國立臺北科技大學 材料及資源工程系

電子材料與元件技術特論

SPECIAL TOPICS IN ELECTRONIC MATERIALS AND DEVICES

Electrical Properties-Part 3

Conduction, Optical Absorption, PN Junction and Solar Cell

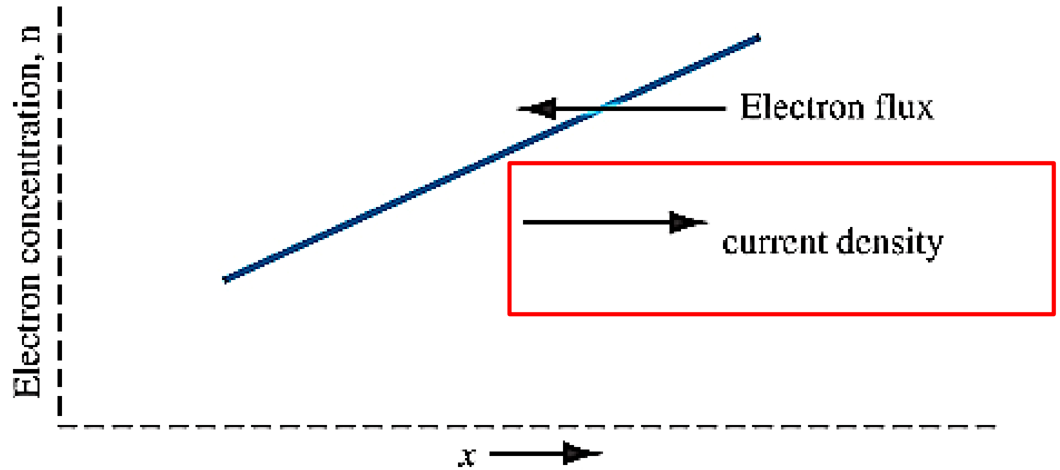
徐曉萱

Diffusion Current Density

- For electrons:

\+ +direction

$$J_{nx|dif} = eD_n \frac{dn}{dx}$$

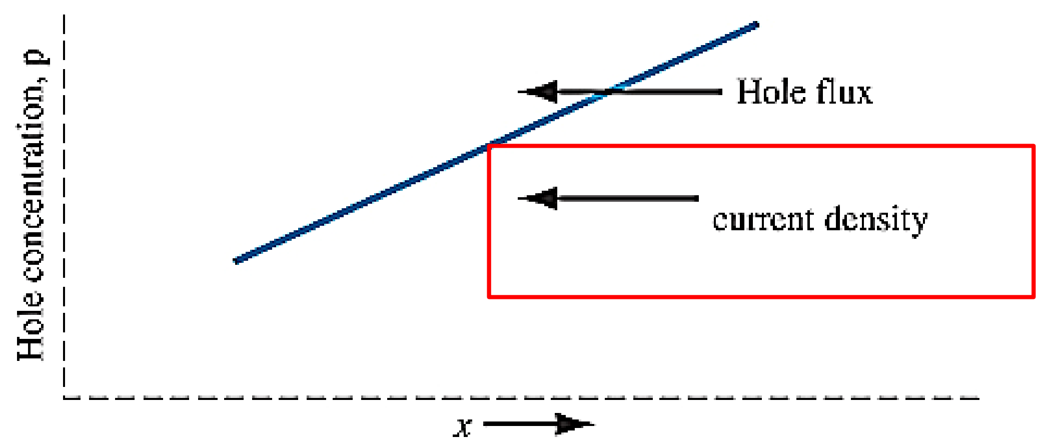


- For holes:

– +

$$J_{px|dif} = -eD_p \frac{dp}{dx}$$

different direction



D is the diffusion coefficient (cm^2/s)

Total Current (Drift & Diffusion)

- Total Electron Current Due to Drift and Diffusion

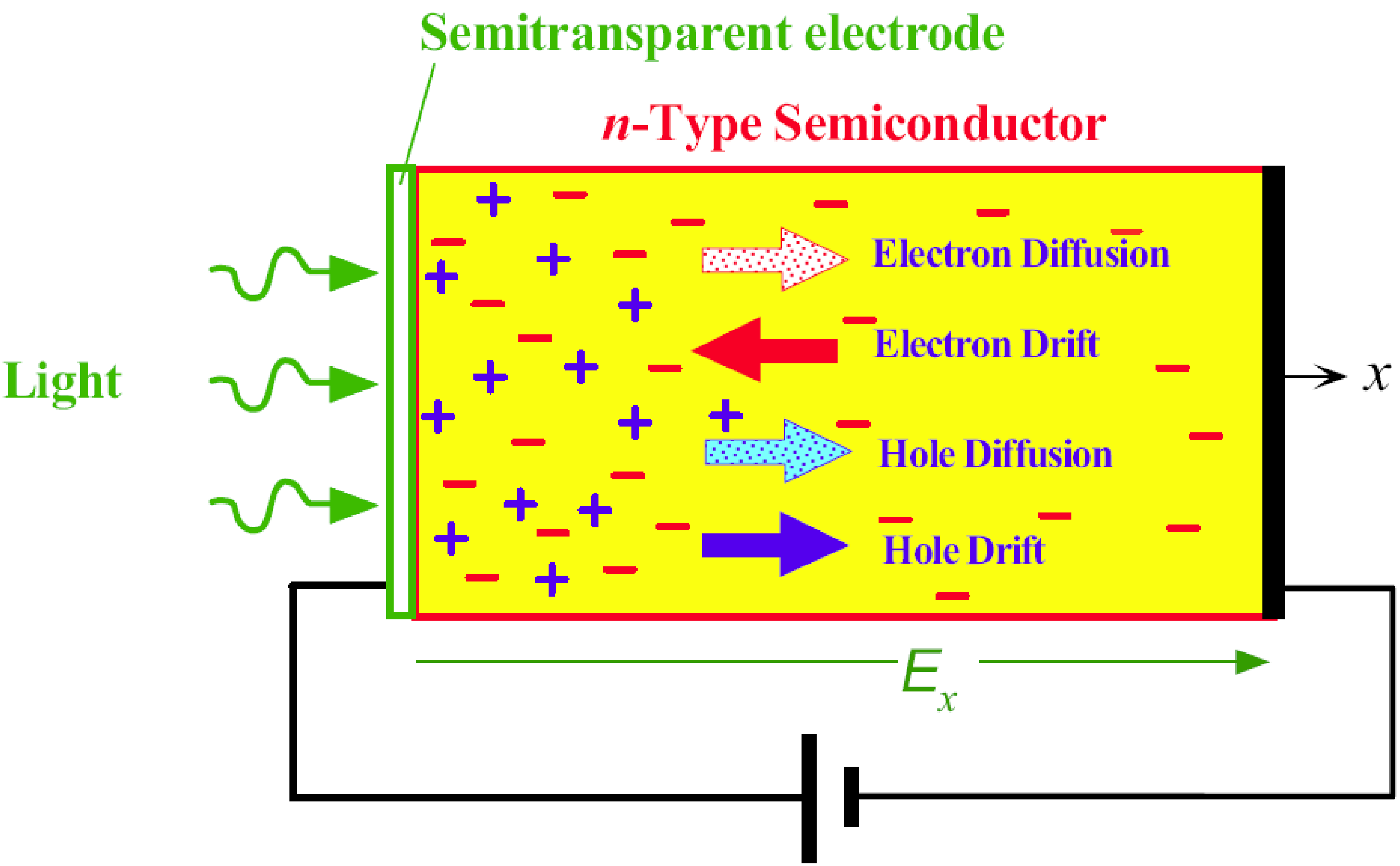
$$J_e = en\mu_e \mathcal{E}_x + eD_e \frac{dn}{dx}$$

- Total Hole Current Due to Drift and Diffusion

$$J_h = ep\mu_h \mathcal{E}_x - eD_h \frac{dp}{dx}$$

- Total current density in 3D:

$$J = \underbrace{en\mu_n \mathrm{E} + ep\mu_p \mathrm{E}}_{\textbf{Drift}} + \underbrace{eD_n \nabla n - eD_p \nabla p}_{\textbf{Diffusion}}$$



When there is an electric field and also a concentration gradient, charge carriers move both by diffusion and drift.

Example

- Given: n-type GaAs at T = 300K. n(x) varies linearly from 10^{18} to 7×10^{17} cm^{-3} over a distance of 0.1 cm. Calculate the diffusion current density due to the electron diffusion. Assume D_n = 225 cm^2/s.

$$J_{n|dif} = eD_n \frac{dn}{dx} \approx eD_n \frac{\Delta n}{\Delta x}$$

$$= (1.6 \times 10^{-19})(225)\left(\frac{1 \times 10^{18} - 7 \times 10^{17}}{0.10}\right) = 108 \text{ A/cm}^2$$

Einstein Relation

- Diffusion coefficient is a measure of the ease with which the diffusing charge carriers move in the medium.
- So is drift mobility
- Two quantities are related through **Einstein Relation**：

$$\boxed{\frac{D_e}{\mu_e} = \frac{kT}{e}} \qquad \boxed{\frac{D_h}{\mu_h} = \frac{kT}{e}}$$

Example

- Consider a homogeneous GaAs semiconductor at T=300K with $N_d=10^{16}$ cm^{-3}, and $N_a=0$. (a) Calculate the thermal–equilibrium values of electron and hole concentrations. (b) For an applied electric field of 10 V/cm, calculate the drift current density. ($n_i=1.8\text{x}10^6$ cm^{-3}, $\mu_n=7500$ cm^2/V-s, $\mu_p=310$ cm^2/V-s)

$$n_o = N_d = 10^{16}\ \text{cm}^{-3}$$

$$p = \frac{n_i^2}{n} = \frac{(1.8 \times 10^6)^2}{10^{16}} = 3.24 \times 10^{-4}\ \text{cm}^{-3}$$

$$J_{drf} = e(\mu_n n + \mu_p p)\text{E} \approx e\mu_n N_d \text{E}$$

$$J = \left(1.6 \times 10^{-19}\right)\left(7500\right)\left(10^{16}\right)\left(10\right) = 120\ \text{A/cm}^2$$

Example

- If a carrier mobility is 1000 cm^2/V-s, please calculate the diffusion coefficient.

$$D = \left(\frac{kT}{e}\right)\mu = (0.0259)(1000) = 25.9 \text{ cm}^2/\text{s}$$

Table 5.2 | Typical mobility and diffusion coefficient values at $T = 300$ K ($\mu = \text{cm}^2/\text{V-s}$ and $D = \text{cm}^2/\text{s}$)

	μ_n	D_n	μ_p	D_p
Silicon	1350	35	480	12.4
Gallium arsenide	8500	220	400	10.4
Germanium	3900	101	1900	49.2

0.0259x at 300K

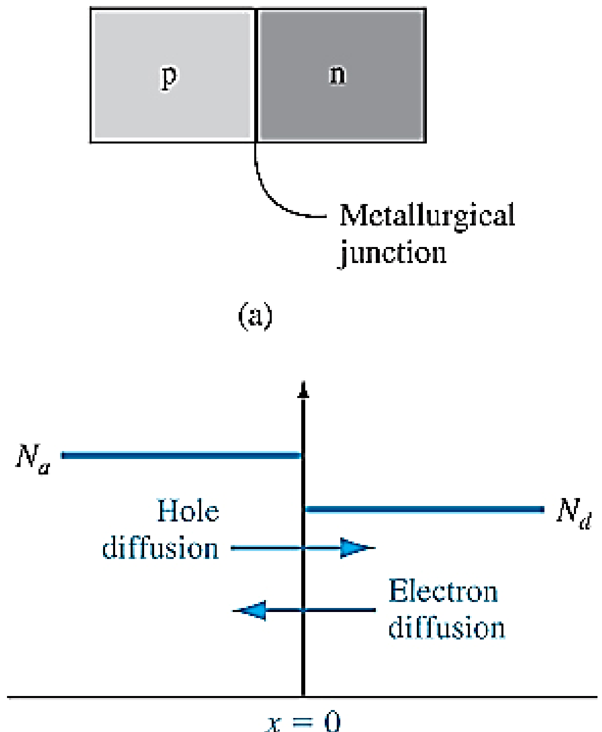
Solar Cell

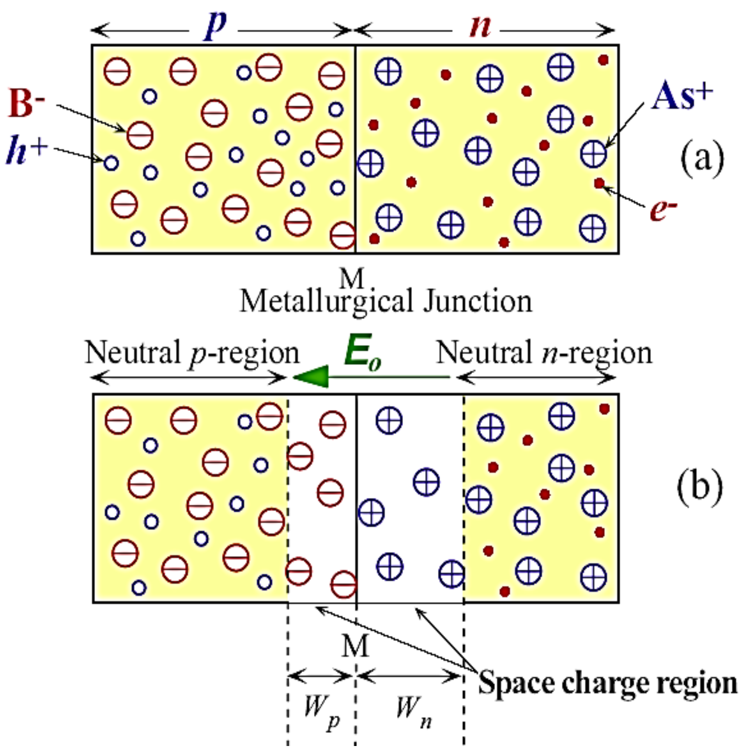
pn Junction Si solar cells at work. Honda's two seated Dream car is powered by photovoltaics. The Honda Dream was first to finish 3,010 km in four days in the 1996 World Solar Challenge.

Solar cell inventors at Bell Labs (left to right) Gerald Pearson, Daryl Chapin and Calvin Fuller are checking a Si solar cell sample for the amount of voltage produced (1954).

