國立臺北科技大學 材料及資源工程系

電子材料與元件技術特論

SPECIAL TOPICS IN ELECTRONIC MATERIALS AND DEVICES

Electrical Properties-Part 1

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Outline

- Electrical Conduction
- Mobility Concept
- Electrical Properties of Conductor
- Electrical Properties of Semiconductor
- Temperature Dependence of Electrical Conductivity

Introduction

- Electrical conductivity and resistivity are:
  - ✓ material parameters
  - ✓ geometry independent
- Electrical resistance is:
  - ✓ Geometry and material dependent parameter
- Conductors, semiconductors, and insulators...
  - ✓ Differ in accessibility of energy states for electrons conduction (Eg)
- How electrons move in materials: electrical conduction
- How many moveable electrons are there in a material *(carrier density)*, how easily do they move *(mobility)*

Ohm's Law

- **Ohm's law:**

$$V = I \cdot R$$

V = voltage (V)

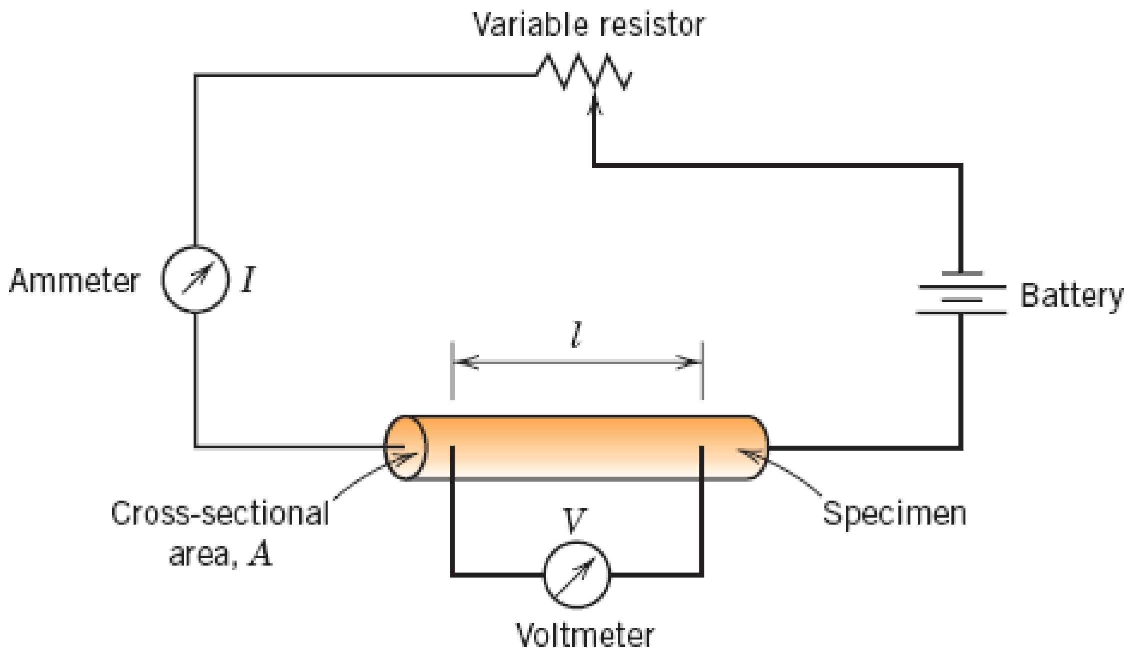
I=current (A)

R=resistance (Ω)

