電子材料與元件技術特論

SPECIAL TOPICS IN ELECTRONIC MATERIALS AND DEVICES

The pn Junction

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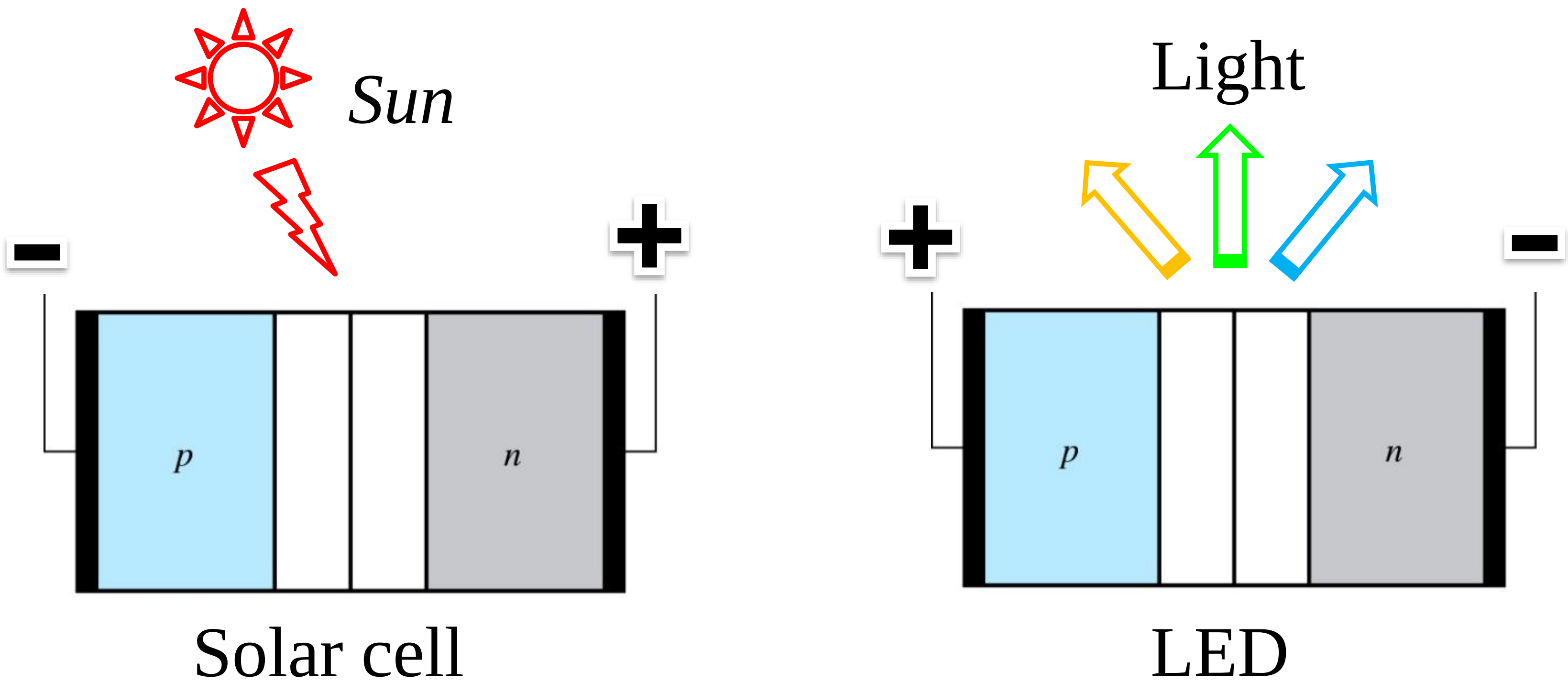
Outline

- **The pn Junction**
- **Basic Structure of the pn Junction**
- **Zero Applied Bias**
  - Electric Field in the Depletion Region
  - Potential in in the Depletion Region
  - Space Charge Width
- **Reverse Applied Bias**
  - Space Charge Width and Electric Field
  - Junction Capacitance
  - One-Sided Junctions

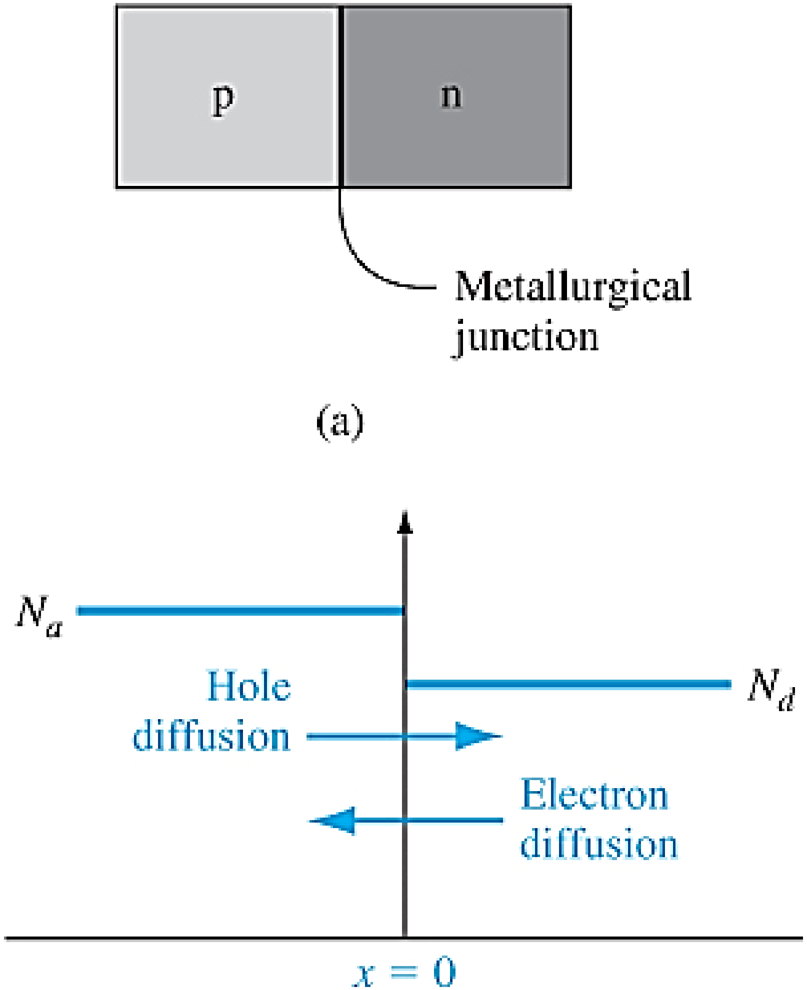
Carrier Transport in Optoelecronics



- Junction carrier transport is responsible for device performance and efficiency in temperature-sensitive solar cell & LED.