A PN Junction

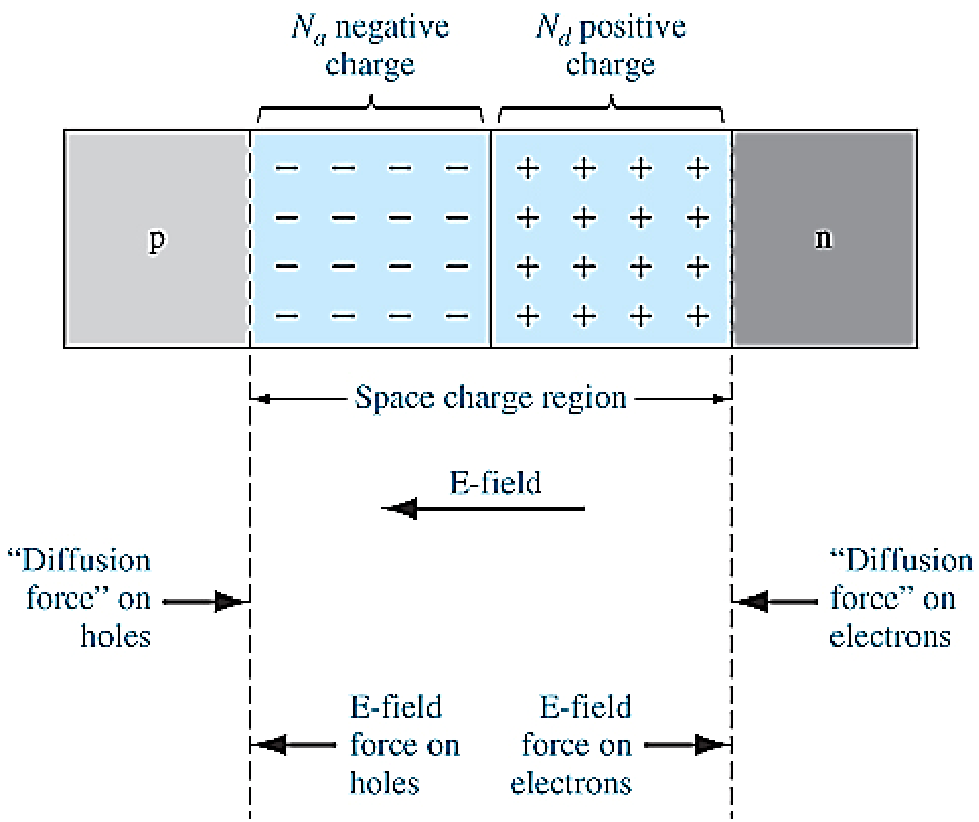
- A step junction with uniform doping in each region and an abrupt change in doping at the interface.
- Electrons diffuse from the n-region to p-region and holes diffuse in the reverse direction.





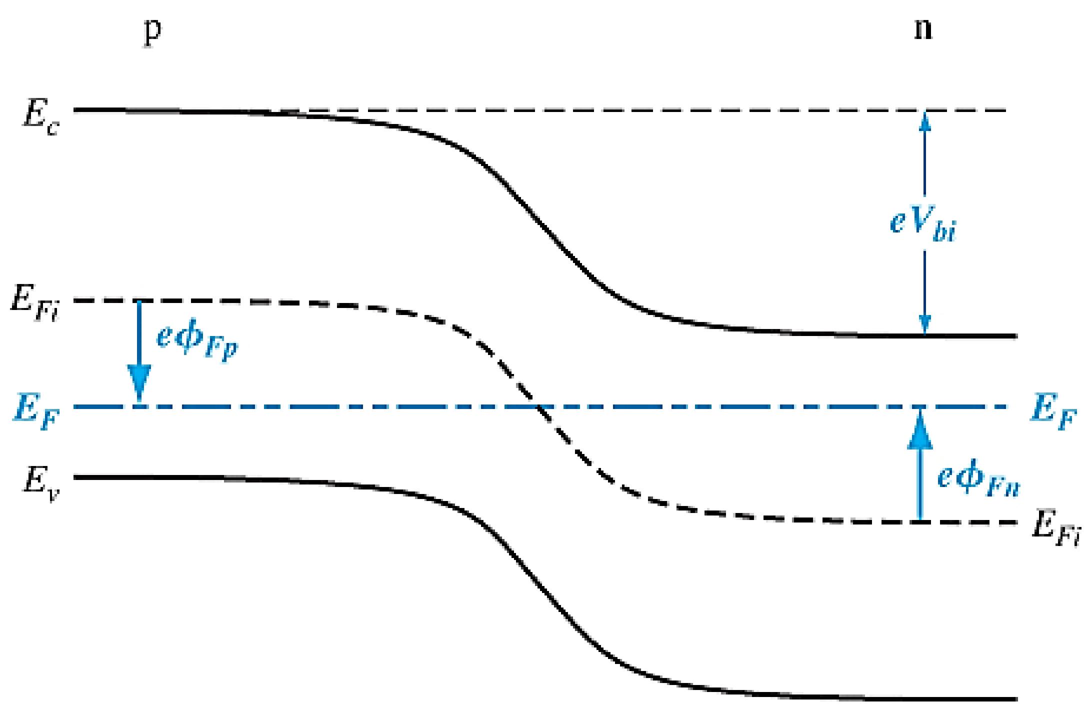
Basic Structure of the pn Junction

- The negatively and positively charged regions are called the space charge region, or the depletion region.
- An electric field is established in the direction from the n- to the p-region.



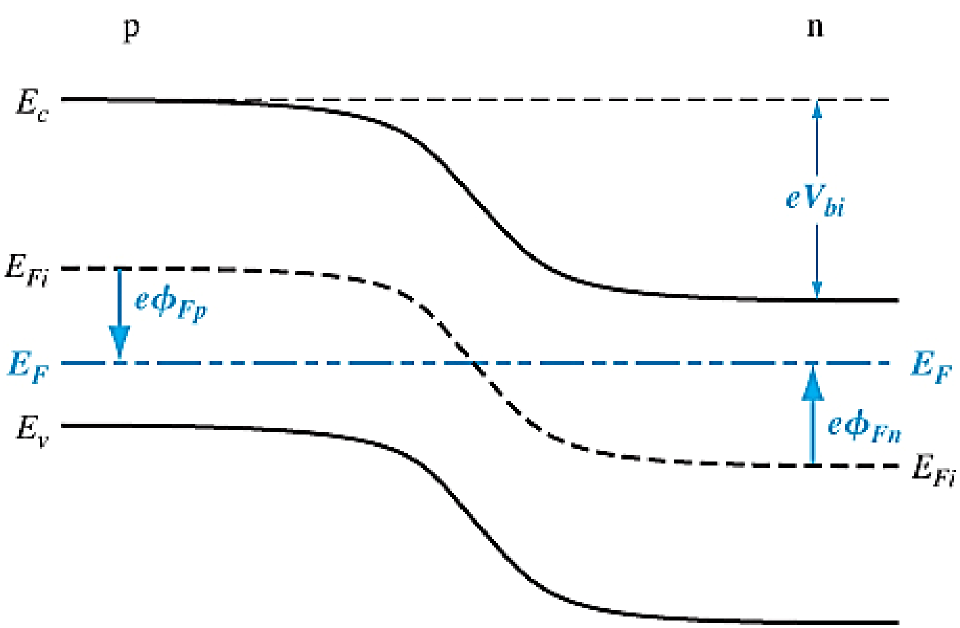
Built-in Potential Barrier

- Energy-band diagram of a pn junction in thermal equilibrium ➔ No current, No external excitation, Constant Fermi energy



Energy Band Diagram of PN Junction

- The CB and VB must bend since the relative position of the Fermi level to the CB and VB is different for the p-type and n-type semiconductor.
- The **band bending** produces a potential barrier, which is referred to as the **built-in potential barrier**.

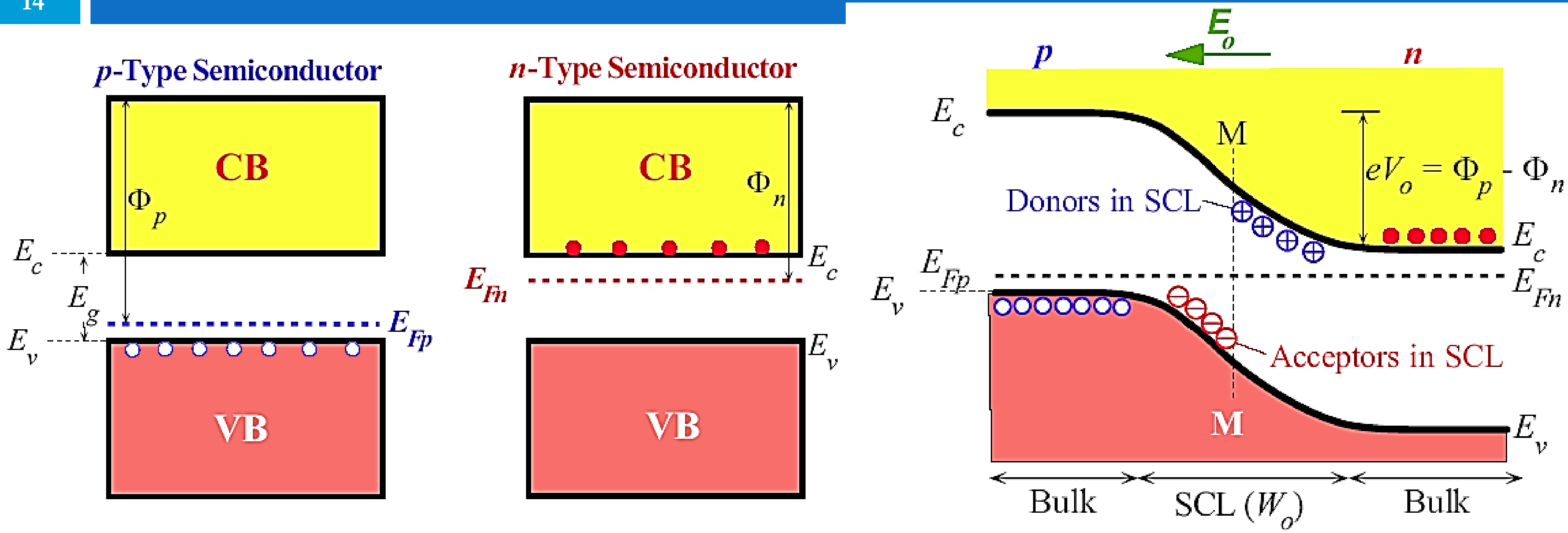


Built-in Potential Barrier:

$$V_{bi} = \frac{kT}{e}\ln\left(\frac{N_a N_d}{n_i^2}\right) = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

V_t is the thermal voltage.

Energy Band Diagram of PN Junction

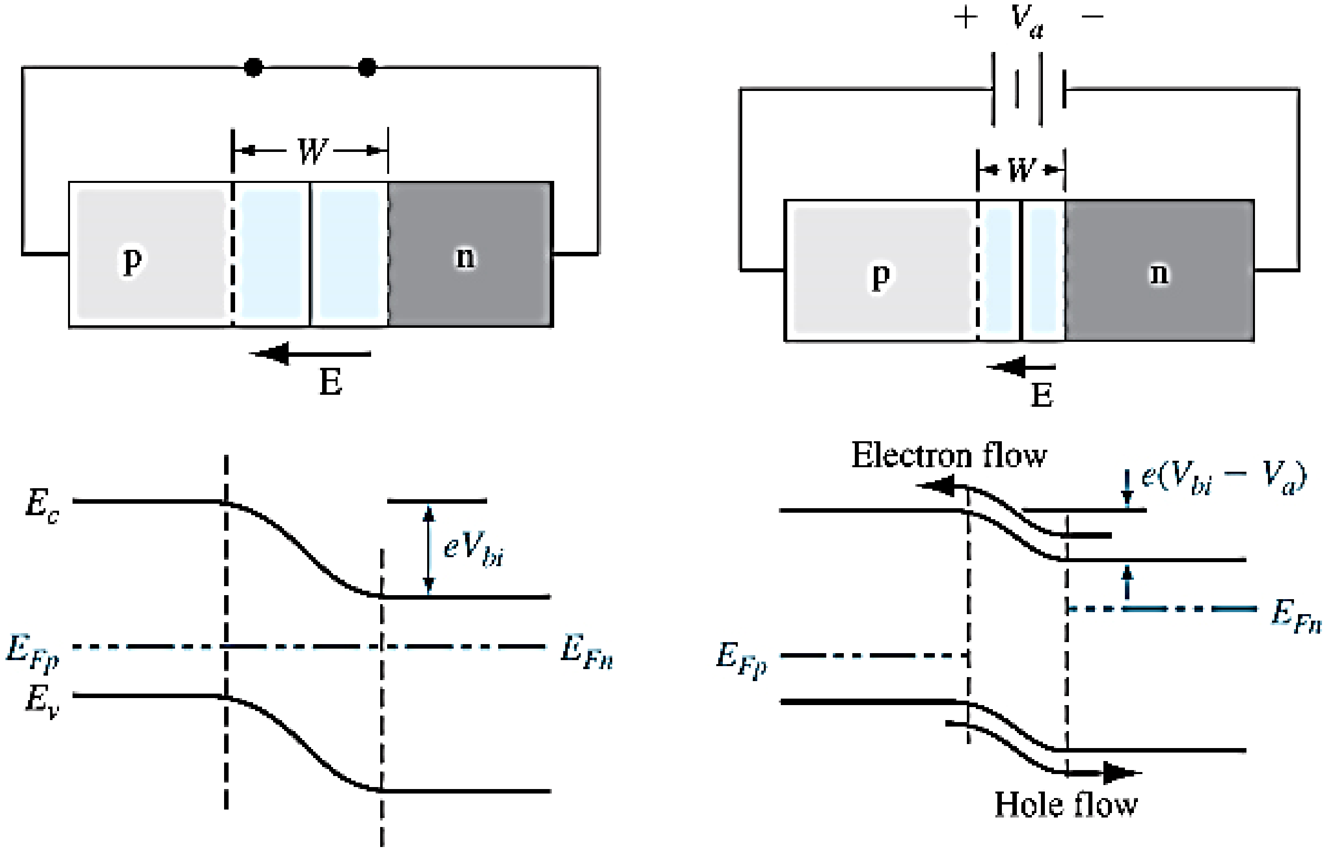


(a) Two isolated *p* and *n*-type semiconductors (same material).

(b) A pn are in contact. The Fermi level must be uniform in equilibrium. The metallurgical junction is at M. The region around M contains the space charge layer (SCL). On the *n*-side of M, SCL has the exposed positively charged donors whereas on the p-side it has the exposed negatively charged acceptors.

