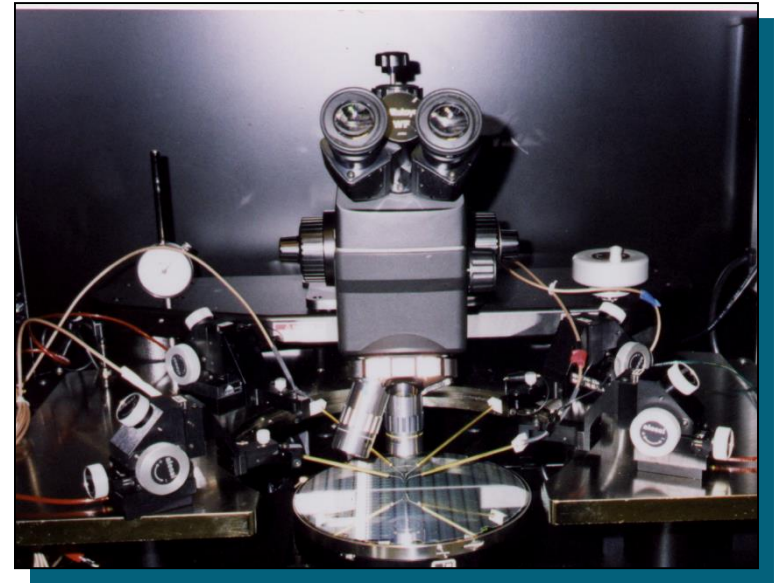


Photo courtesy of Advanced Micro Devices

# References

1. Slides from Prof. Jaeger, Auburn University
2. Slides from Profs. W. Hu and J. B. Lee, UTD
3. Donald A. Neamen, Semiconductor Physics and Devices: Basic Principles, 4th ed., McGraw-Hill
4. Principles of Electronic Materials and Devices, Third Edition, S. O. Kasap, McGraw-Hill, 2006