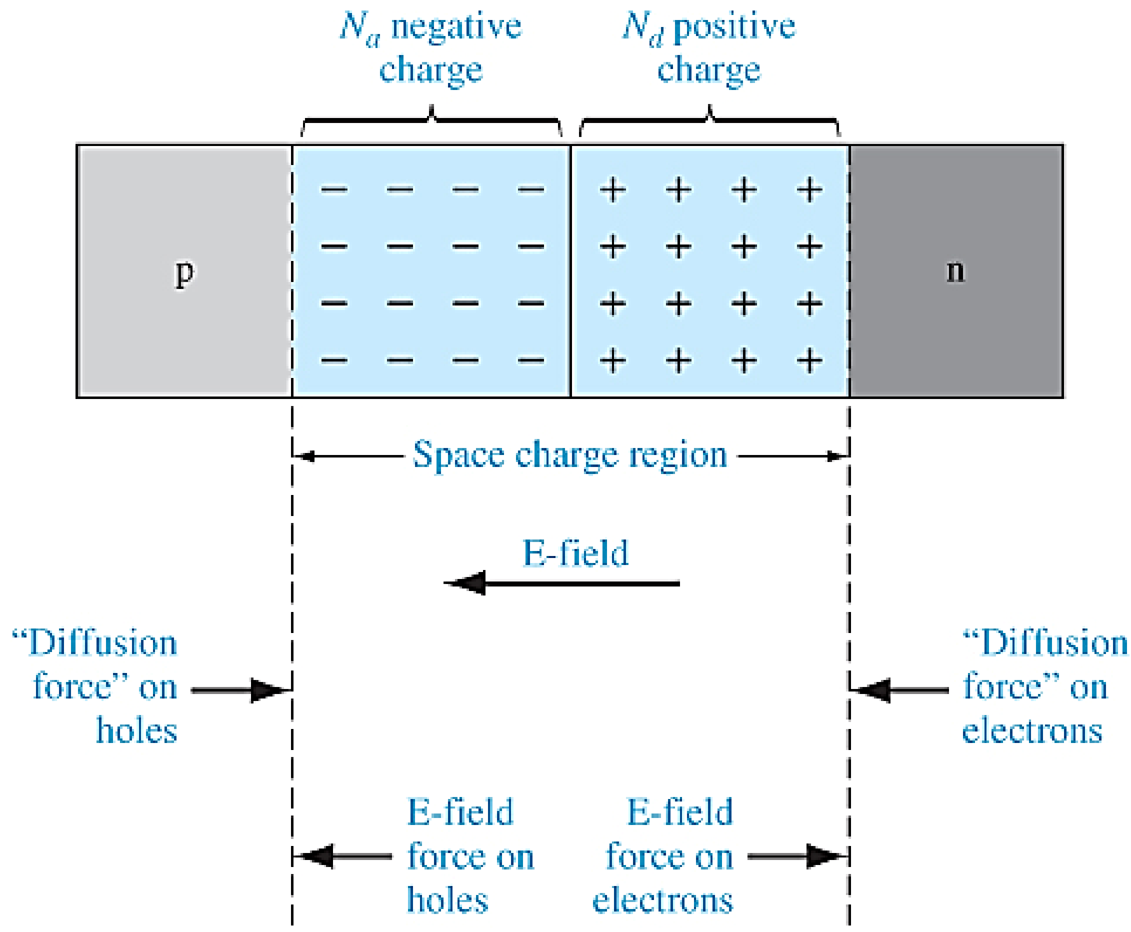
The pn Junction

- The entire semiconductor is a single crystal.
- The interface is called the metallurgical junction.
- A step junction with uniform doping in each region and an abrupt change in doping at the interface.
- Electrons diffuse from the __________ to __________ and holes diffuse in the reverse direction.



Basic Structure of the pn Junction

- The negatively and positively charged regions are called the space charge region, or the ________________.

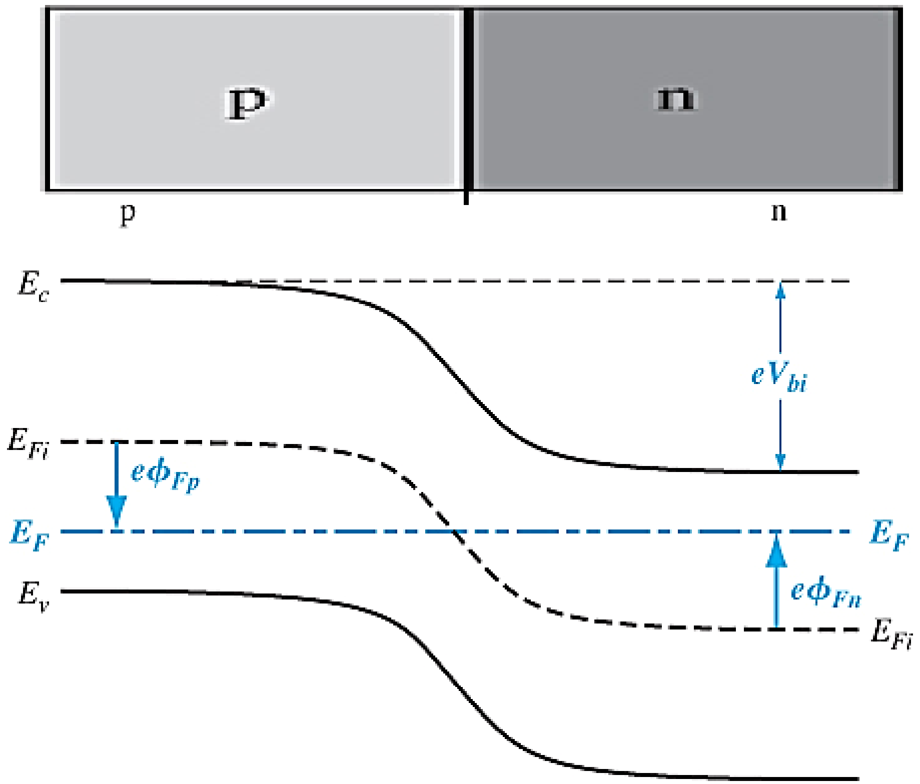


- An electric field is established in the direction from the ___________ to the ___________.

Built-in Potential Barrier

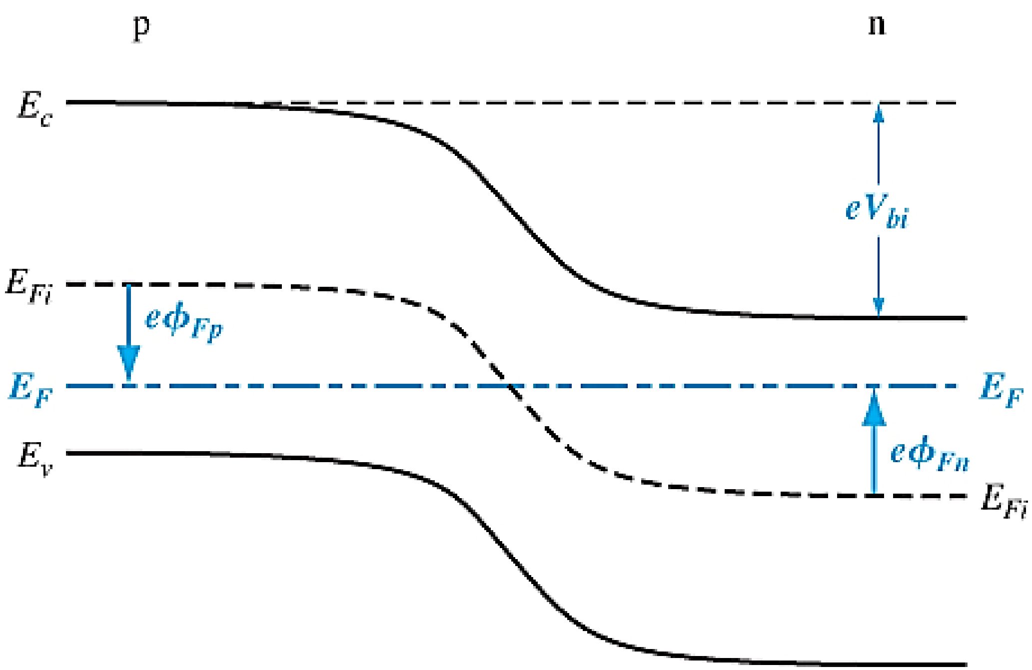
❖ Energy-band diagram of a pn junction in thermal equilibrium ➔ No current, No external excitation, Constant Fermi energy

❖ The fermi level of pn junction must be uniform in equilibrium.



Built-in Potential Barrier

- The CB and VB must bend since the relative position of the Fermi level to the CB and VB is different for the p-type and n-type semiconductor.
- The band bending produces a potential barrier, which is referred to as the **built-in potential barrier**.



$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$$

$$e\phi_{\mathrm{Fn}} = E_{\mathrm{F}} - E_{\mathrm{Fi}}$$

$$e\phi_{Fp} = E_{\mathrm{Fi}} - E_{\mathrm{F}}$$

Built-in Potential Barrier

❖ Previously $\boldsymbol{N_a}$ and $\boldsymbol{N_d}$ denoted the concentrations in the same region. From now on, they will denote the net concentrations in the individual p- and n-regions, respectively.

For the n-region:

$$n_0 = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = n_i \exp\left[\frac{E_F - E_{Fi}}{kT}\right] = N_d$$

$$e\phi_{Fn} = E_F - E_{Fi} = kT \ln\left(\frac{N_d}{n_i}\right)$$

For the p-region:

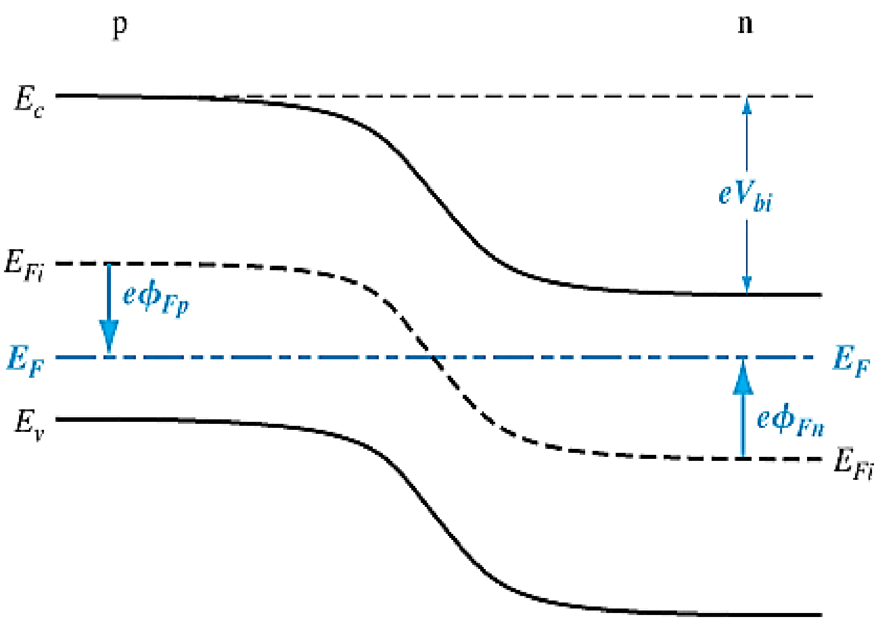
$$p_0 = N_v \exp\left[\frac{-(E_F - E_v)}{kT}\right] = n_i \exp\left[\frac{E_{Fi} - E_F}{kT}\right] = N_a$$