Energy Band Diagram

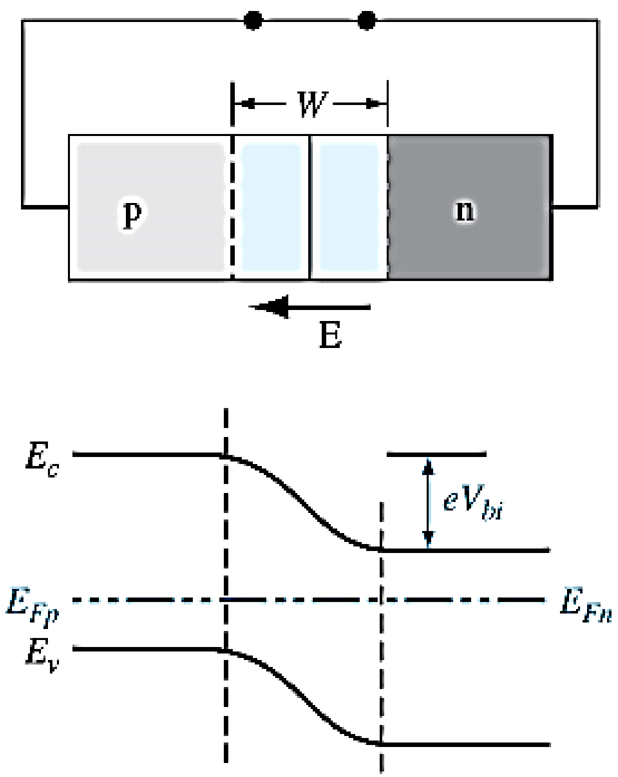
- When a **forward bias** is applied to a pn junction, a **current** will be induced in the device.



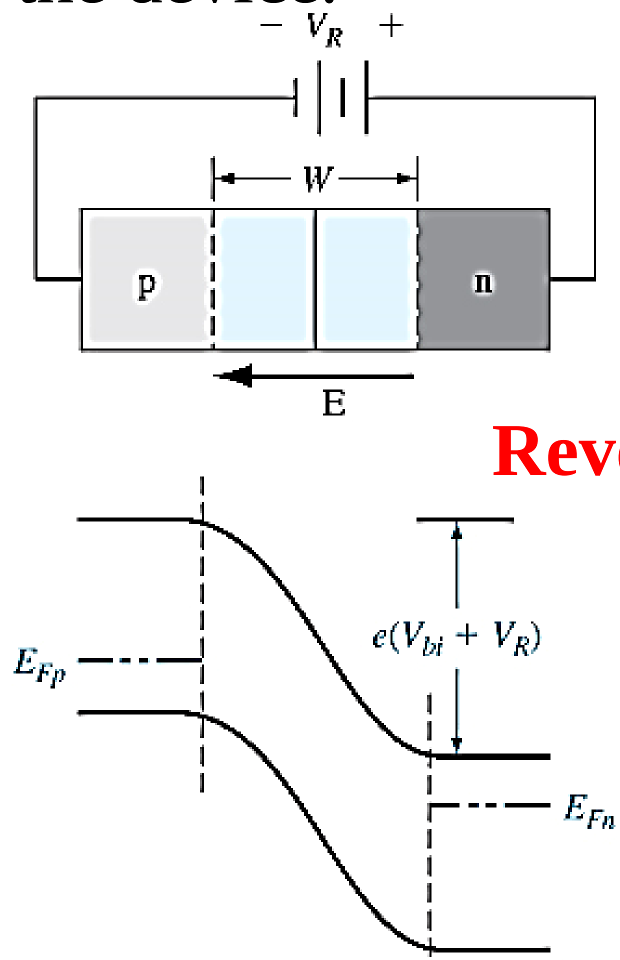
Zero Applied Bias **Forward Bias**

Energy Band Diagram

- When a **reverse bias** is applied to a pn junction, **no current** will be induced in the device.



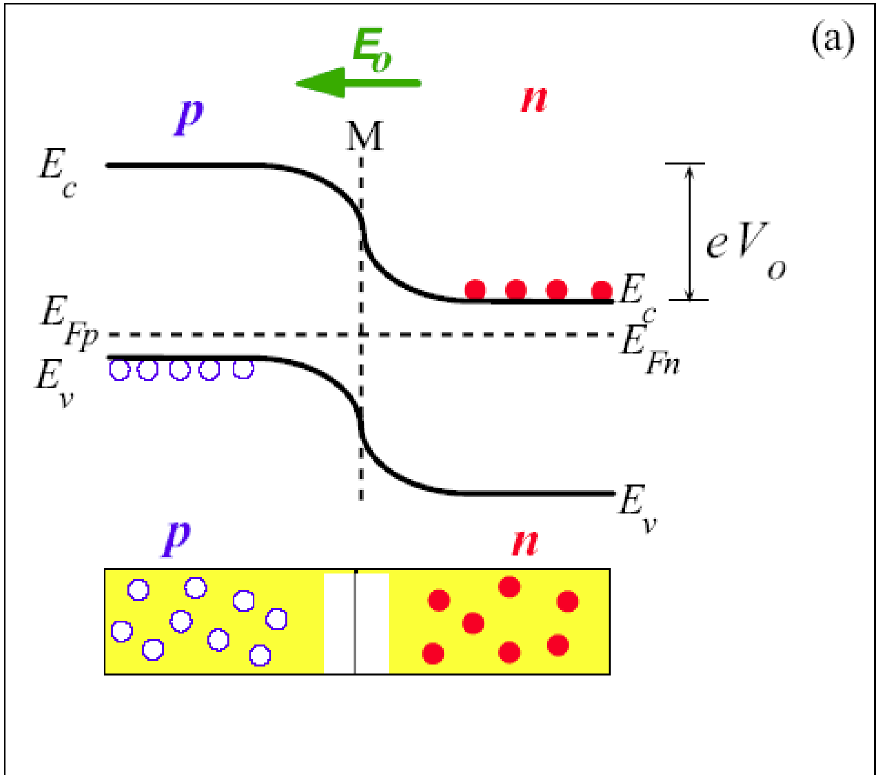
Zero Applied Bias



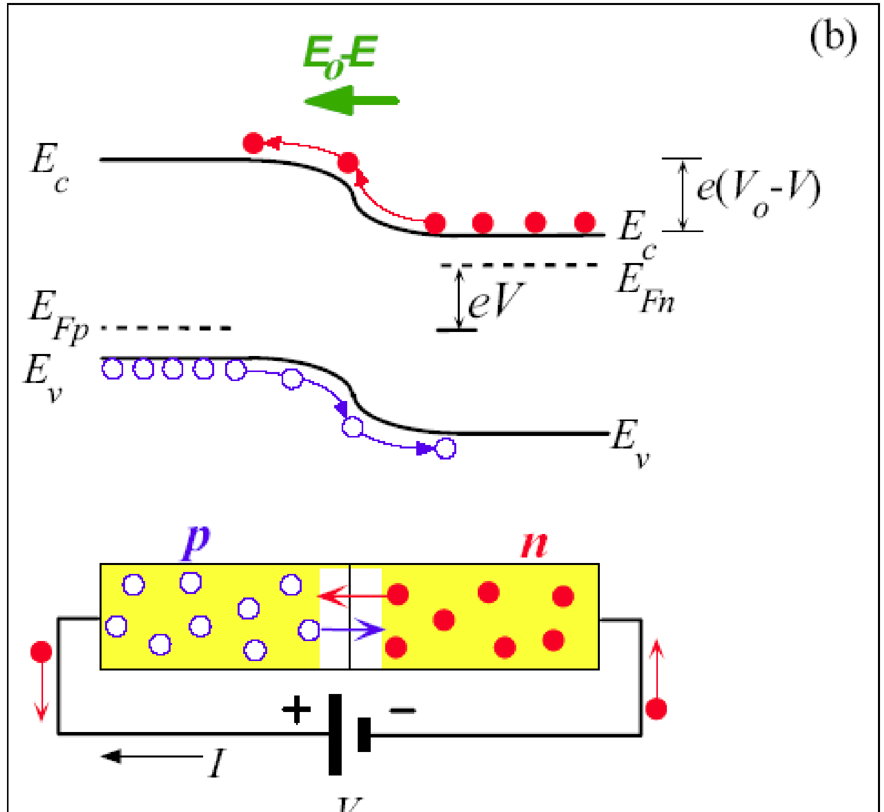
Reverse Bias

Energy Band Diagram of PN Junction

- The Forward bias opposes the built in potential and reduces the barrier for carrier injection across the junction.



(a) open circuit



(b) forward bias

Forward Bias: Diffusion Current

Ideal diode (Shockley) equation:

$$J = J_{so}\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

Reverse saturation current density, J_{S0} :

$$J_{so} = \left[\left(\frac{eD_h}{L_h N_d}\right) + \left(\frac{eD_e}{L_e N_a}\right)\right] n_i^2$$

Forward Bias: Diffusion Current

- **Intrinsic concentration**

$$n_i^2 = (N_c N_v)\exp\left(-\frac{eV_g}{kT}\right)$$

E_g

where $V_g = E_g / e$ is the bandgap energy expressed in volts

$V_g =$ 0.67 V for Ge, 1.12 V for Si, and 1.42 V for GaAs

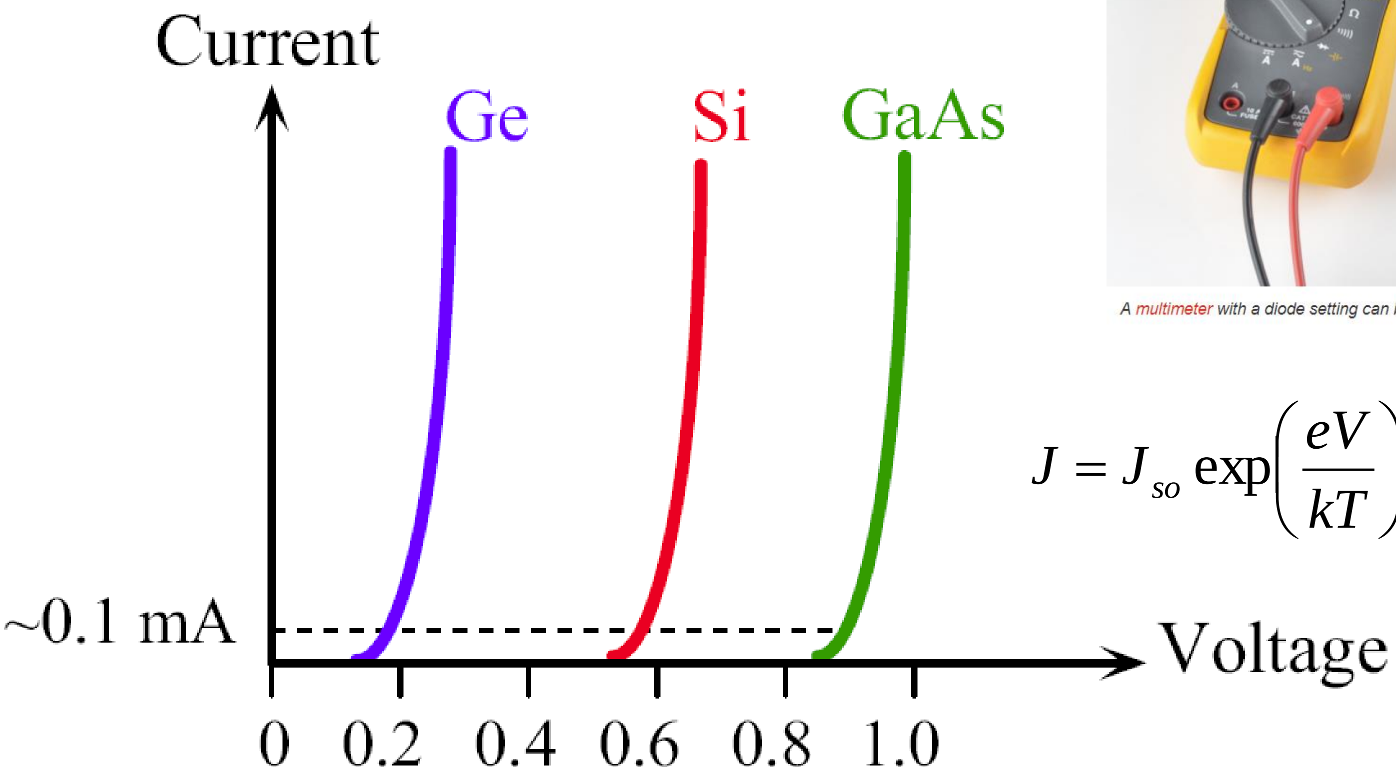
$$J = J_{so}\exp\left(\frac{eV}{kT}\right) = C\exp\left(\frac{e(V - V_g)}{kT}\right) \qquad V > kT/e$$

$$J_{so} = \left[\left(\frac{eD_h}{L_h N_d}\right) + \left(\frac{eD_e}{L_e N_a}\right)\right] n_i^2$$

We can plot I-V curve for Ge, Si and GaAs

Forward Bias: Diffusion Current

- I-V characteristics of Ge, Si, and GaAs pn junctions.