- **Resistivity, ρ:**
- **Resistance, R:**

$$R = \frac{\rho L}{A}$$

ρ : Ω.m



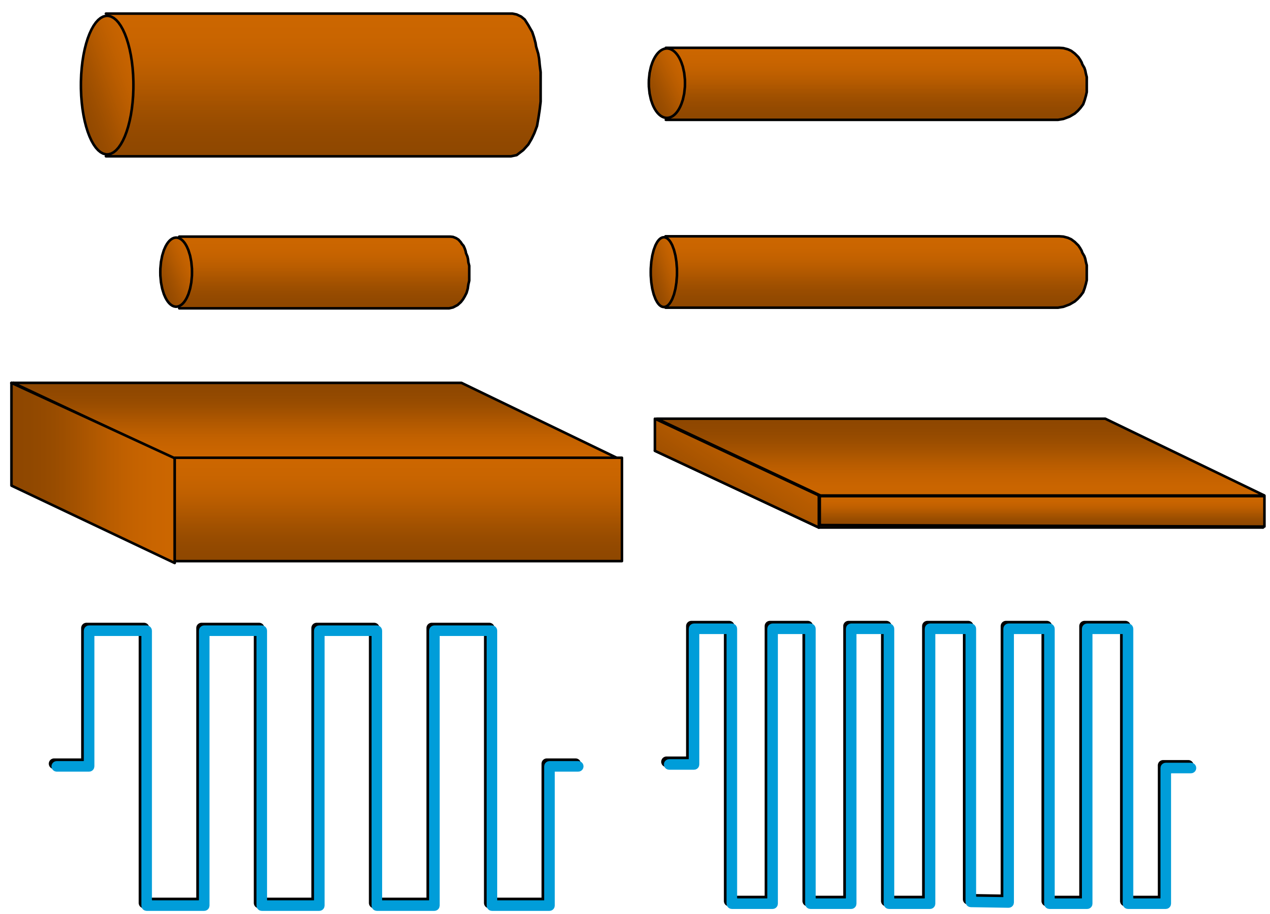
Schematic diagram for measuring electrical resistivity

Adapted from Callister 7e

How Sizes Affect Resistance

$$R = \frac{\rho \, L}{A}$$

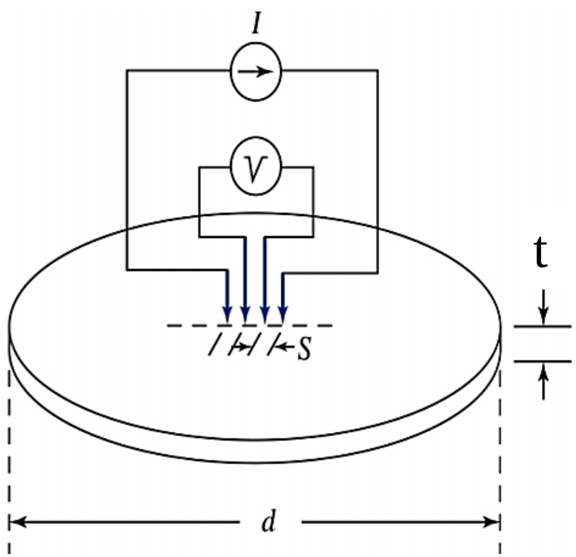
Low Resistance **High Resistance**



Resistance is determined by length, area, and the resistivity of the material.