$$\phi_{Fp} = +\frac{kT}{e} \ln\left(\frac{N_a}{n_i}\right)$$

Built-in Potential Barrier

$$\phi_{\mathrm{Fn}} = E_{\mathrm{F}} - E_{\mathrm{Fi}} = \frac{kT}{e}\ln\left(\frac{N_{\mathrm{d}}}{n_{\mathrm{i}}}\right)$$

$$\phi_{Fp} = +\frac{kT}{e}\ln\left(\frac{N_a}{n_i}\right)$$



$$V_{bi} = |\phi_{Fn}| + |\phi_{Fp}|$$

Built-in Potential Barrier:

$$\boxed{V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)}$$

V_t is the thermal voltage

Example

Q: At 300K, the doping concentration on the p-side and n-side of a silicon diode are 2 x 10^{17}cm^{-3} and 10^{15} cm^{-3}, respectively. What is the built-in voltage of Si diode?

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left[\frac{(2\times 10^{17})(10^{15})}{(1.5\times 10^{10})^2}\right] = 0.713\ \text{V}$$

The built-in potential of a P N junction diode is 0.7 V at room temperature. What will be the approximate value of built-in potential if the doping concentrations on both sides are doubled?

Given built-in voltage of the diode is 0.7 V

Now doping concentrations on both sides are doubled. i.e, $N_A' = 2N_A$ and $N_D' = 2N_D$

Newly built-in potential is:

$$V_0' = \frac{KT}{q}\ln\left(\frac{2N_A \times 2N_D}{n_i^2}\right)$$

$$V_0' == \frac{KT}{q}\ln\left(\frac{4N_A N_D}{n_i^2}\right)$$

$$V_0' = \frac{KT}{q}\ln(4) + \frac{KT}{q}\ln\left(\frac{N_A N_D}{n_i^2}\right)$$

$V_0' = 0.03604 + 0.7$

$V_0' = 0.73604$ V

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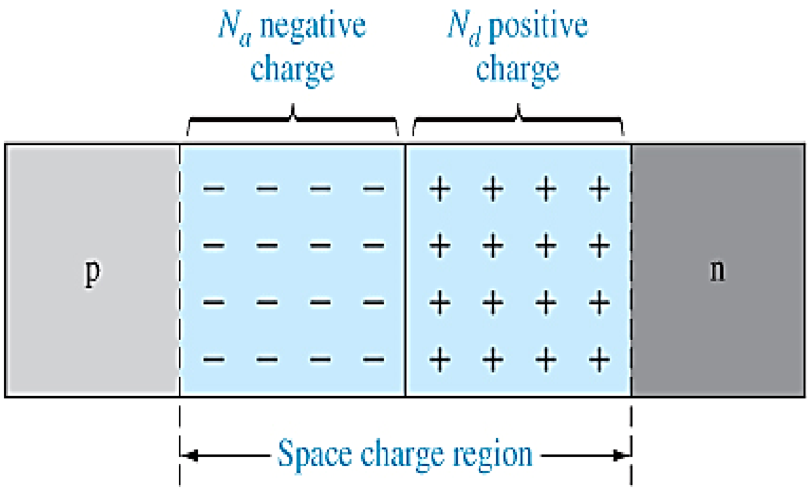
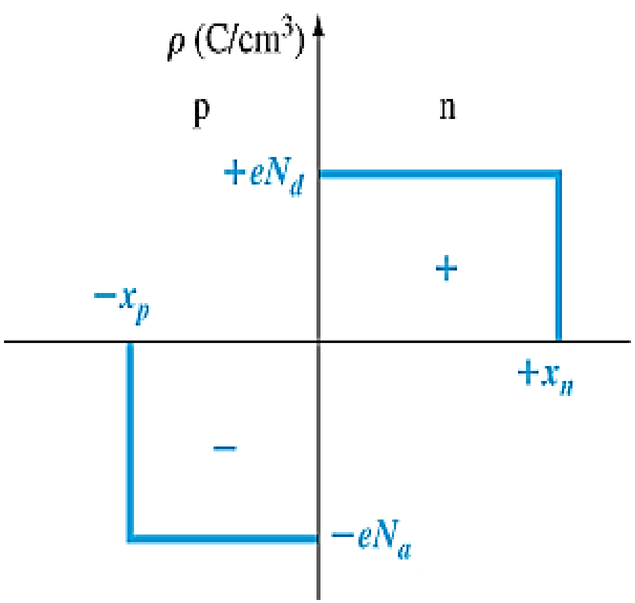
Electric Field





❖ Assume the space charge region abruptly ends in the n-region at x = +x_n and in the p-region at x = -x_p.

Charge Density (C/cm^3):

$$\rho(x) = eN_d \qquad 0 < x < x_n$$

$$\rho(x) = -eN_a \qquad -x_p < x < 0$$

Electric Field

❖ **Poisson's equation**:

$$\frac{d^2\phi(x)}{dx^2} = -\frac{d\mathrm{E}(x)}{dx} = \frac{-\rho(x)}{\epsilon_s}$$

Si permittivity: $\varepsilon_s = 11.7 \times (8.85 \times 10^{-14})$ F/cm.

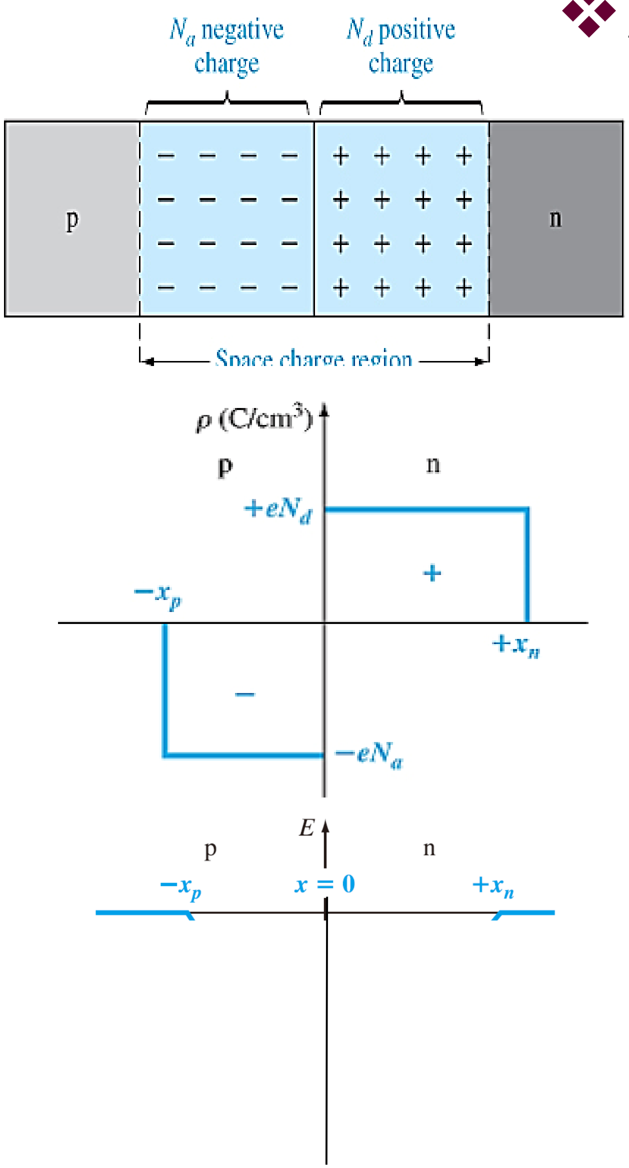
❖ In the p-region:

$$\rho(x) = -eN_a \qquad -x_p < x < 0$$

$$\mathrm{E} = \int \frac{\rho(x)}{\epsilon_s} dx = -\int \frac{eN_a}{\epsilon_s} dx = \frac{-eN_a}{\epsilon_s} x + C_1$$

Electric Field in the p-region

❖ At $x < -x_p$, E = 0 in the neutral p-region:



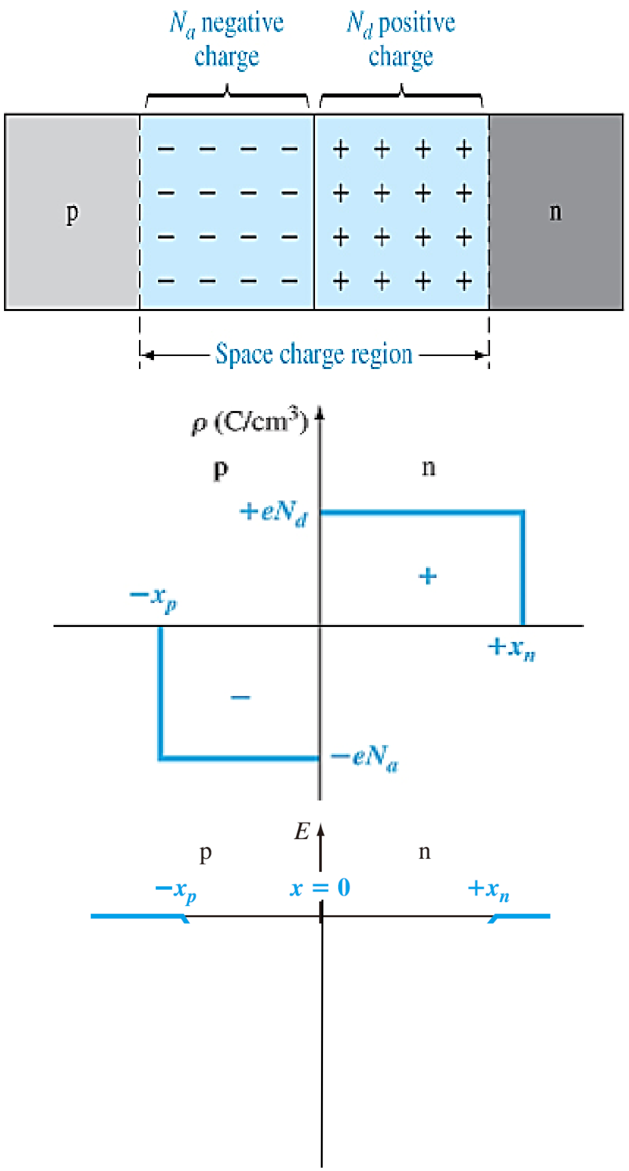
$$E = \int \frac{\rho(x)}{\epsilon_s} dx = -\int \frac{eN_a}{\epsilon_s} dx = \frac{-eN_a}{\epsilon_s} x + C_1$$

$$\Rightarrow \quad C_1 = -\frac{eN_a}{\varepsilon_s} x_p$$

$$\boxed{E = \frac{-eN_a}{\varepsilon_s}\left(x + x_p\right)}$$

$$-x_p \le x \le 0$$

Electric Field in the n-region



- At $x > x_n$, E=0 in the neutral n-region:

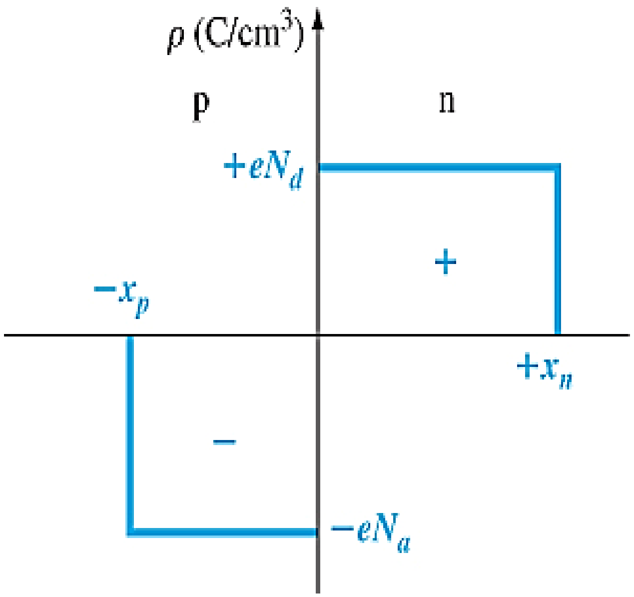
$$E = \int \frac{(eN_d)}{\epsilon_s} dx = \frac{eN_d}{\epsilon_s} x + C_2$$

$$C_2 = -\frac{eN_d}{\varepsilon_s} x_n$$

$$\boxed{E = \frac{-eN_d}{\epsilon_s}(x_n - x)}$$

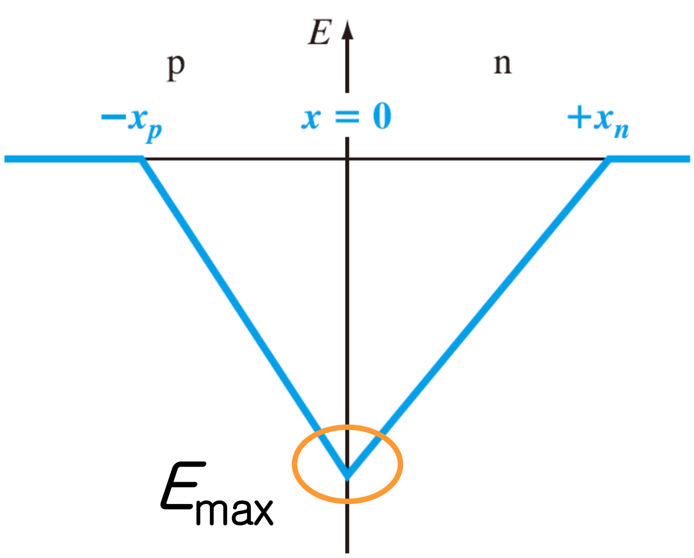
$$0 \le x \le x_n$$

Electric Field in Depletion Region



$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_s} = -\frac{dE(x)}{dx}$$

- The electric field must be continuous as there are no surface charge densities within the pn junction structure.



$$\boxed{E_{\max} = \frac{-eN_d x_n}{\varepsilon_s} = \frac{-eN_a x_p}{\varepsilon_s}}$$

$$\boxed{N_a x_p = N_d x_n}$$

Potential in Depletion Region

❖ **Poisson's equation**:

$$\frac{d^2\phi(x)}{dx^2} = \frac{-\rho(x)}{\epsilon_s} = -\frac{d\mathrm{E}(x)}{dx}$$

❖ **The potential in the p-region:**

$$\phi(x) = -\int E(x)dx = \int \frac{eN_{\mathrm{a}}}{\varepsilon_{\mathrm{s}}}\left(x + x_{\mathrm{p}}\right)dx$$

$$\phi(x) = \frac{eN_{\mathrm{a}}}{\varepsilon_{\mathrm{s}}}\left(\frac{x^2}{2} + x_{\mathrm{p}} \cdot x\right) + C_1'$$

❖ Setting the potential to be zero at x = -x $_p$: $C_1' = \frac{eN_a}{2\epsilon_s} x_p^2$

$$\phi(x) = \frac{eN_a}{2\epsilon_s}(x + x_p)^2 \qquad (-x_p \le x \le 0)$$

Potential in Depletion Region

❖ **The potential in the n-region:**

$$\phi(x) = -\int E(x)dx = \int \frac{eN_\mathrm{d}}{\varepsilon_\mathrm{s}}(x_\mathrm{n} - x)dx$$

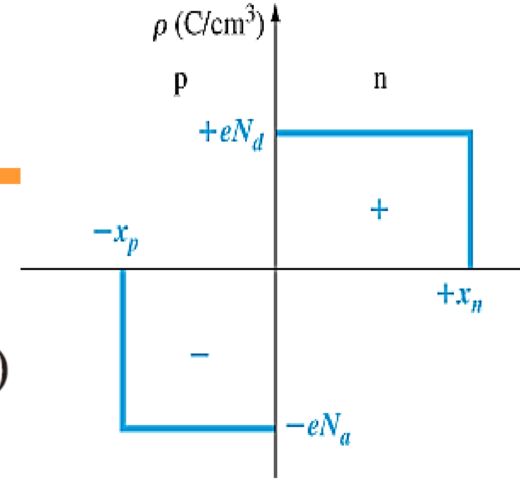
$$\phi(x) = \frac{eN_\mathrm{d}}{\varepsilon_\mathrm{s}}\left(x_\mathrm{n} \cdot x - \frac{x^2}{2}\right) + C_2'$$

❖ The potential is continuous at x = 0:

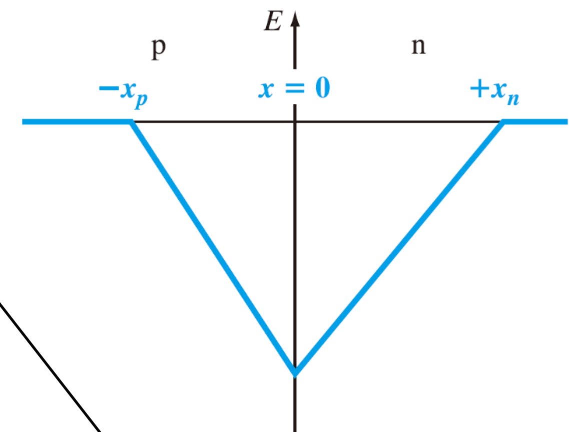
$$C_2' = \frac{eN_\mathrm{a}}{2\varepsilon_\mathrm{s}} x_\mathrm{p}^2$$

$$\phi(x) = \frac{eN_d}{\epsilon_s}\left(x_n \cdot x - \frac{x^2}{2}\right) + \frac{eN_a}{2\epsilon_s} x_p^2 \qquad (0 \le x \le x_n)$$