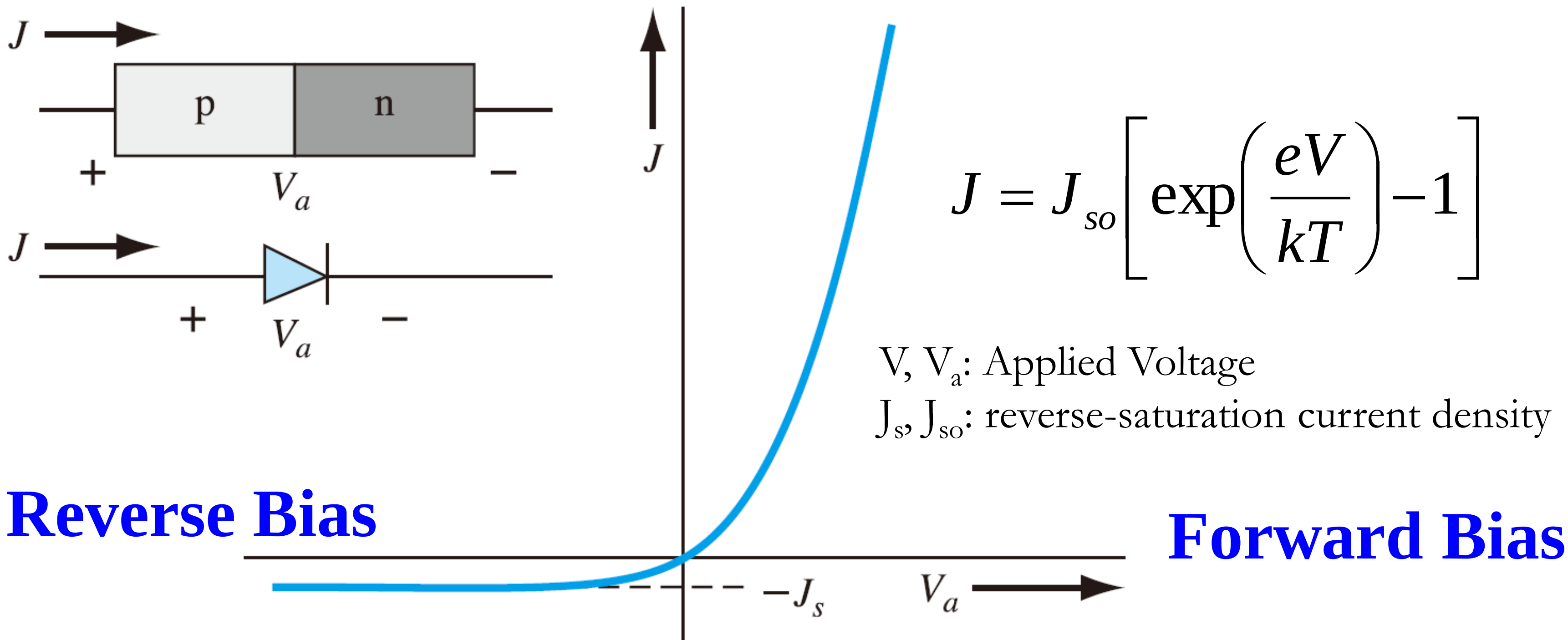
A multimeter with a diode setting can be used to measure (the minimum of) a diode's forward voltage drop.

$$J = J_{so}\exp\left(\frac{eV}{kT}\right) = C\exp\left(\frac{e(V - V_g)}{kT}\right)$$

https://learn.sparkfun.com/tutorials/diodes/real-diode-characteristics

Ideal Current-Voltage Relationship

- If V_a becomes negative by a few kT/e volts, then the reverse-bias current density becomes independent of the reverse-bias voltage. J_{so} is called the **reverse-saturation current density**.

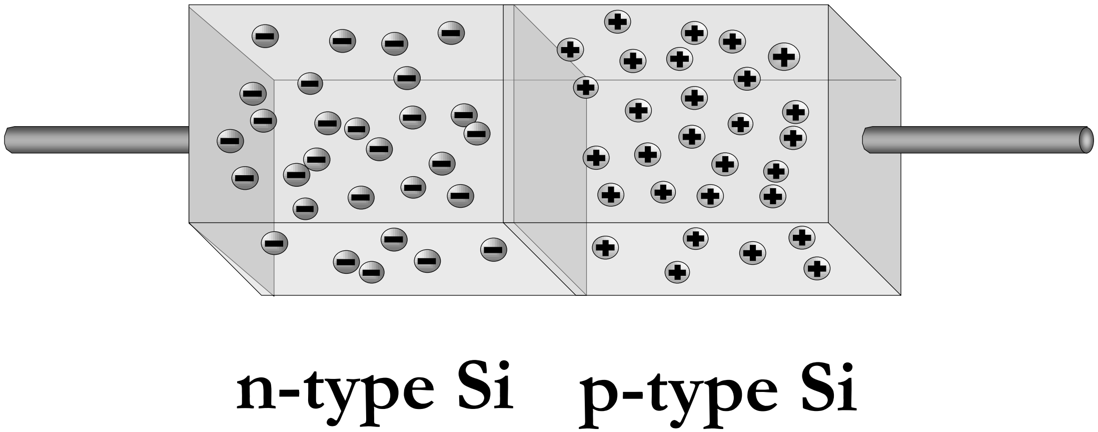


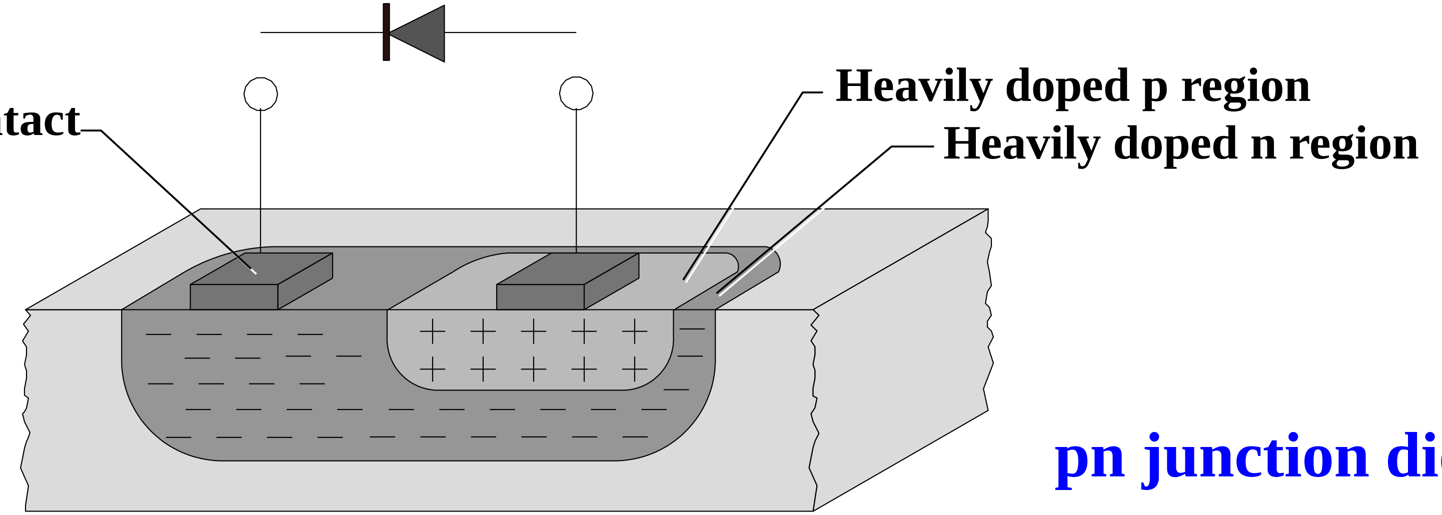
$$J = J_{so}\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

V, V_a: Applied Voltage

J_s, J_{so}: reverse-saturation current density

Cross Section of pn Junction

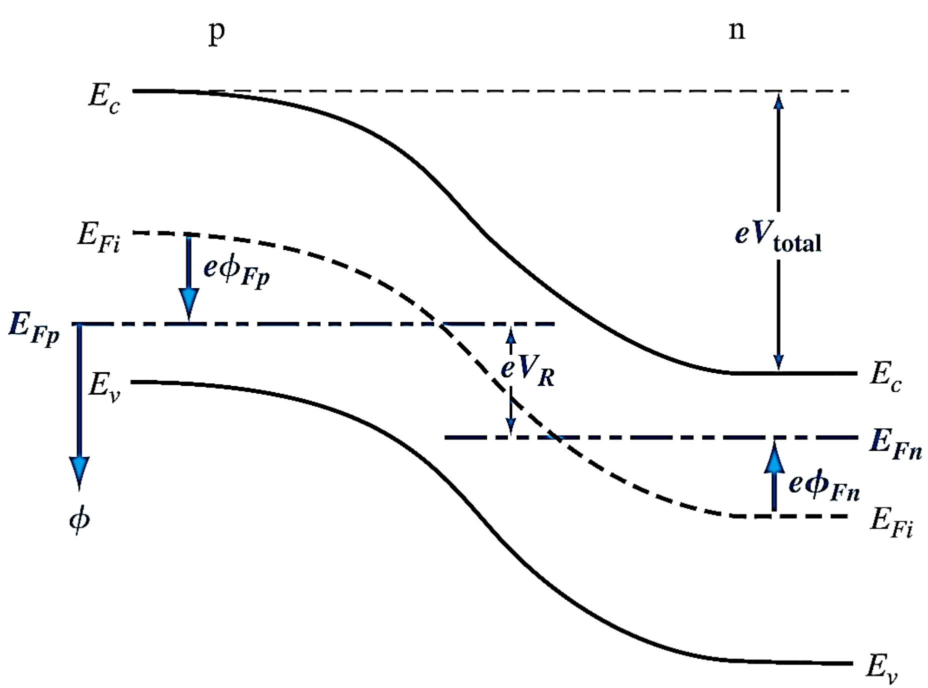




pn junction diode

Reverse Applied Bias

- A positive voltage is applied to the n-region; a negative voltage is applied to the p-region.
- The Fermi level in the n-region moves further downward.

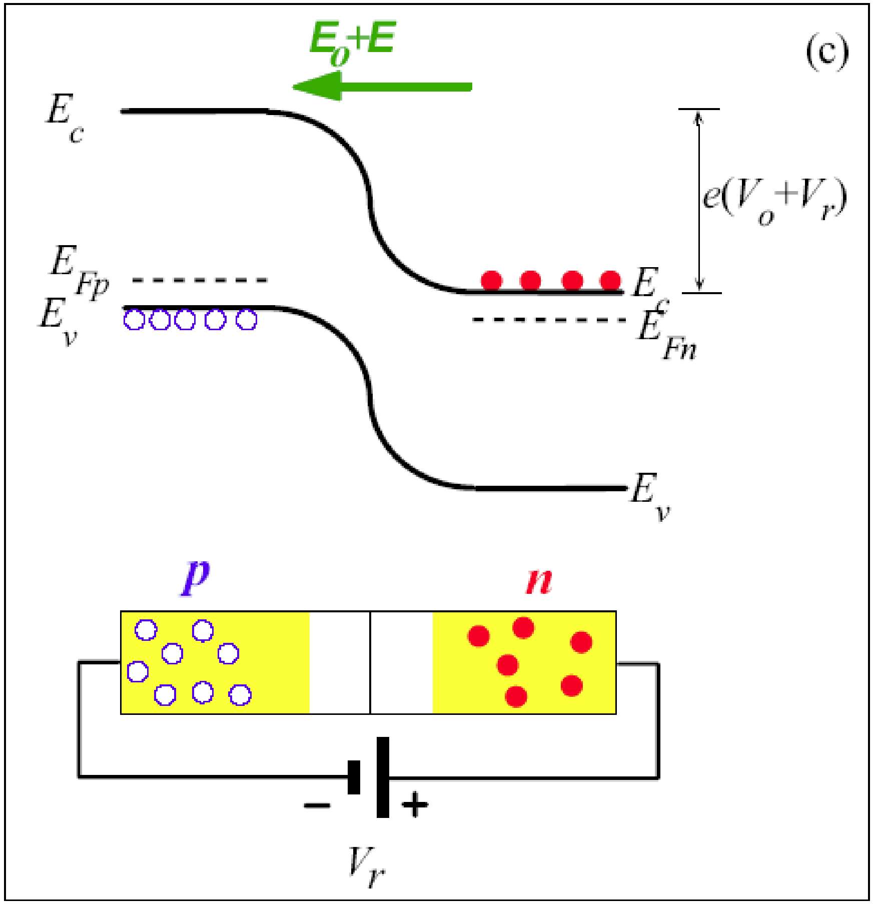


Total potential barrier:

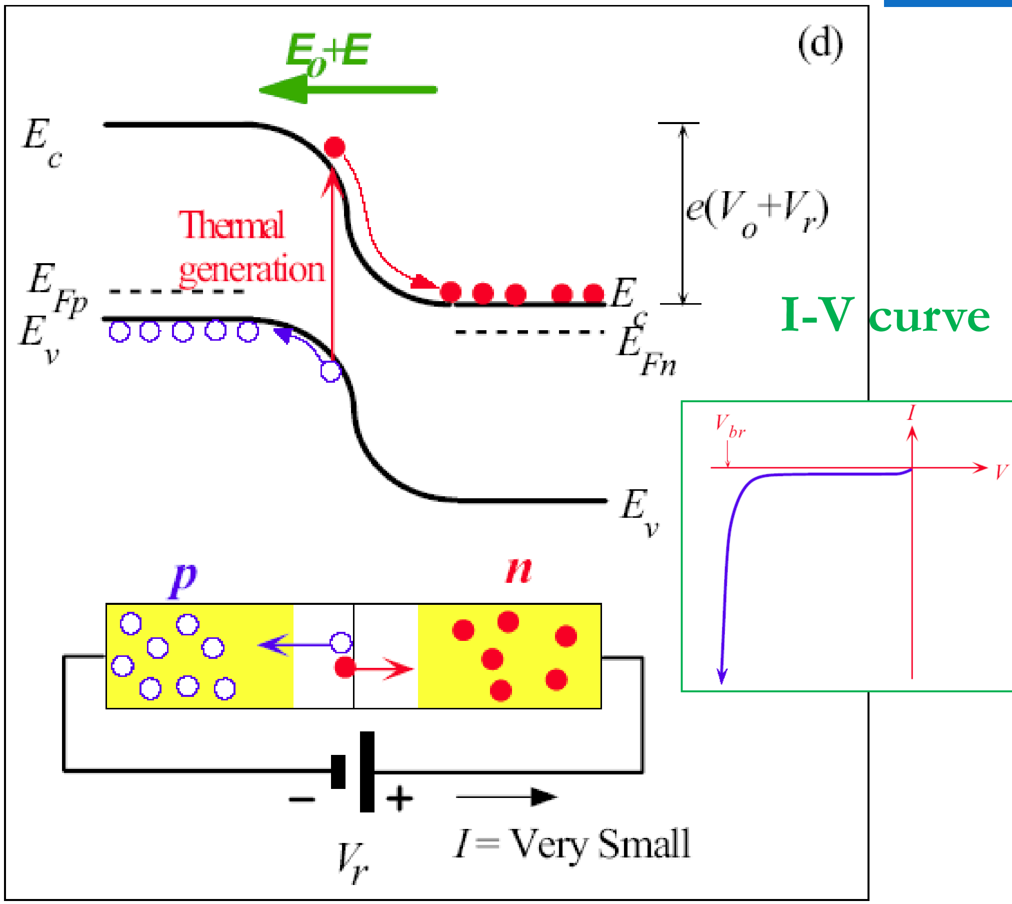
$$V_{\text{total}} = V_{bi} + V_R$$

V_R is the reverse bias

Reverse Bias Conditions



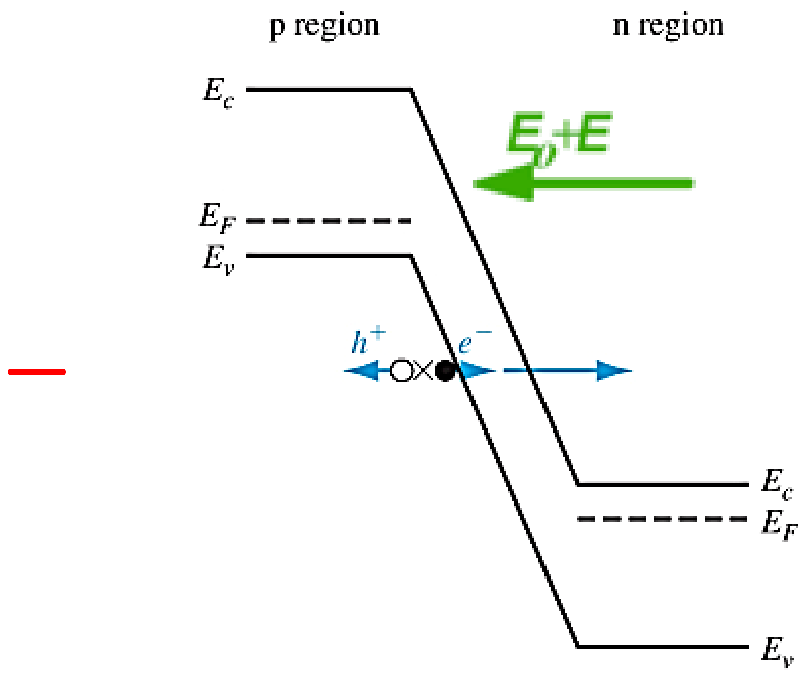
(c) Reverse bias conditions.