Sheet Resistance Measurement

- To measure the sheet resistance & resistivity, four-point probe is applied. A current is passed through the outer probes and induces a voltage in the inner probes.

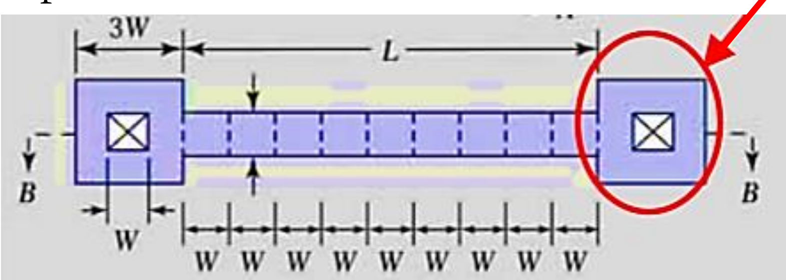


when L=W, $R = \rho\frac{L}{A} = \rho\frac{L}{t.w} = \frac{\rho}{t}\,(\Omega)$

Sheet resistance = $R_{sh} = \frac{\rho}{t} \quad (\Omega/\square)$

contact resistance

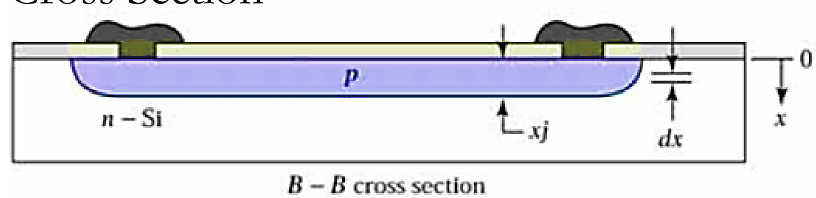
Top view



$$\rho = \frac{\pi}{ln\,(2)}t\left(\frac{V}{I}\right) = 4.523t\left(\frac{V}{I}\right)$$

Only valid for wafer thickness less than half the probe spacing (t < s/2)

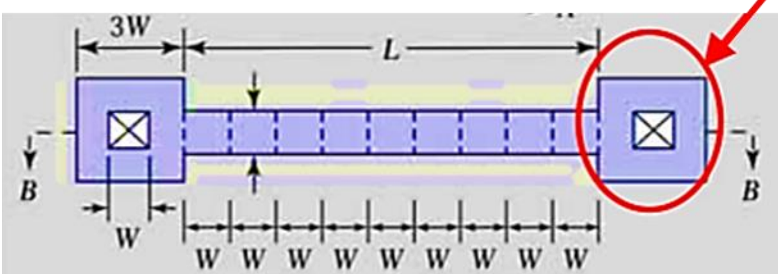
Cross Section



Resistance Calculation

- Please calculate the resistance of the following test structure. With L=90μm, W=10μm, sheet resistance=1 KΩ/□, and contact resistance (Rc)=0.6 KΩ.