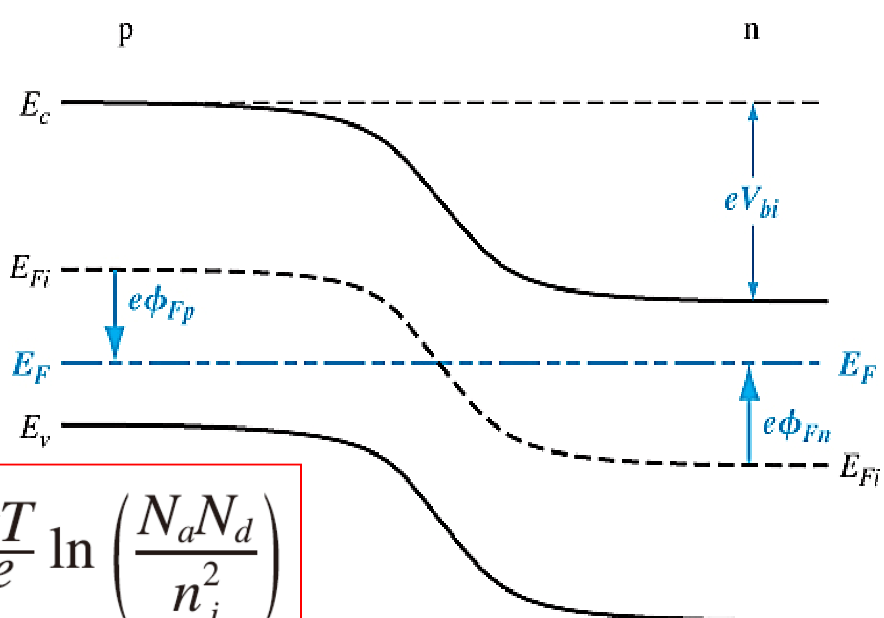
Potential Distribution

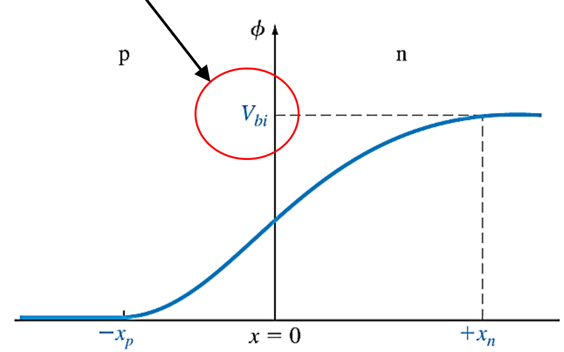


$$\phi(x) = \frac{eN_d}{\epsilon_s}\left(x_n \cdot x - \frac{x^2}{2}\right) + \frac{eN_a}{2\epsilon_s}x_p^2 \qquad (0 \le x \le x_n)$$

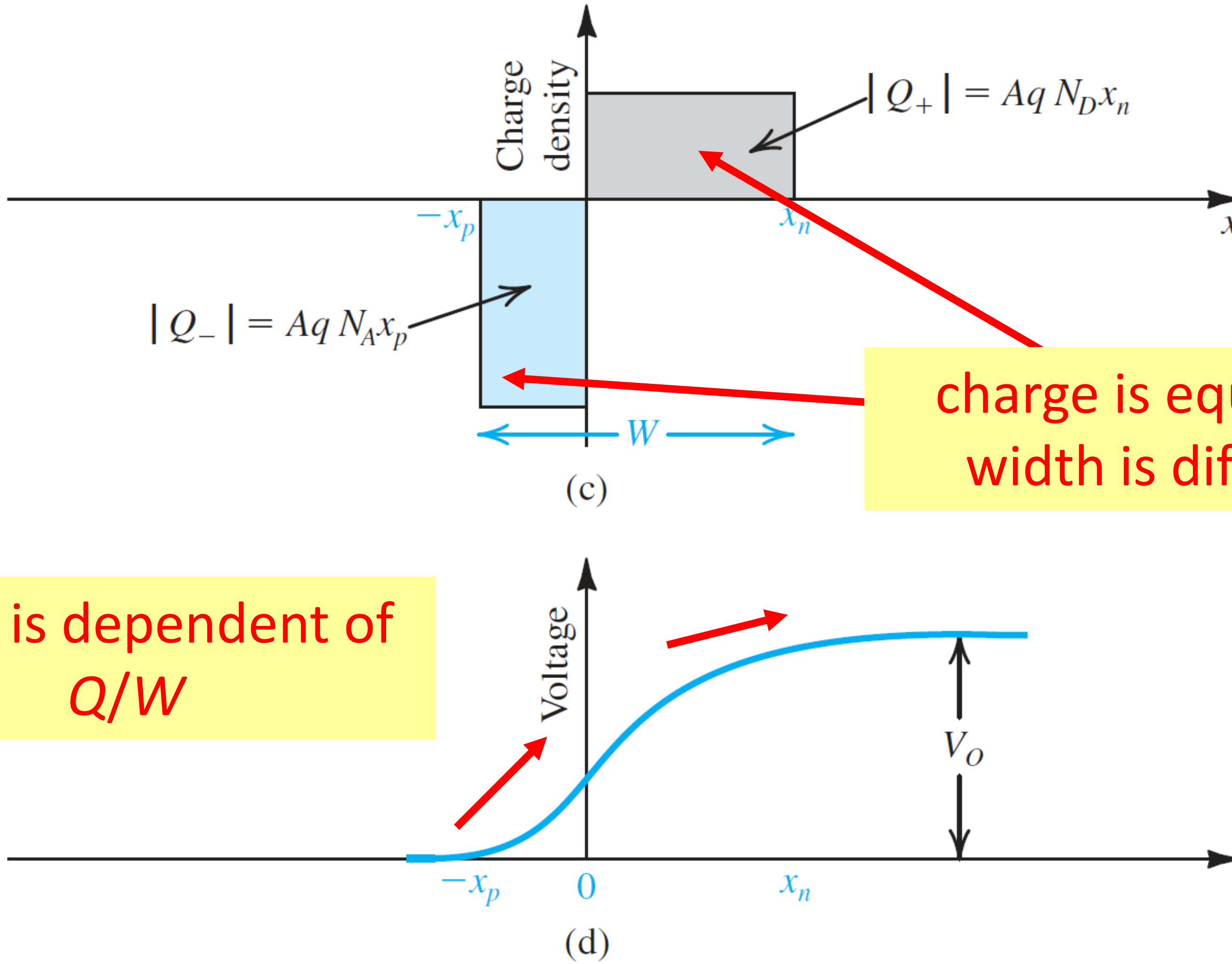


$$\boxed{V_{bi} = |\phi(x = x_n)| = \frac{e}{2\epsilon_s}\left(N_d x_n^2 + N_a x_p^2\right)}$$





$$\boxed{V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right)}$$



charge is equal, but width is different

dv/dx is dependent of *Q/W*

Figure 1.37 (c) The charge stored in both sides of the depletion region; $Q_J = |Q_+| = |Q_-|$. **(d)** The built-in voltage V_0.