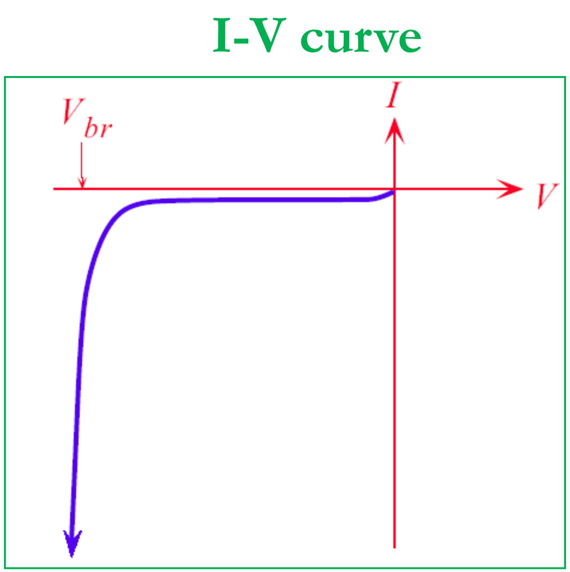


(d) Thermal generation of electron hole pairs in the depletion region results in a small reverse current

Zener Breakdown

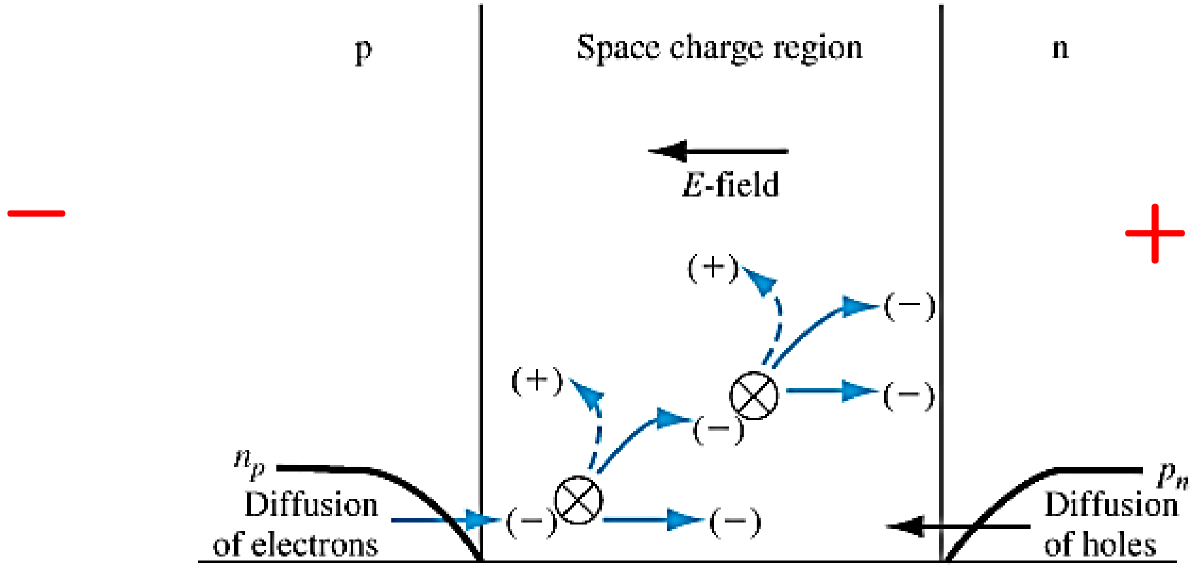
- In highly doped pn junctions through a **tunneling** mechanism. The CB and VB on the opposite sides of highly doped pn junctions are very close under reverse bias so that electrons may tunnel directly from the VB on p-side into the CB on n-side.



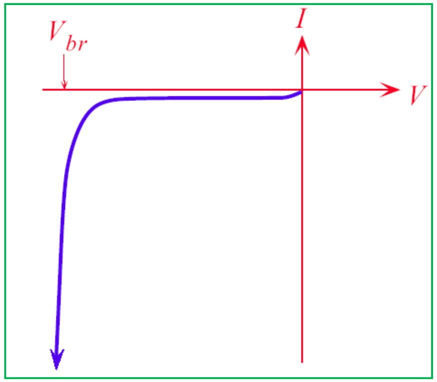


Avalanche Breakdown

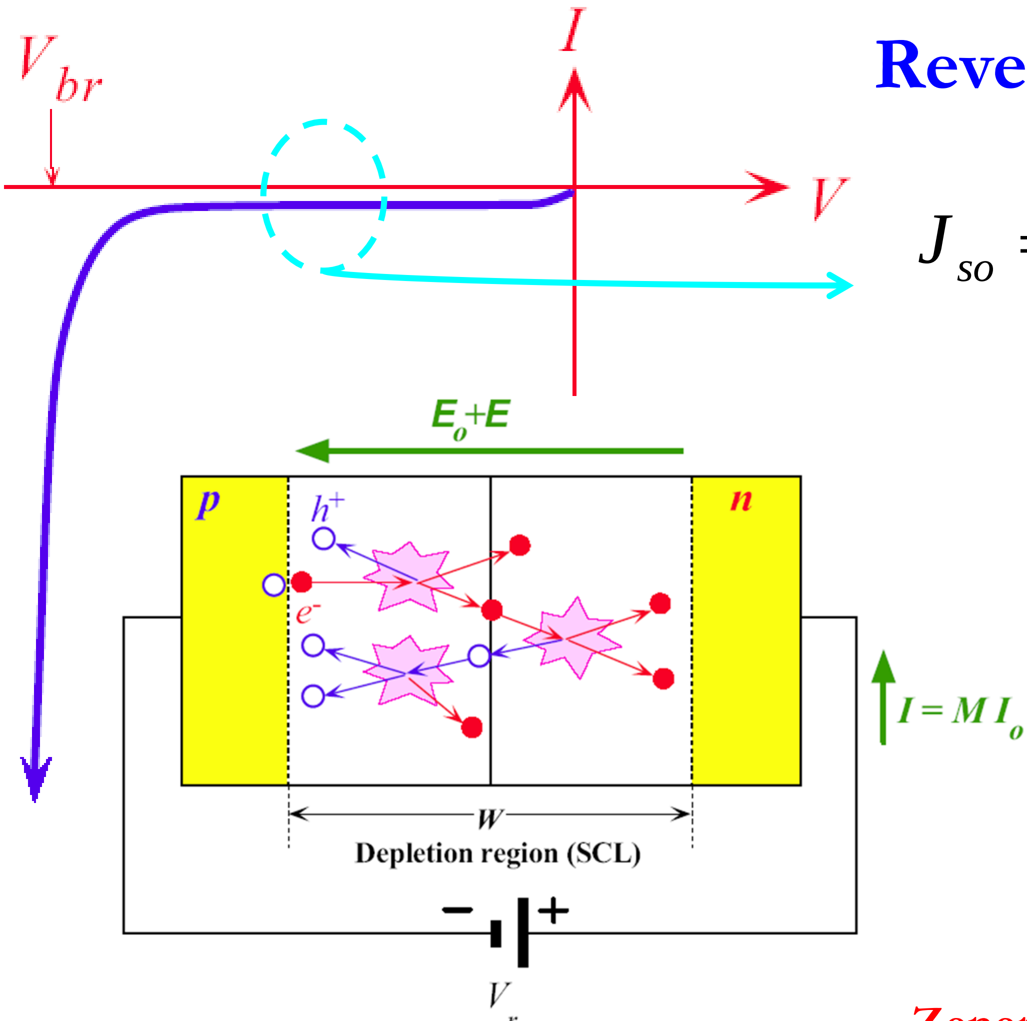
- Electrons/holes moving through space charge region acquire sufficient energy from electric field (V_R) to **create electron–hole pairs by colliding with atoms**. The newly created e-/h+ move in opposite directions due to electric field and thus contribute to the **reverse-bias current**. The newly generated e-/h+ may acquire sufficient energy to ionize other atoms, leading to the avalanche process.



I-V curve

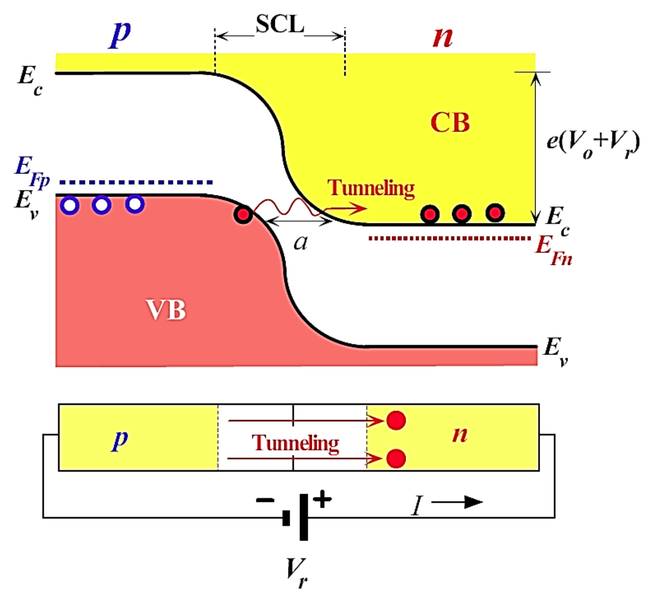


Reverse *I*-*V* of a *PN* Junction



Reverse saturation current:

$$J_{so} = \left[\left(\frac{eD_h}{L_h N_d}\right) + \left(\frac{eD_e}{L_e N_a}\right)\right] n_i^2$$

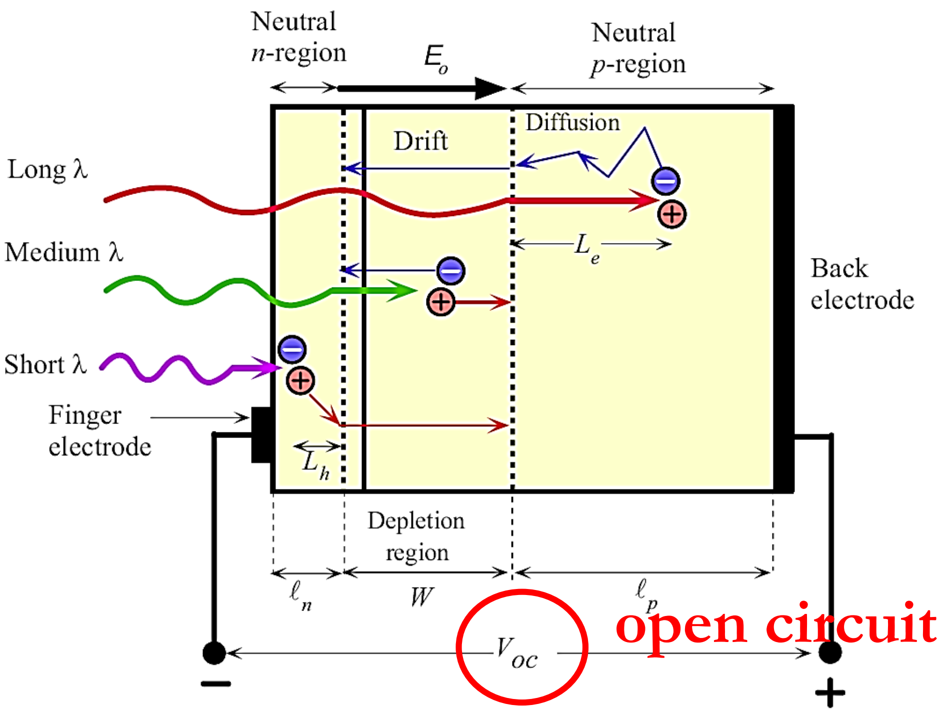


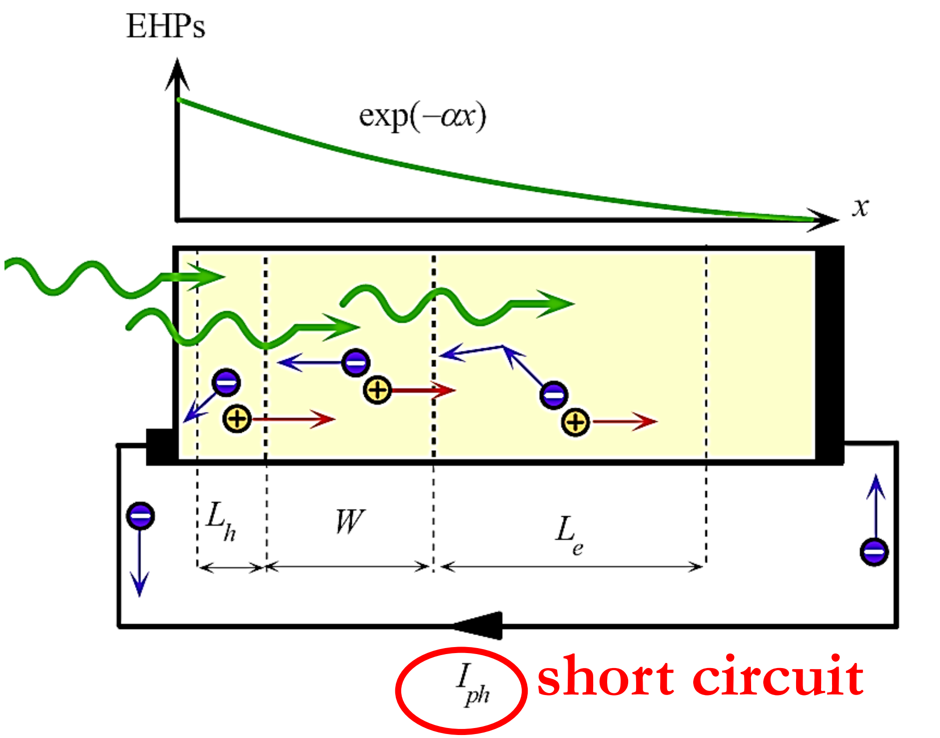
Avalanche breakdown: impact ionization.