0.6 KΩ



$$R = \left(R_{sh} \times \frac{L}{W}\right) + 2R_C$$

$=(1\ \text{K}\Omega/\square \times 9\ \square) + 2 \times 0.6$

$=10.2\ \text{K}\Omega$

Electrical Conductivity

- **Electrical conductivity, σ:**

$$\sigma = \frac{1}{\rho}$$

σ: $(\Omega\text{-m})^{-1}$

- **Electric field, E:**

$$E = \frac{V}{L}$$

- **Current density, J:**

$$J = \frac{I}{A} = \sigma E$$

Three major group classifications according to electrical conductivity:

- ✓ Conductors (metals)
- ✓ Semiconductors
- ✓ Insulators

$$R = \frac{\rho l}{A} \Rightarrow V = I\frac{\rho l}{A}$$

$$\boxed{\rho = 1/\sigma}$$

$$\boxed{\frac{V}{l}} = \boxed{\frac{I}{A}}\frac{1}{\sigma} \Rightarrow \boxed{E} = \frac{\boxed{J}}{\sigma}$$

$$J = \sigma E$$

Current Density

$\boxed{J = \sigma \, \boldsymbol{E}}$ <= another way to state Ohm's law

$$J \equiv \text{current density} = \frac{\text{current}}{\text{surface area}} = \frac{I}{A}$$ like a flux

$\boldsymbol{E}$ $\equiv$ electric field potential $= V/\ell$ or $(\Delta V/\Delta \ell)$

$$J = \sigma \, (\Delta V/\Delta \ell)$$

Electron flux conductivity voltage gradient

Resistivity of interconnect

- Resistivity of interconnect

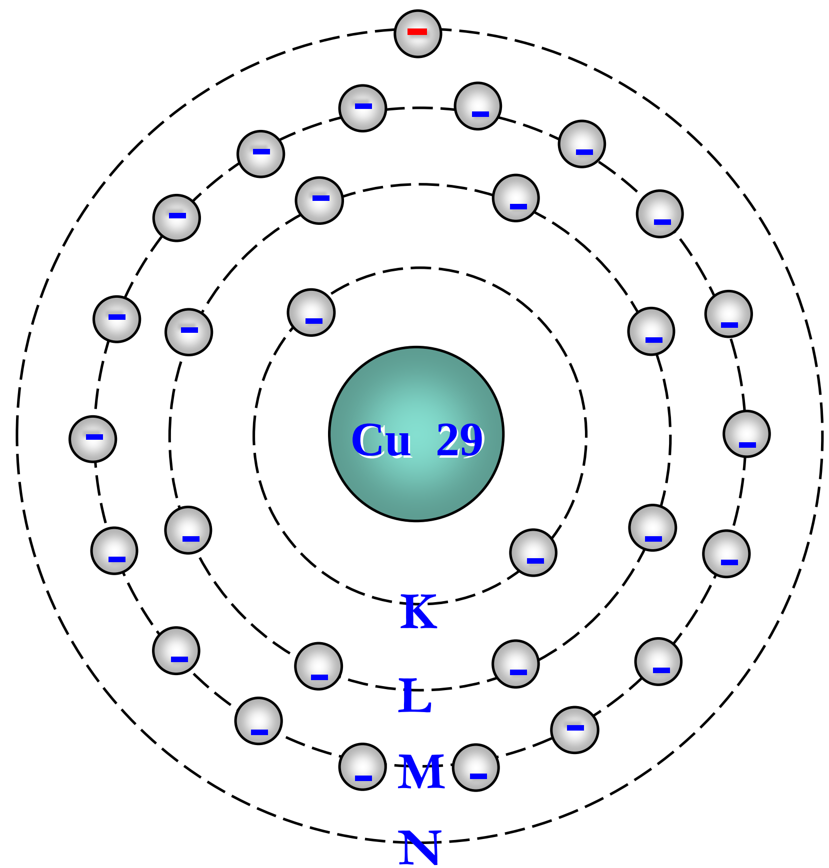
Material	ρ (Ω-m)
Silver (Ag)	1.6×10^{-8}
Copper (Cu)	1.7×10^{-8}
Gold (Au)	2.2×10^{-8}
Aluminum (Al)	2.7×10^{-8}
Tungsten (W)	5.5×10^{-8}

Flow of Free Electrons in Copper

Copper (Cu) atom

Shell #	Maximum # e^- per shell	Actual # e^- per shell
K	2	2
L	8	8
M	18	18
N	32	1
Total #	**60**	**29**

One electron in the valence shell



A good conductor generally has few valance electrons in valance shell that are loosely bound and easily given up by the atom.