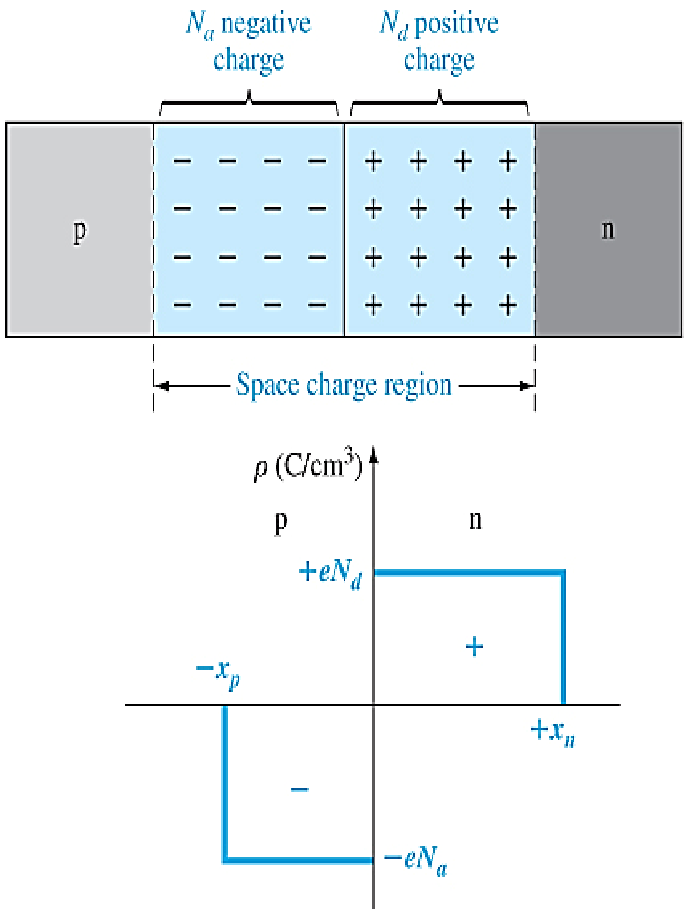
Space Charge Width

$$N_a x_p = N_d x_n \Rightarrow x_p = \frac{N_d x_n}{N_a}$$

p.15

$$V_{bi} = \frac{e}{2\epsilon_s}\left(N_d x_n^2 + N_a x_p^2\right)$$

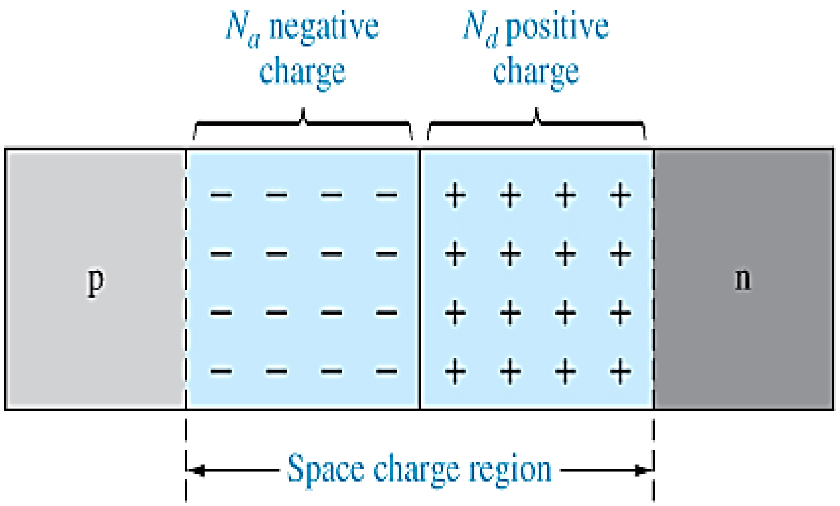
$$V_{bi} = \frac{e}{2\varepsilon_s}\left(N_d x_n^2 + \frac{N_d^2 x_n^2}{N_a}\right)$$

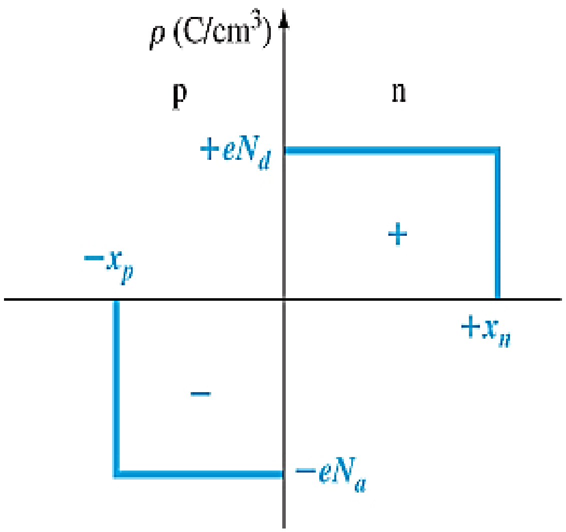


$$\boxed{x_n = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{1/2}}$$

$$\boxed{x_p = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a + N_d}\right]\right\}^{1/2}}$$

Space Charge Width





$$\boxed{x_n = \frac{N_a}{N_a + N_d} W \qquad x_p = \frac{N_d}{N_a + N_d} W}$$

$$W = x_n + x_p$$

$$\boxed{W = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}}$$

Example

❖ Consider a silicon pn junction at T=300 K with doping concentrations of $N_a=10^{16}$ cm^{-3} and $N_d=10^{15}$ cm^{-3}. Calculate the space charge width and maximum electric field in the junction.

$$V_{bi} = \left(\frac{kT}{e}\right)\ln\left(\frac{N_a N_d}{n_i^2}\right) = (0.0259)\ln\left[\frac{(10^{16})(10^{15})}{(1.5\times10^{10})^2}\right] = 0.635\text{V}$$

$$W = \left\{\frac{2\epsilon_s V_{bi}}{e}\left[\frac{N_a + N_d}{N_a N_d}\right]\right\}^{1/2}$$

$$= \left\{\frac{2(11.7)(8.85\times10^{-14})(0.635)}{1.6\times10^{-19}}\left[\frac{10^{16}+10^{15}}{(10^{16})(10^{15})}\right]\right\}^{1/2}$$

$$= 0.951\times10^{-4}\text{ cm} = 0.951\ \mu\text{m}$$

$$E_{max} = -\frac{eN_d x_n}{\epsilon_s} = -\frac{(1.6\times10^{-19})(10^{15})(0.8644\times10^{-4})}{(11.7)(8.85\times10^{-14})} = -1.34\times10^4\text{ V/cm}$$

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  - **Space Charge Width and Electric Field**
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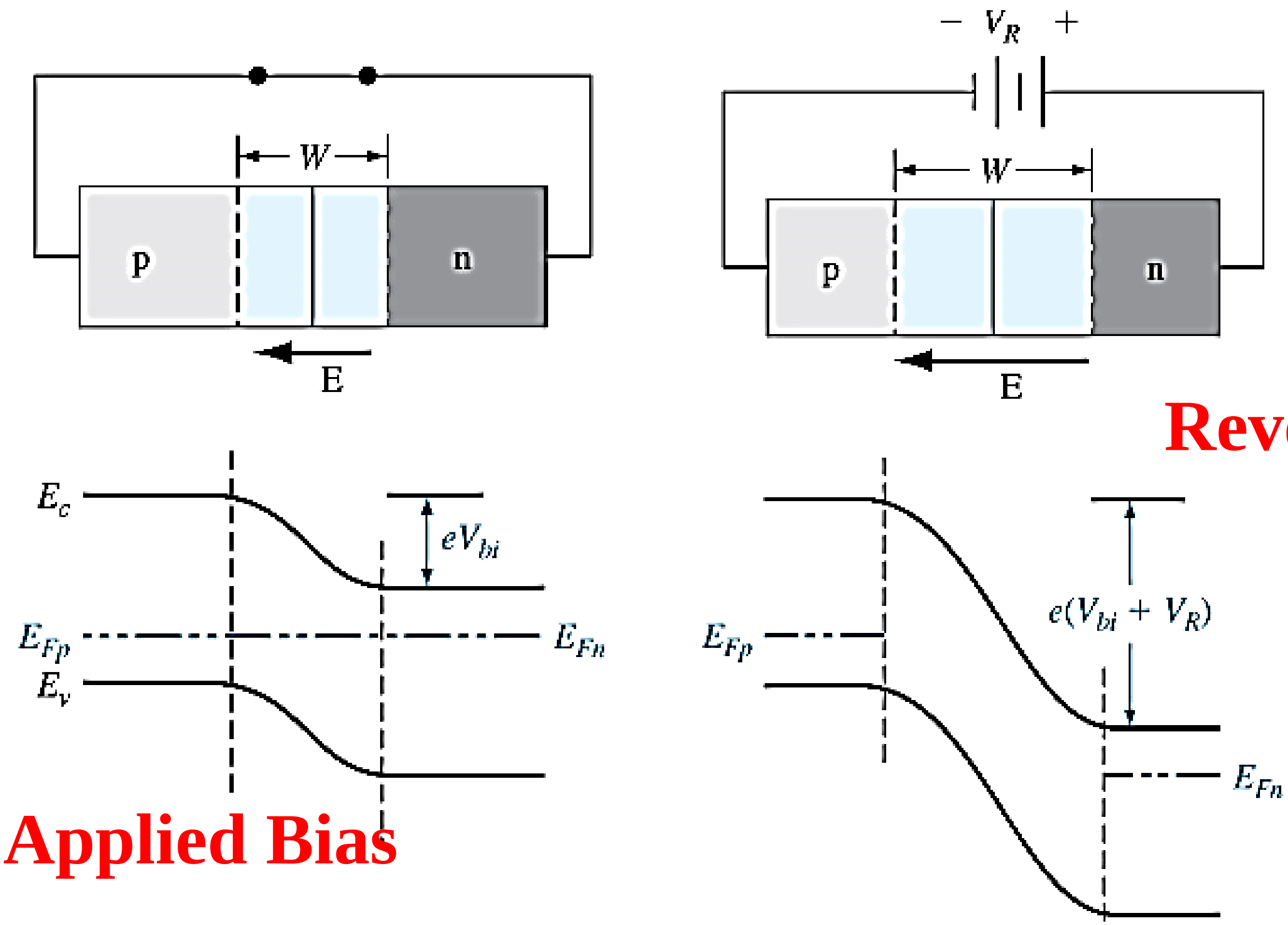
Energy Band Diagram

❖ When a reverse bias is applied to a pn junction, no current will be induced in the device.