Zener breakdown: electrons tunneling from the VB of p-side to the CB of *n*-side

Principle of Operation of Solar Cell

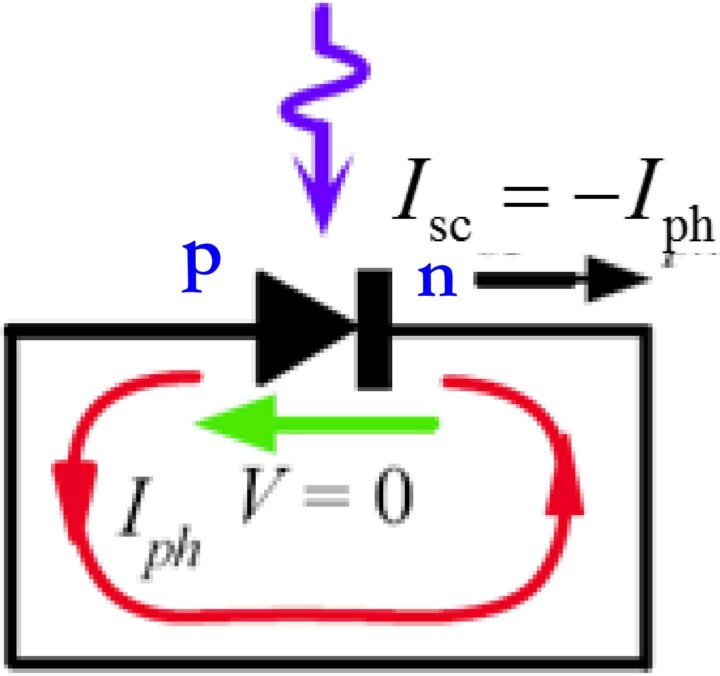
- Light is absorbed in the depletion region and produces e^--h^+ pair. These are separated by the built-in potential.
- The amount of absorption reduces with depth and hence depletion region must be close to the surface to maximize absorption.

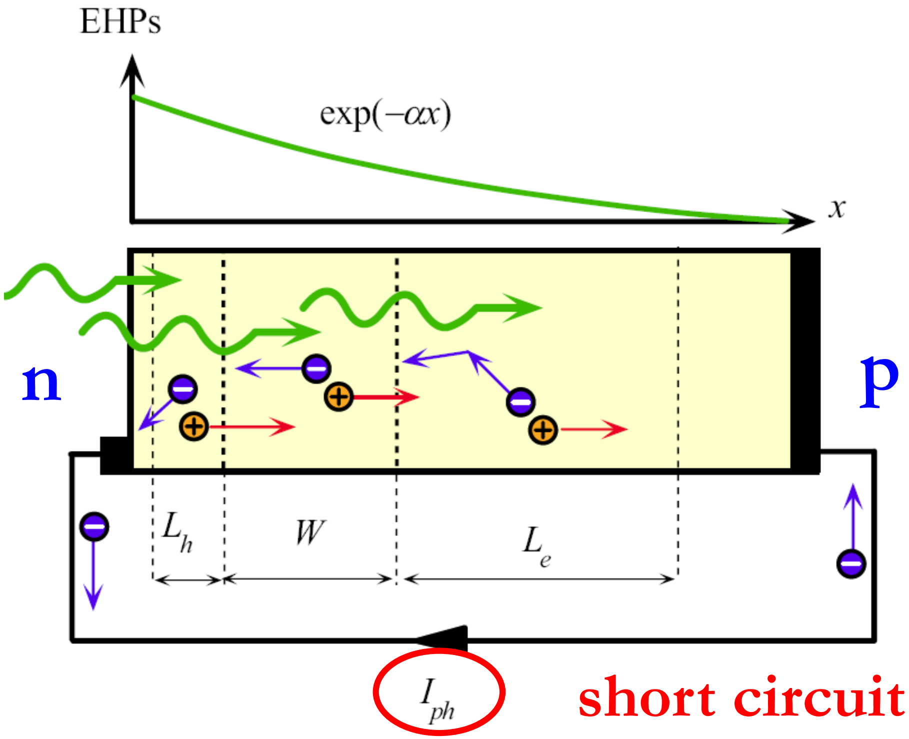




Principle of Operation of Solar Cell

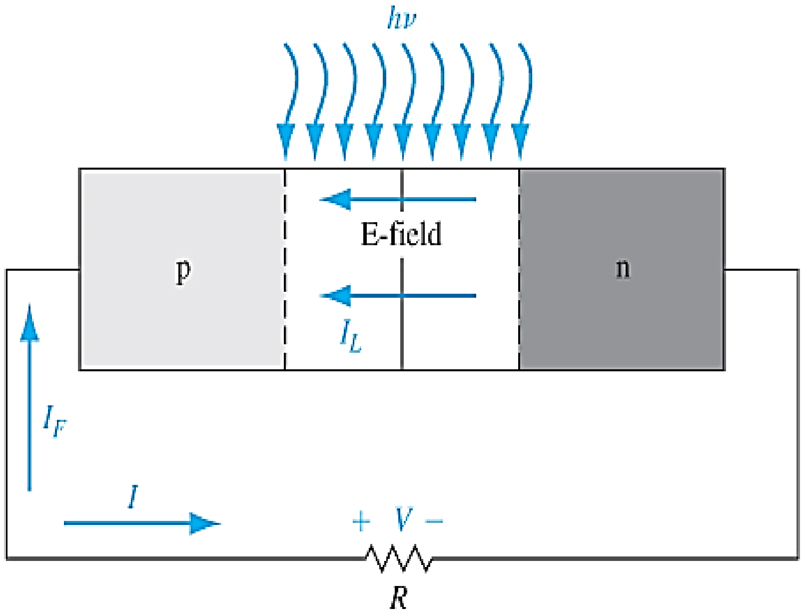
- If the external circuit is a short circuit (external load resistance is zero) then the only current is due to the generated EHPs (e^{-}-h^{+} pair) by the incident light. This is called the photocurrent, I_{ph} or short circuit current, I_{sc}.





A PN Junction Solar Cell

- The current from the forward biased pn junction (I_F) opposes the photocurrent, I_L (I_{ph}).
- I_{ph} is due to electrons going to the *n*-side and holes to the *p*-side by the electric field within the depletion region, i.e. drift current, while the forward bias current, I_F is due to diffusion current caused by the injection of minority carriers.

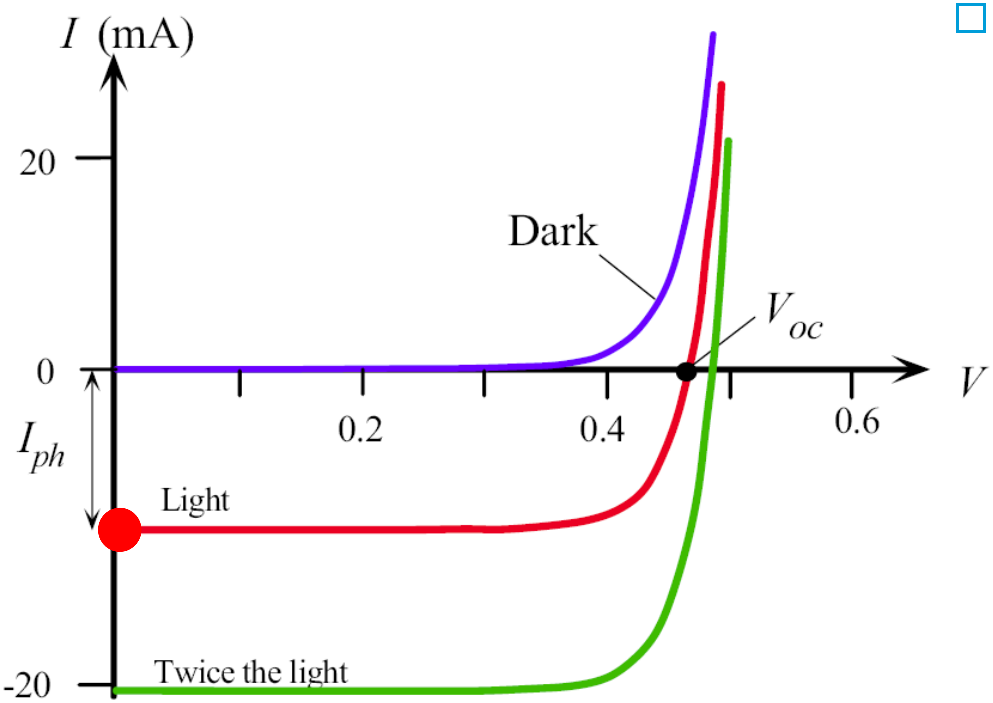


Solar cell *I-V*

$$I = I_L - I_F = I_L - I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

Photocurrent generated by light, I_{ph}

I-V of a Si Solar Cell



- Photocurrent, I_{ph} is the current when the external voltage is zero and an open circuit voltage, V_{oc}, which is the voltage when the net current in the circuit is zero.

$$I = -I_{\mathrm{ph}} + I_o\left[\exp\left(\frac{eV}{\eta kT}\right) - 1\right]$$

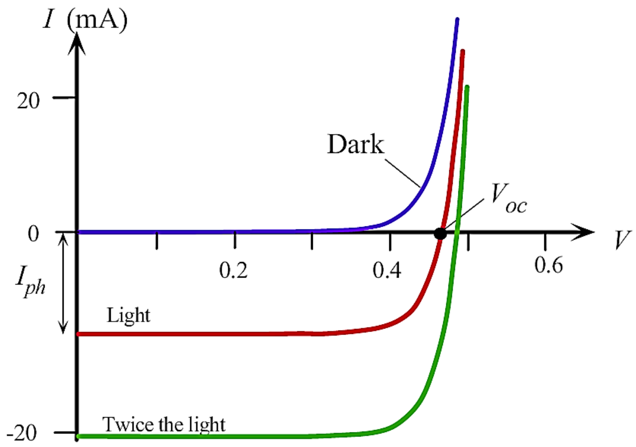
where I_o is the reverse saturation current and is the ideality factor: 1-2

The dark characteristics is similar to a pn junction I-V. Photovoltaic operation is always in the negative current region.

Example

- For a Si pn junction at T=300K, the reverse saturation current density is $3.6\text{x}10^{-11}$ A/cm^2, and the photocurrent density is 15 mA/cm^2, please calculate the open circuit voltage V_{oc}.

Answer: An open circuit voltage, V_{oc}, which is the voltage when the net current in the circuit is zero.



$$I = I_L - I_F = I_L - I_S\left[\exp\left(\frac{eV}{kT}\right) - 1\right]$$

$$I = 0 = I_L - I_S\left[\exp\left(\frac{eV_{oc}}{kT}\right) - 1\right]$$

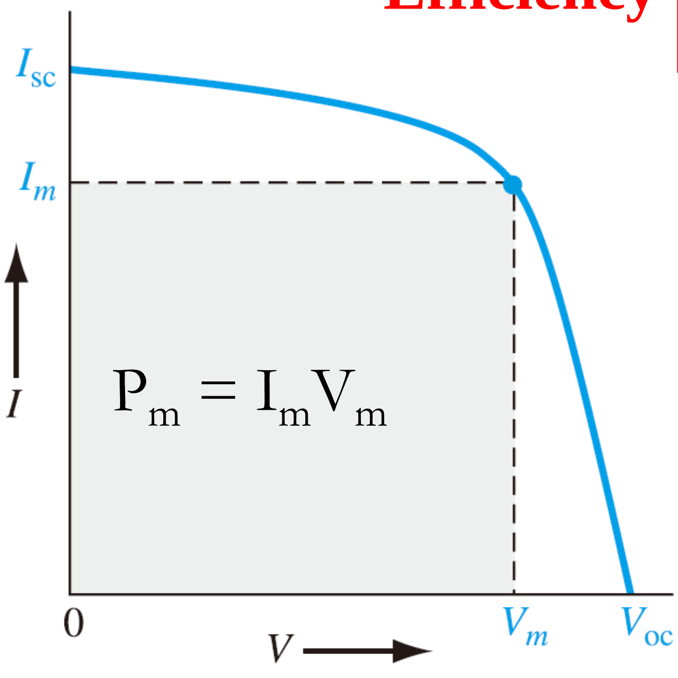
$$V_{oc} = V_t \ln\left(1 + \frac{I_L}{I_S}\right) = V_t \ln\left(1 + \frac{J_L}{J_S}\right) = (0.0259)\ln\left(1 + \frac{15\times10^{-3}}{3.6\times10^{-11}}\right) = 0.514\ \text{V}$$

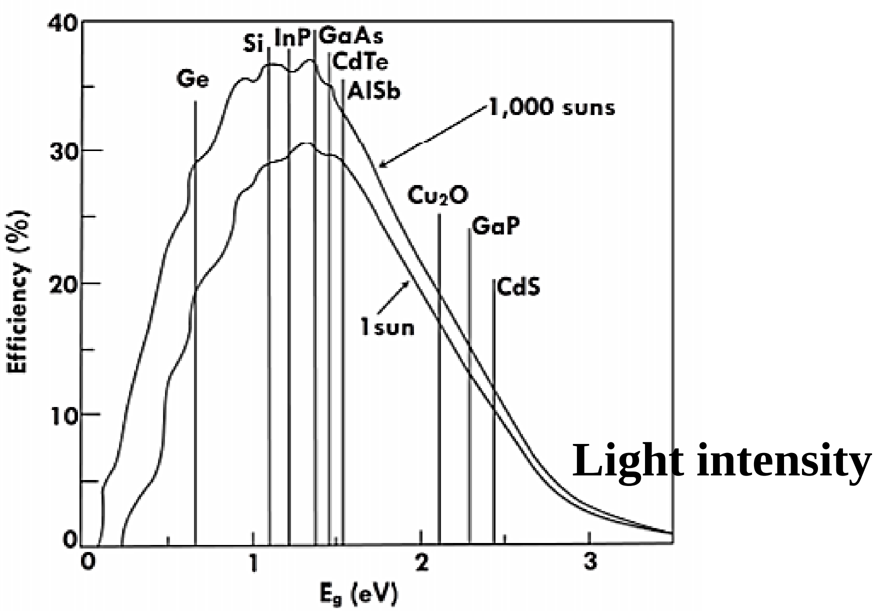
I-*V* of a Solar Cell

- I-V for a solar cell with **maximum power, P_m** indicated by the shaded area. The corresponding voltage & current are V_m and I_m.
- The efficiency of the solar cell depends on the band gap of the material.