Example

(a) Compute the electrical conductivity of a 7.0-mm diameter cylindrical silicon specimen 57 mm long in which a current of 0.25 A passes in an axial direction. A voltage of 24 V is measured across two probes that are separated by 45 mm.

Substitute $0.25\ \mathrm{A}$ for I, $45\times10^{-3}\ \mathrm{m}$ for l, $24\ \mathrm{V}$ for V and $7\times10^{-3}\ \mathrm{m}$ for d

$$\boldsymbol{\sigma} = \frac{\mathbf{1}}{\boldsymbol{\rho}} = \frac{\boldsymbol{L}}{\boldsymbol{R\,A}} = \frac{\boldsymbol{I}\,\boldsymbol{L}}{\boldsymbol{V}\,\boldsymbol{A}}$$

$$\sigma = \frac{0.25\times45\times10^{-3}}{24\times\left(\frac{\pi}{4}\times\left(7\times10^{-3}\right)^2\right)}$$

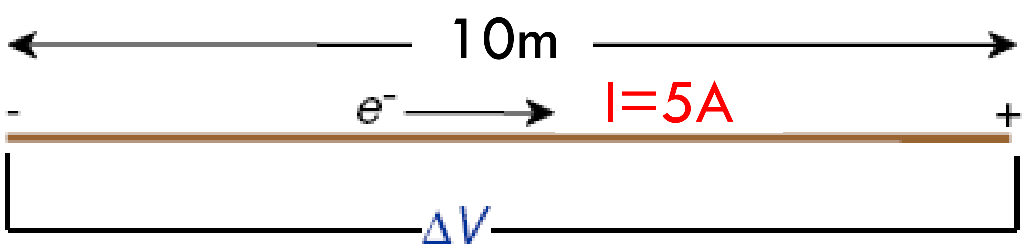
$$= 12.2\,(\Omega\cdot\mathrm{m})^{-1}$$

(b) Compute the resistance over the entire 57 mm of the specimen.

Example

Question: An aluminum (Al) wire 10 m long must experience a voltage drop of less than 1.0 V when a current of 5 A passes through it. Compute the minimum diameter of the wire. (Al $\sigma = 3.8 \times 10^7\ (\Omega\text{-m})^{-1}$)

Al wire



$$A = \pi\left(\frac{d}{2}\right)^2$$

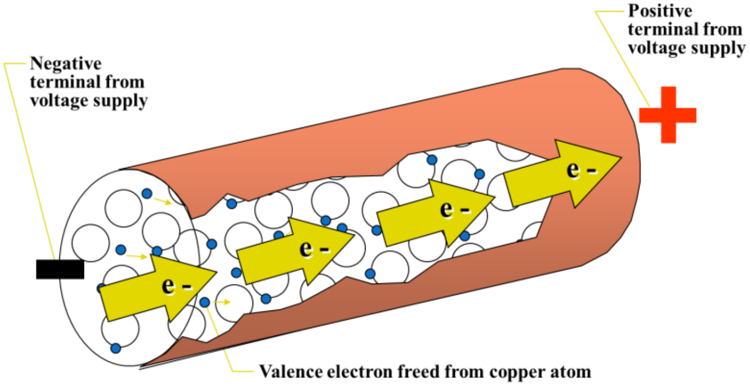
$$\sigma = \frac{1}{\rho} = \frac{L}{R\,A} = \frac{I}{V}\frac{L}{A}$$

Example

- An aluminum (Al) wire 10 m long must experience a voltage drop of less than 1.0 V when a current of 5 A passes through it. Compute the minimum diameter of the wire. (Al: σ = 3.8 x 10^7 $(\Omega\text{-m})^{-1}$)