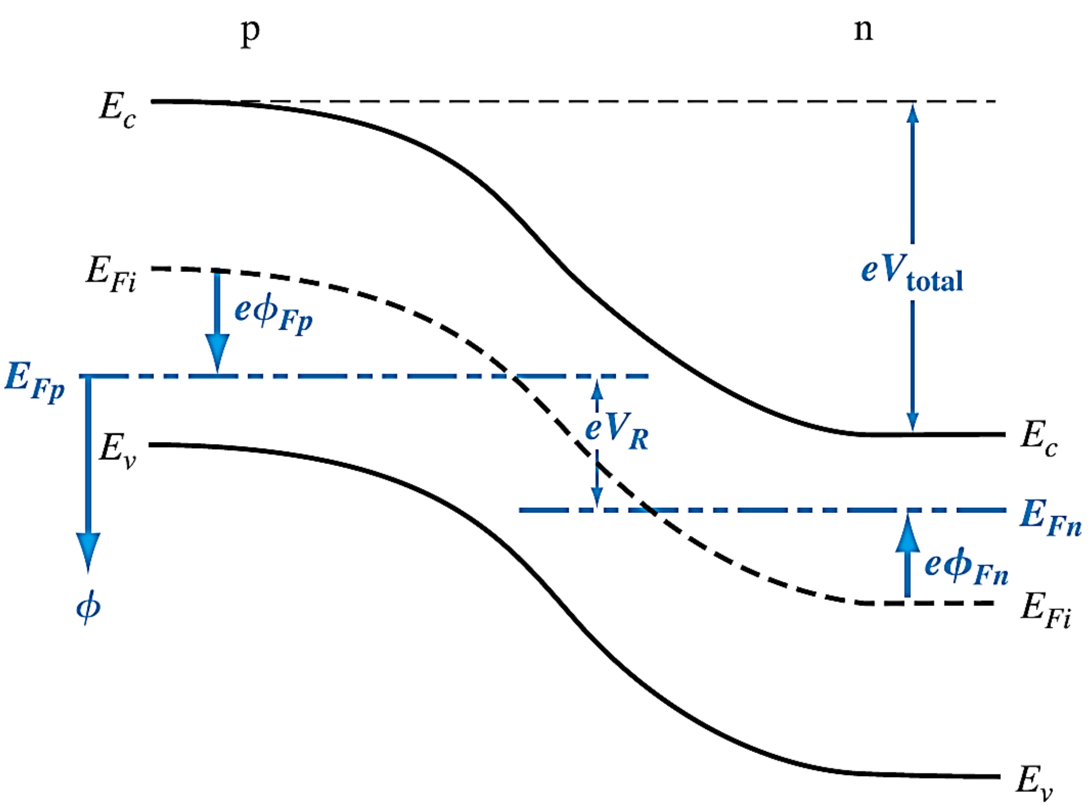


Reverse Bias

Zero Applied Bias

Reverse Applied Bias

- A positive voltage is applied to the n-region; a negative voltage is applied to the p-region.
- The Fermi level in the n-region moves further downward.



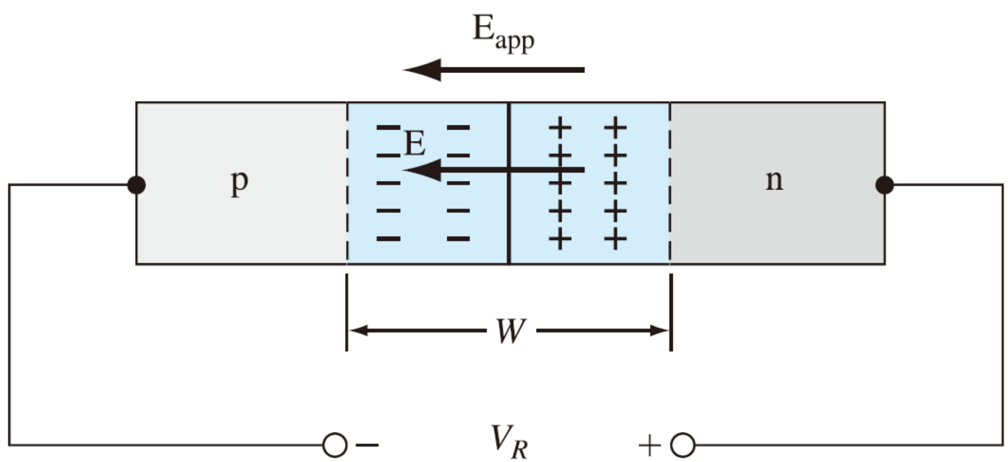
Total potential barrier:

$$V_{total} = V_{bi} + V_R$$

V_R is the reverse bias

Space Charge Width under Reverse Bias

- The charge densities are constant under complete ionization.
- Under a reverse bias, the space charge width must increase in order to increase the number of positive and negative charges within the space charge region.
- Therefore, electric field in the depletion region must increase.



$$W = \left\{ \frac{2\epsilon_s V_{bi}}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

$$W = \left\{ \frac{2\epsilon_s (V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}$$

Example

❖ Consider a silicon pn junction at T=300 K with doping concentrations of $N_a=10^{16}$ cm^{-3} and $N_d=10^{15}$ cm^{-3}. Calculate the space charge under a reverse bias of 5V. $n_i=1.5 \times 10^{10}$ cm^{-3}.

$$\boxed{W = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[\frac{N_a + N_d}{N_a N_d} \right] \right\}^{1/2}}$$

$$W = \left\{ \frac{2(11.7)(8.85 \times 10^{-14})(0.635 + 5)}{1.6 \times 10^{-19}} \left[\frac{10^{16} + 10^{15}}{(10^{16})(10^{15})} \right] \right\}^{1/2}$$

$$W = 2.83 \times 10^{-4} \text{ cm} = 2.83\ \mu\text{m}$$

$W = 0.95$ µm without reverse bias.

Electric Field under Reverse Applied Bias

- The electric field is still a linear function of distance.
- The maximum electric field is still at the metallurgical junction.

$$\mathrm{E}_{\max} = -\left\{\frac{2e(V_{bi}+V_R)}{\epsilon_s}\left(\frac{N_aN_d}{N_a+N_d}\right)\right\}^{1/2}$$

$$x_n = \left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{N_a}{N_d}\right]\left[\frac{1}{N_a+N_d}\right]\right\}^{1/2}$$

$$\mathrm{E}_{\max} = \frac{-2(V_{bi}+V_R)}{W}$$

$$x_p = \left\{\frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a+N_d}\right]\right\}^{1/2}$$

$$W = \left\{\frac{2\epsilon_s(V_{bi}+V_R)}{e}\left[\frac{N_a+N_d}{N_aN_d}\right]\right\}^{1/2}$$

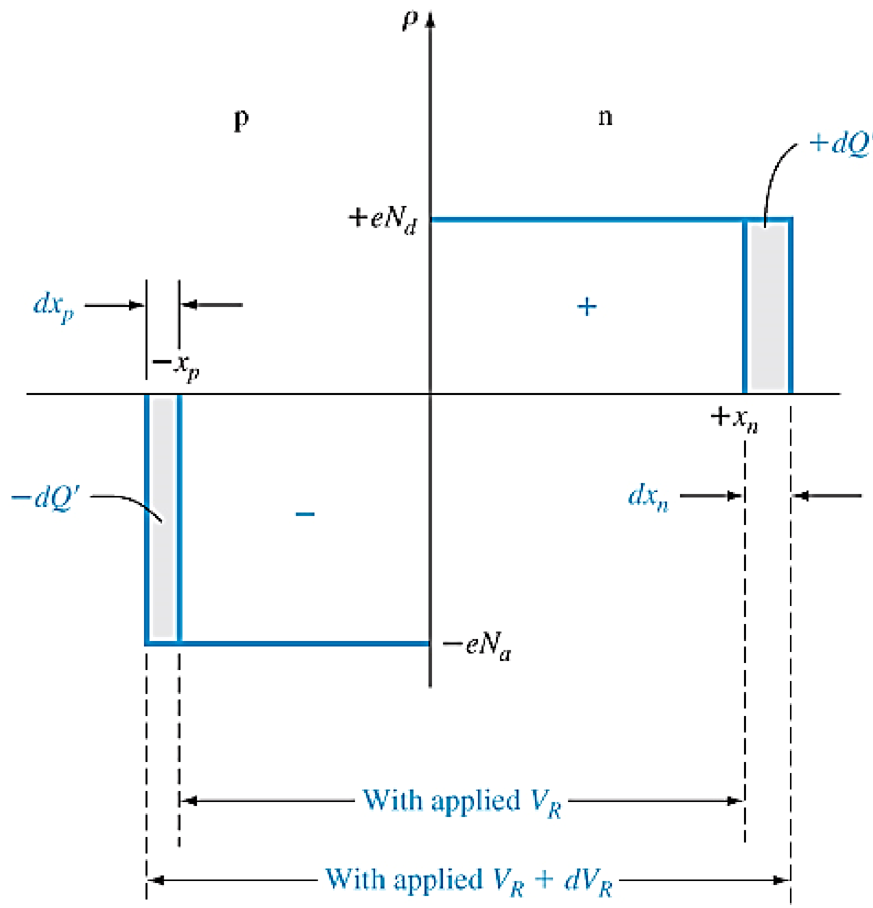
$$\mathrm{E}_{\max} = \frac{-eN_dx_n}{\epsilon_s} = \frac{-eN_ax_p}{\epsilon_s}$$