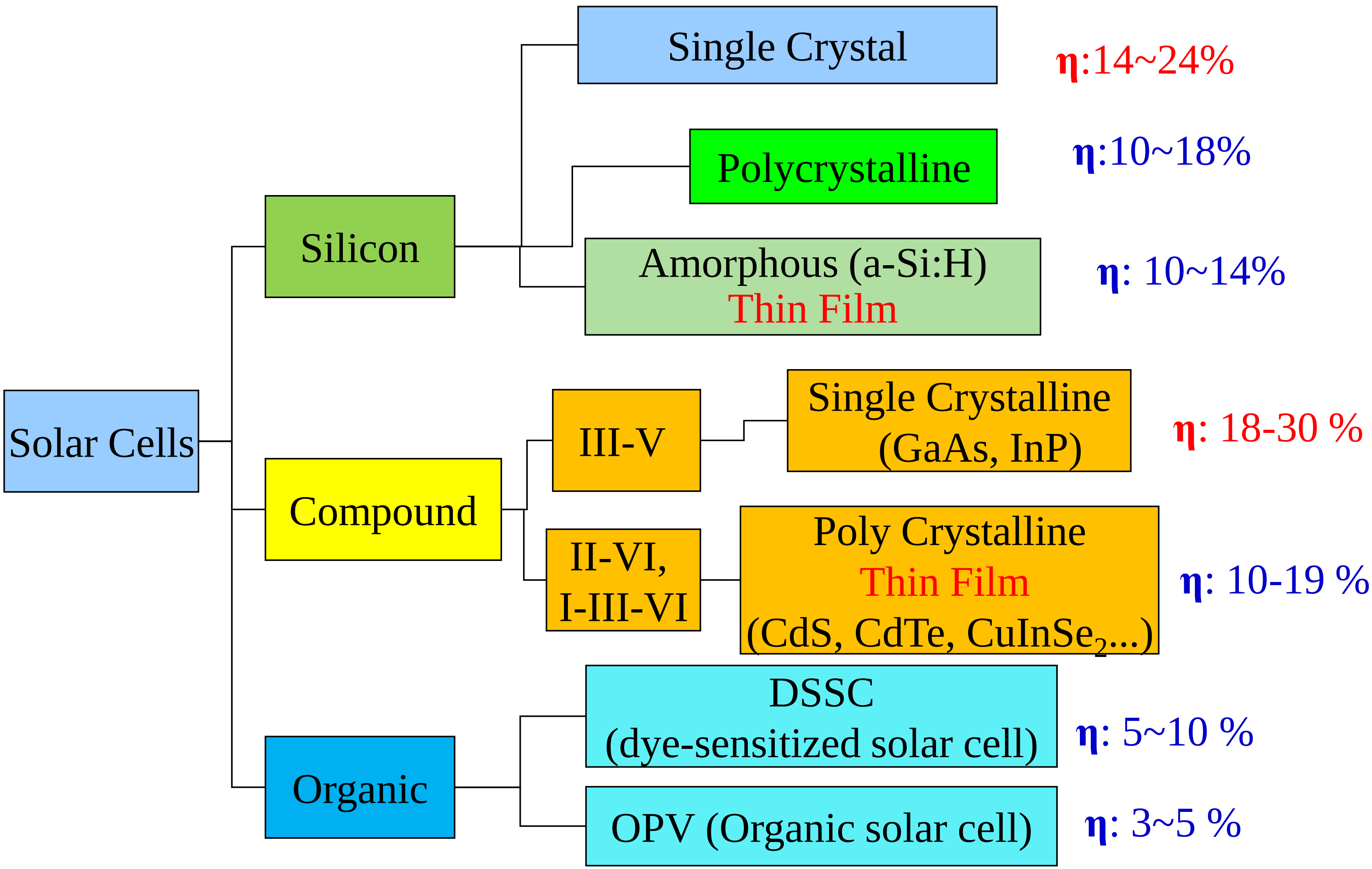
Efficiency

$$\eta = \frac{P_m}{P_{in}} \times 100\% = \frac{I_m V_m}{P_{in}} \times 100\%$$





The Category of Solar Cell



The Category of Solar Cell

- Conventional solar cells are made of Si single crystal and have an efficiency of around 22-24%, while polycrystalline Si cells have an efficiency of 18%.

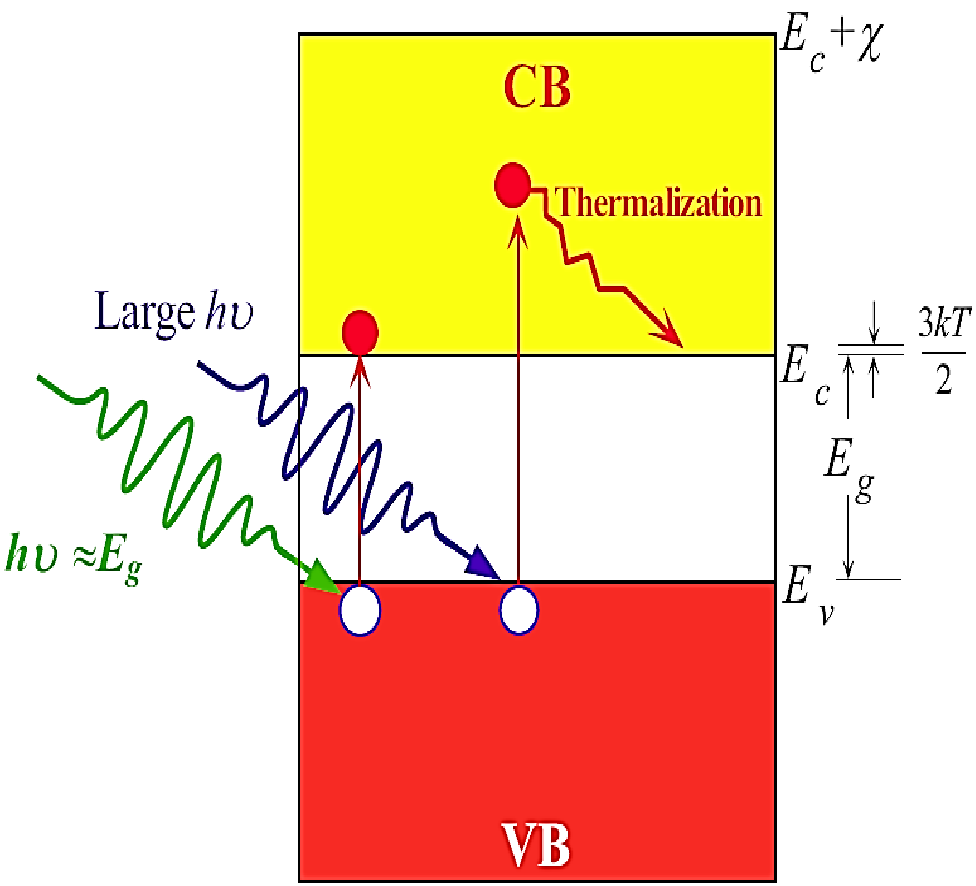
- Polycrystalline solar cells are cheaper to manufacture but have a lower efficiency since the microstructure introduces defects in the material that can trap carriers.

- Poly-crystalline II-VI-group materials are used for the manufacturing of low-cost thin-film solar cells. However, ultra-high-efficiency solar cells are generally fabricated by single-crystal III-V-group materials.

Optical Absorption in Semiconductor

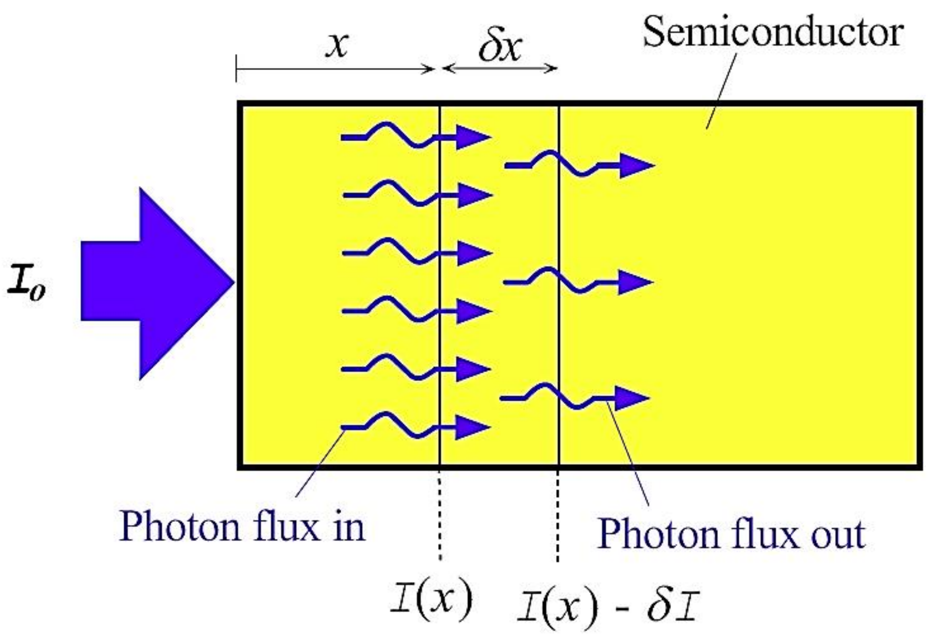
- Optical absorption generates electron-hole pairs.

- Energetic electrons must lose their excess energy to lattice vibrations until their average energy is $(3/2)kT$ in the CB.



Optical Absorption

- Absorption of photons within a small elemental volume of width (δx)



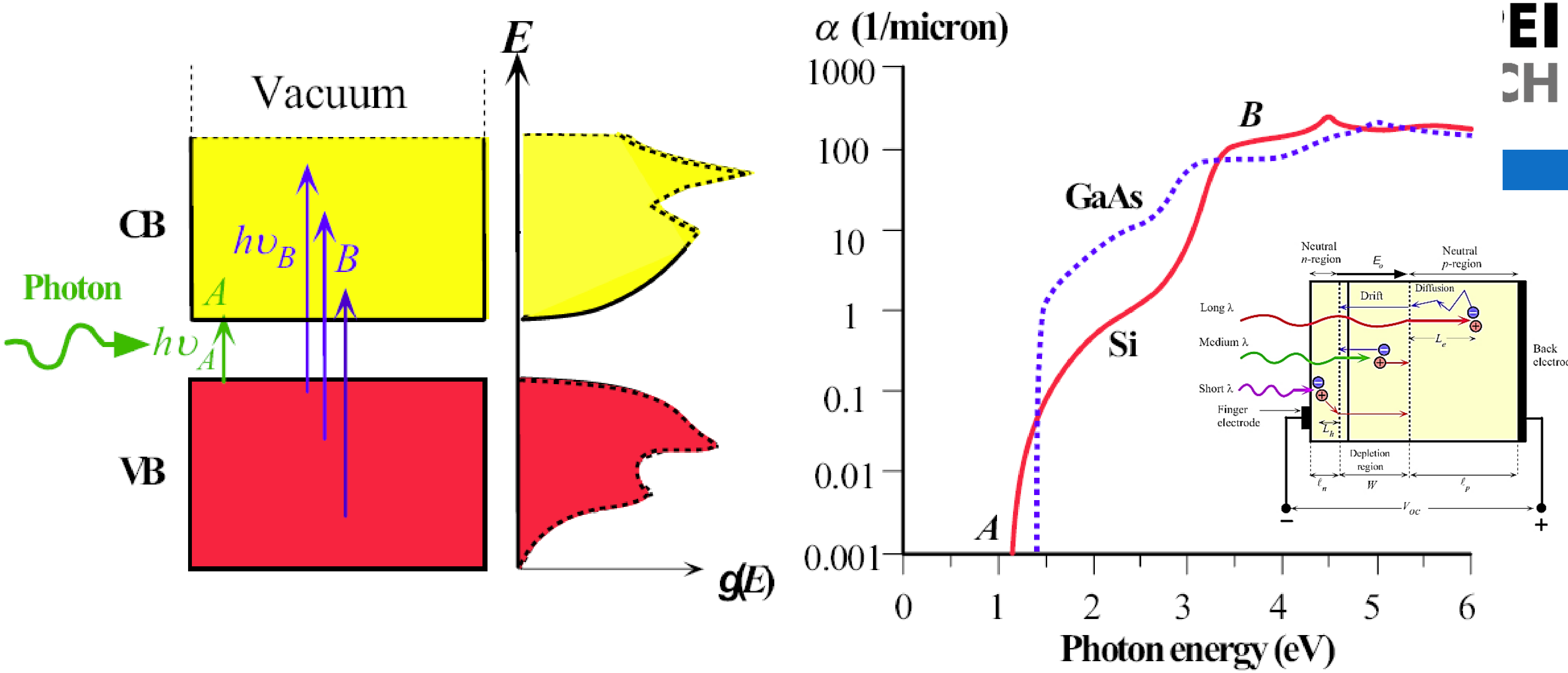
$$\alpha = -\frac{\delta I}{I\,\delta x}$$

α = optical absorption coefficient, I = light intensity, δI = change in the light intensity in a small elemental volume of thickness δx at x

Beer-Lambert Law

$$I(x) = I_o \exp(-\alpha x)$$

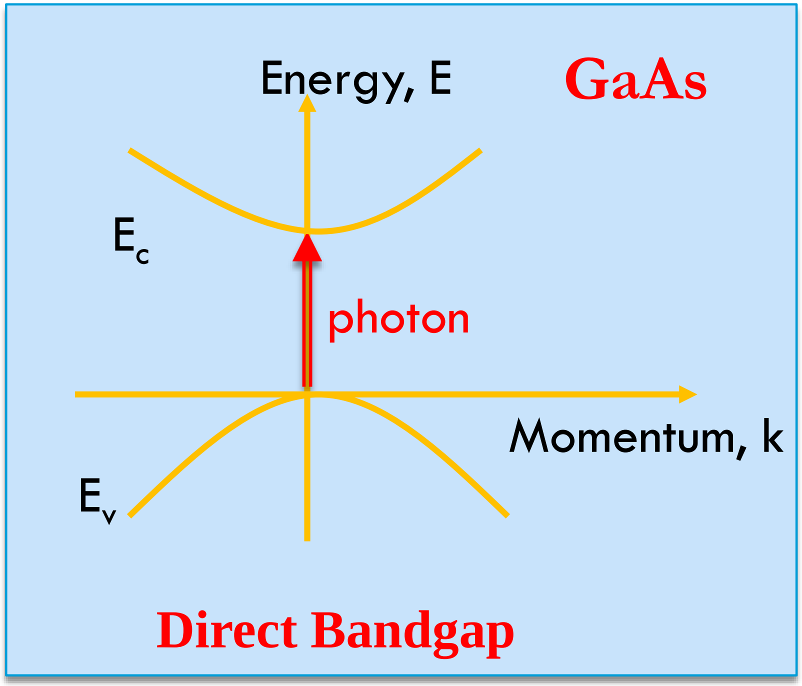
$I(x)$ = light intensity at x, I_o = initial light intensity, α = absorption coefficient, x = distance from the surface (location) where $I = I_o$. Note: Light propagates along x.