$$\pi\left(\frac{d}{2}\right)^2 = \frac{I\,l}{V\sigma}$$

$$d = \sqrt{\frac{4\,I\,l}{\pi V \sigma}}$$



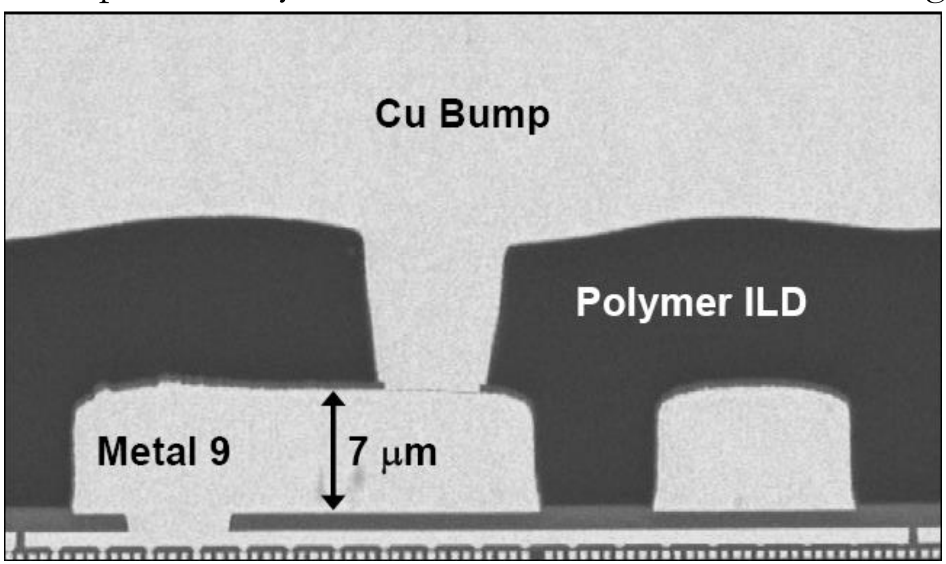
$$= \sqrt{\frac{(4)(5\ \text{A})(10\ \text{m})}{(\pi)(1\ \text{V})\left[3.8\ \times 10^7\ (\Omega - \text{m})^{-1}\right]}}$$

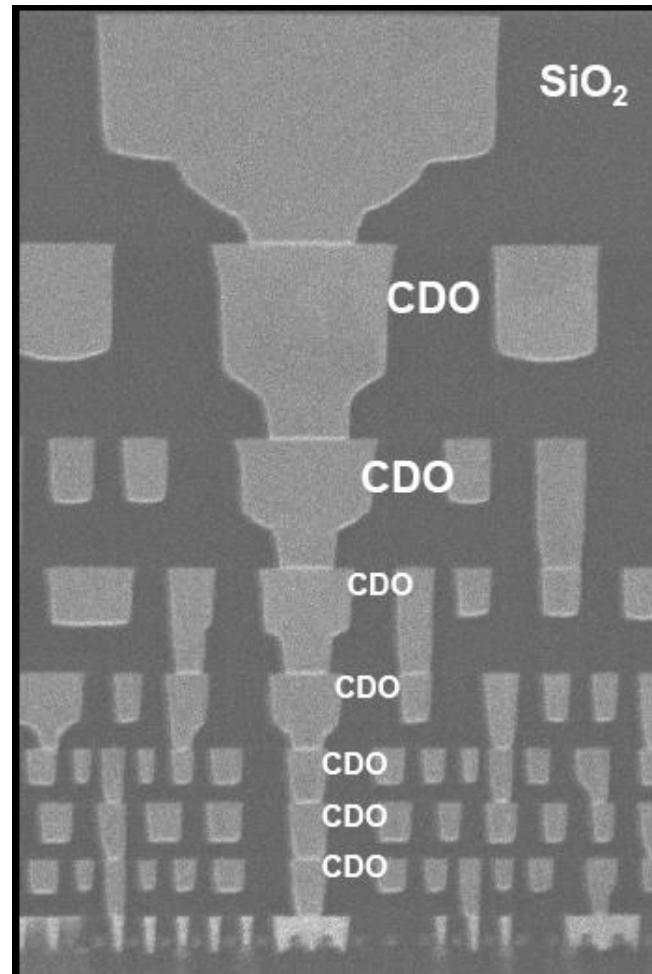
$$= 1.3 \times 10^{-3}\ \text{m} = 1.3\ \text{mm}$$