Junction Capacitance

- An increase in the applied reverse bias will lead to an increase in the positive and negative charges in the n and p-regions, respectively.
- The junction capacitance **per unit area** is obtained from:

$$C' = \frac{dQ'}{dV_R} \qquad \text{F/cm}^2$$

$$dQ' = eN_d\,dx_n = eN_a\,dx_p$$



Junction Capacitance

$$C' = \frac{dQ'}{dV_R} = eN_d \frac{dx_n}{dV_R}$$

$$x_n = \left\{ \frac{2\epsilon_s(V_{bi} + V_R)}{e} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$\frac{dx_n}{dV_R} = \left\{ \frac{\varepsilon_s}{2e(V_{bi} + V_R)} \left[\frac{N_a}{N_d} \right] \left[\frac{1}{N_a + N_d} \right] \right\}^{1/2}$$

$$\boxed{C' = \left\{ \frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)} \right\}^{1/2}}$$

$$\boxed{C' = \frac{\epsilon_s}{W}}$$

Example

❖ Consider a silicon pn junction at T=300 K with doping concentrations of $N_a=10^{16}$ cm^{-3} and $N_d=10^{15}$ cm^{-3}. Calculate the junction capacitance under a reverse bias of 5V. Junction area is 10^{-4} cm^2.

$$\boxed{C' = \left\{\frac{e\epsilon_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}\right\}^{1/2}}$$

$$C' = \left\{\frac{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(10^{16})(10^{15})}{2(0.635 + 5)(10^{16} + 10^{15})}\right\}^{1/2}$$

$$C' = 3.66 \times 10^{-9} \text{ F/cm}^2$$

$$C = C' \cdot A = 0.366 \times 10^{-12} \text{ F} = 0.366 \text{ pF}$$

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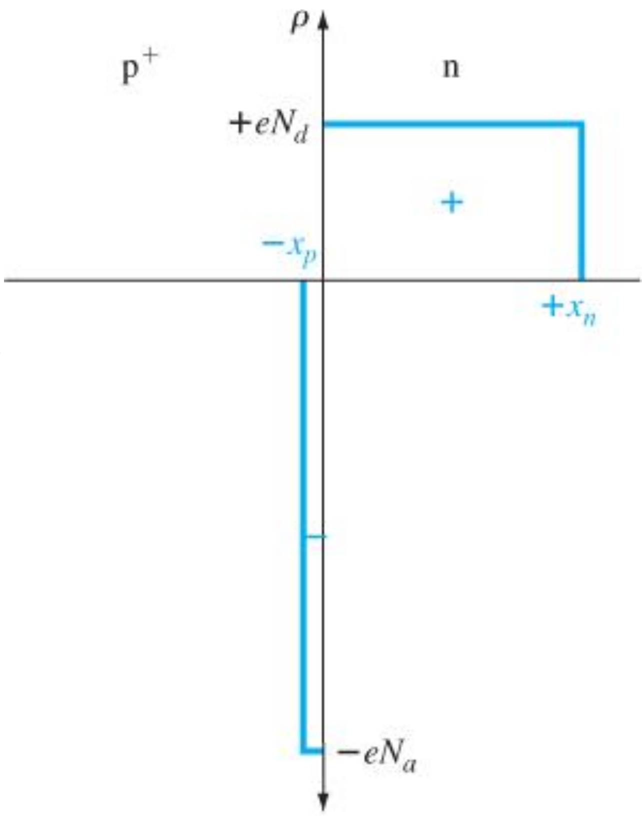
One-Sided Junctions

❖ If $N_a >> N_d$, a p^+n junction is formed.

$$\boxed{W = \left\{ \frac{2\epsilon_s(V_{bi}+V_R)}{e}\left[\frac{\cancel{N_a}+\cancel{N_d}}{\cancel{N_a}N_d}\right] \right\}^{1/2}} \approx \left\{ \frac{2\epsilon_s(V_{bi}+V_R)}{eN_d} \right\}^{1/2}$$

$$x_n = \left\{ \frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{\cancel{N_a}}{N_d}\right]\left[\frac{1}{\cancel{N_a}+\cancel{N_d}}\right] \right\}^{1/2} \approx \left\{ \frac{2\epsilon_s(V_{bi}+V_R)}{eN_d} \right\}^{1/2}$$

$$x_p = \left\{ \frac{2\varepsilon_s(V_{bi}+V_R)}{e}\left[\frac{N_d}{N_a}\right]\left[\frac{1}{N_a+\cancel{N_d}}\right] \right\}^{1/2}$$



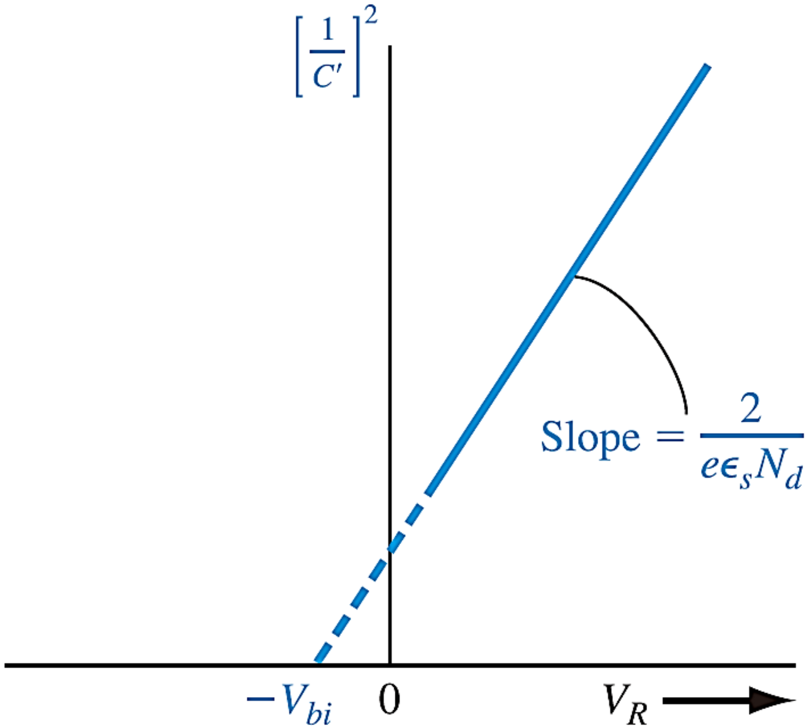
$$x_p \ll x_n$$

$$\boxed{W \approx x_n}$$

One-Sided Junctions

❖ If $N_a >> N_d$, a p^+n junction is formed.

$$C' = \left\{ \frac{e\varepsilon_s \cancel{N_a} N_d}{2(V_{bi}+V_R)(\cancel{N_a}+\cancel{N_d})} \right\}^{1/2} \approx \left\{ \frac{e\varepsilon_s N_d}{2(V_{bi}+V_R)} \right\}^{1/2}$$



$$\left(\frac{1}{C'}\right)^2 = \frac{2(V_{bi}+V_R)}{e\epsilon_s N_d}$$

This curve can be used to determine the doping concentration and the built-in potential.

Example

❖ Assume a silicon p^+n junction at T = 300 K with $n_i = 1.5\text{x}10^{10}$ cm^{-3}. Assume that the intercept of the $(1/C')^2$ versus V_R curve gives V_{bi} = 0.725 V and the slope is 6.15 x10^{15} [(F/cm^2)$^{-2}$V^{-1}]. Determine the doping concentrations of N_a and N_d.

$$\text{Slope} = \frac{2}{e\epsilon_s N_d}$$

$$N_d = \frac{2}{e\,\epsilon_s} \cdot \frac{1}{\text{斜率}} = \frac{2}{(1.6 \times 10^{-19})(11.7)(8.85 \times 10^{-14})(6.15 \times 10^{15})}$$

$$N_d = 1.96 \times 10^{15}\ \text{cm}^{-3}$$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$N_a = \frac{n_i^2}{N_d} \exp\left(\frac{V_{bi}}{V_t}\right) = \frac{(1.5 \times 10^{10})^2}{1.963 \times 10^{15}} \exp\left(\frac{0.725}{0.0259}\right)$$

$$N_a = 1.64 \times 10^{17}\ \text{cm}^{-3}$$

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