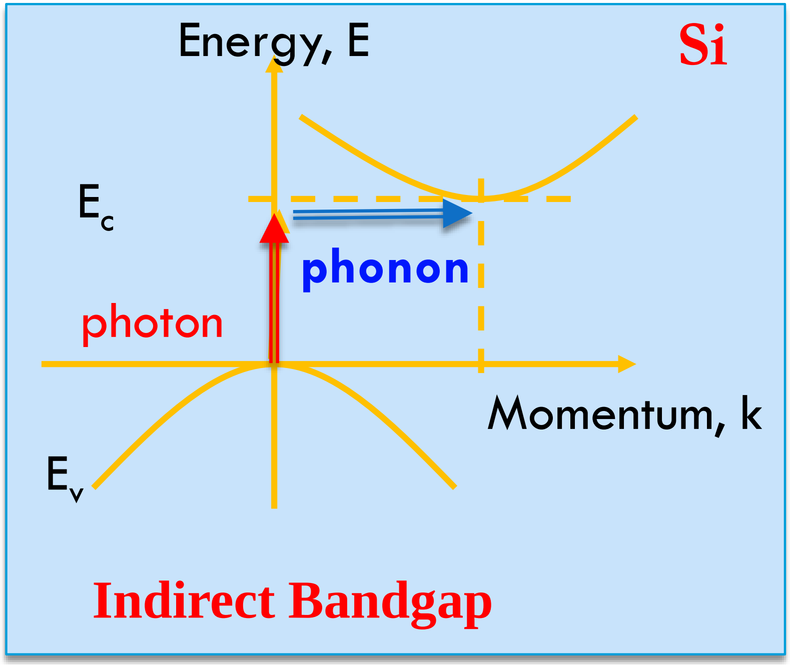


- The absorption coefficient α depends on the photon energy hv and hence on the wavelength.
- Density of states (DOS) increases from band edges and exhibits peaks and troughs. Generally **α increases with the photon energy** greater than E_g because more energetic photons can excite electrons from VB to numerous available states deep in the CB.

Direct & Indirect Bandgap

- The direct and indirect band gap behavior is reflected in absorption coefficients.





- High absorption probability
- Thinner material is required

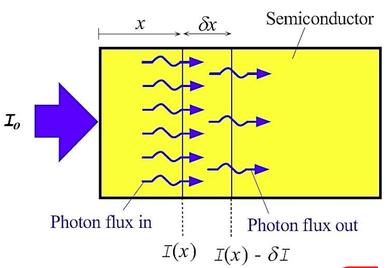
- Low absorption probability
- Thicker material is required

Absorption Length (λ)

- **Absorption length** λ is the distance into a material when the light intensity has dropped to 1/e (36.7%).

$$I(x) = I_o \exp(-\alpha x) \qquad \lambda = \frac{1}{\alpha} \quad \Rightarrow \quad I(\lambda) = I_o \exp(-1)$$

- λ is material and energy dependent.



Visible light

	Absorption Length in Microns (μm) (for absorbing about 73% of incoming light)			
Wavelength (nm)	**c-Si**	**a-Si**	**CIGS** Compound	**GaAs** Compound
400 nm (3.1eV)	0.15	0.05	0.05	0.09
600 nm (2eV)	1.8	0.14	0.06	0.18
800 nm (1.55 eV)	9.3	Not absorbed	0.14	1.1
1000 nm (1.24eV)	180.9	Not absorbed	0.25	Not absorbed

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Example

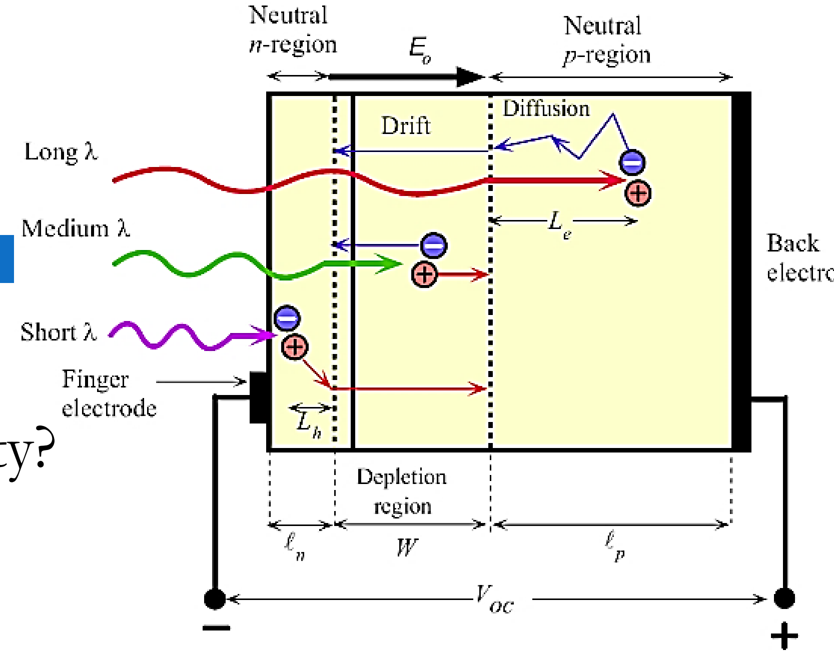
- Consider a light with wavelength of 0.7 μm. (1) Calculate the minimum width of CIGS solar cell to absorb 90% of incident light. (2) Calculate the minimum width of GaAs solar cell to absorb 90% of incident light. (absorption coefficient for CIGS is 10^5 cm^{-1}, while for GaAs is $5\text{x}10^4$ cm^{-1})

$$I(x) = I_o \exp(-\alpha x)$$

For CIGS: $\frac{I(x)}{I_0} = e^{-\alpha x} = e^{-10^5 x} = \frac{10}{100}$ x=0.23 μm

For GaAs: $\frac{I(x)}{I_0} = e^{-\alpha x} = e^{-5x10^4 x} = \frac{10}{100}$ x=0.46 μm

Diffusion Length



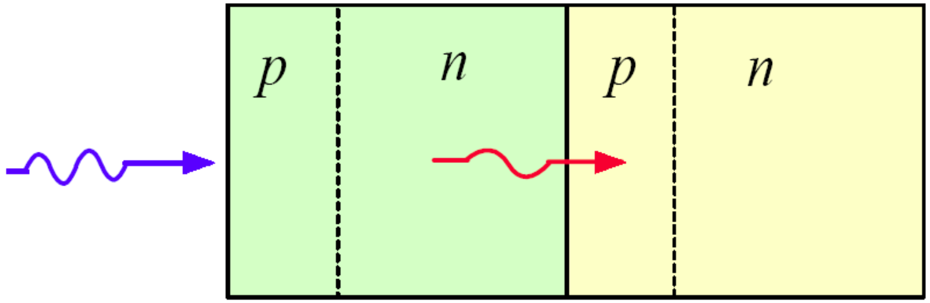
- How much light is absorbed?
- How much light is turned into electricity?

$$L_h = \sqrt{2D_h\tau_h} \qquad L_e = \sqrt{2D_e\tau_e}$$

	c-Si	a-Si:H	CdTe	CIGS
Mobility, μ (cm^2/Vs)	1350 (e^-) 480 (h^+)	10-20 (e^-) 1 – 5 (h^+)	~1000 (e^-) ~50 (h^+)	~400 (e^-) ~ 30 (h^+)
Diffusion Coefficient, D (cm^2/s)	35 (e^-) 12.5 (h^+)	~0.25 (e^-) ~0.05 (h^+)	~25 (e^-) ~2 (h^+)	~10 (e^-) ~ 1 (h^+)
Diffusion length, *L*	Few 100 μm	Few 100 nm	Few μm	Few μm
Lifetime, τ	5- 50 μs	5 – 50 ns	Several 10 ns	Several 10 ns

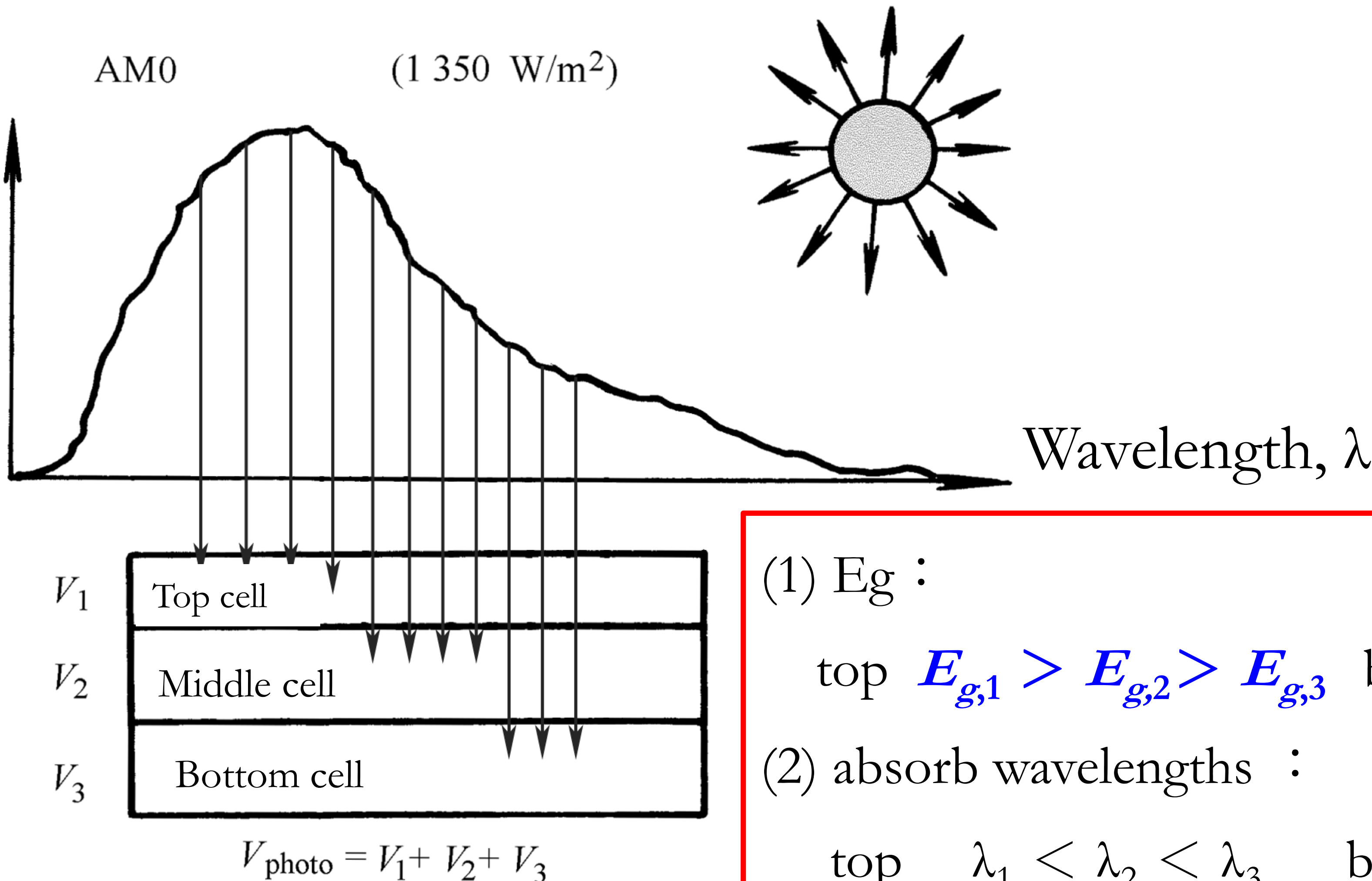
Tandem Solar Cell: The Road to Higher Efficiency

- Another way of improving solar cell efficiency is to have more than one cell in tandem.
- The higher band gap cell is closer to the illuminating surface to absorb the short wavelengths and the smaller band gap cell is at the interior to absorb the longer wavelengths. A larger portion of the solar radiation is absorbed so that tandem cells have high efficiency.



Cell 1 (E_{g1}) Cell 2 ($E_{g2} < E_{g1}$)

Tandem Solar Cell



(1) Eg：

top $E_{g,1} > E_{g,2} > E_{g,3}$ bottom

(2) absorb wavelengths ：

top $\lambda_1 < \lambda_2 < \lambda_3$ bottom

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Table 6.3 Typical characteristics of various solar cells at room temperature under AM1.5 illumination of 1000 W m^{-2}

Semiconductor	E_g (eV)	V_{oc} (V)	J_{sc} (mA cm^{-2})	FF	η (%)	Comments
Si, single crystal	1.1	0.5–0.7	42	0.7–0.8	16–24	Single crystal, PERL
Si, polycrystalline	1.1	0.5–0.65	38	0.7–0.8	12–19	
Amorphous Si:Ge:H film					8–13	Amorphous film with tandem structure, convenient large-area fabrication
GaAs, single crystal	1.42	1.02	28	0.85	24–25	
GaAlAs/GaAs, tandem		1.03	27.9	0.864	24.8	Different bandgap materials in tandem increases absorption efficiency
GaInP/GaAs, tandem		2.5	14	0.86	25–30	Different bandgap materials in tandem increases absorption efficiency
CdTe, thin film	1.5	0.84	26	0.75	15–16	
InP, single crystal	1.34	0.87	29	0.85	21–22	
$CuInSe_2$	1.0				12–13	

NOTE: AM1.5 refers to a solar illumination of "Air Mass 1.5," which represents solar radiation falling on the Earth's surface with a total intensity (or irradiance) of 1000 W m^{-2}. AM1.5 is widely used for comparing solar cells.

References

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