Intel 45nm Interconnects

- **In integrated circuits (ICs), interconnects are formed during the back-end-of-line (BEOL) after the fabrication of the transistors on the substrate**

https://www.youtube.com/watch?v=d9SWNLZvA8g



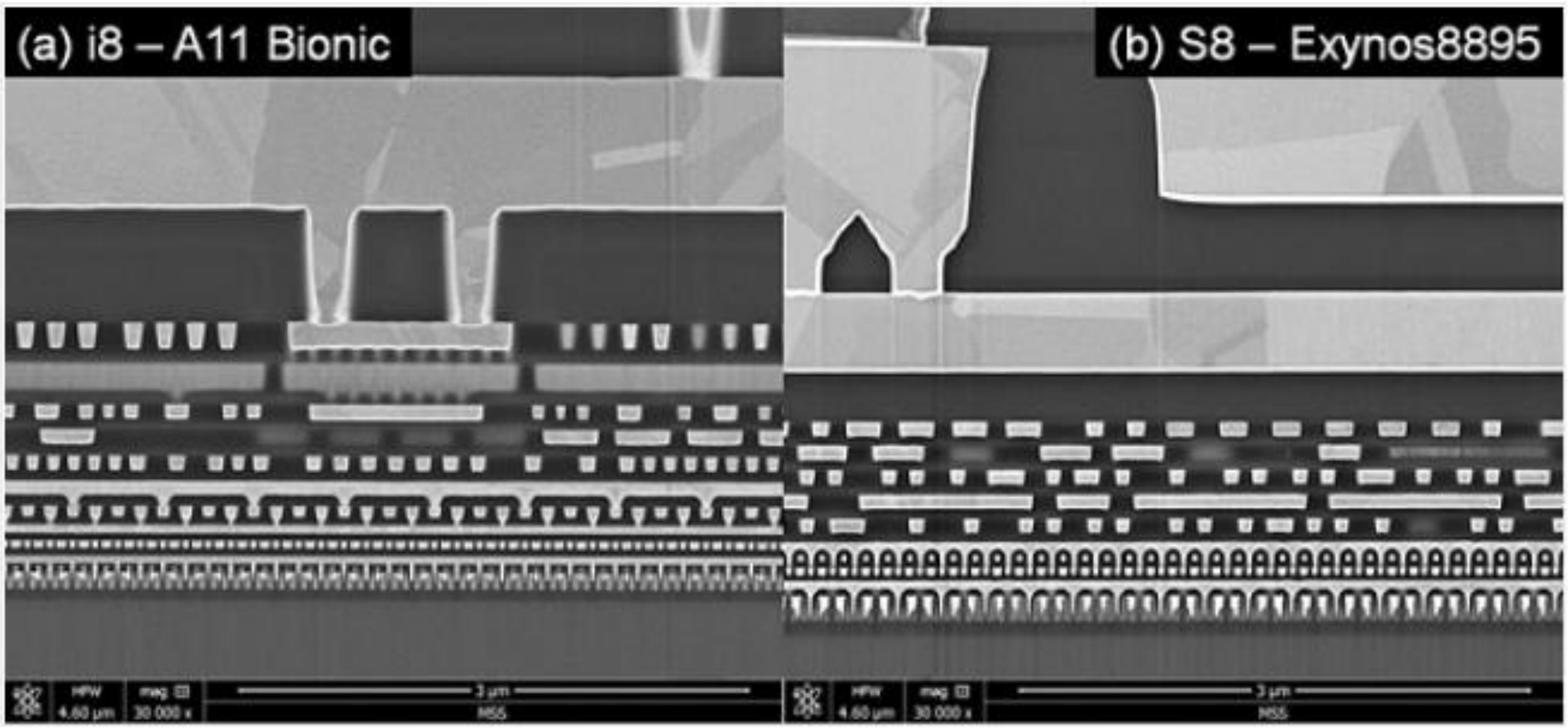


1P9M

From :Intel

TSMC vs. Samsung 10nm Interconnects

- The metal thickness from M1 to M11 for A11 and Exynos8895 is 2.88 μm and 3.21 μm, respectively.



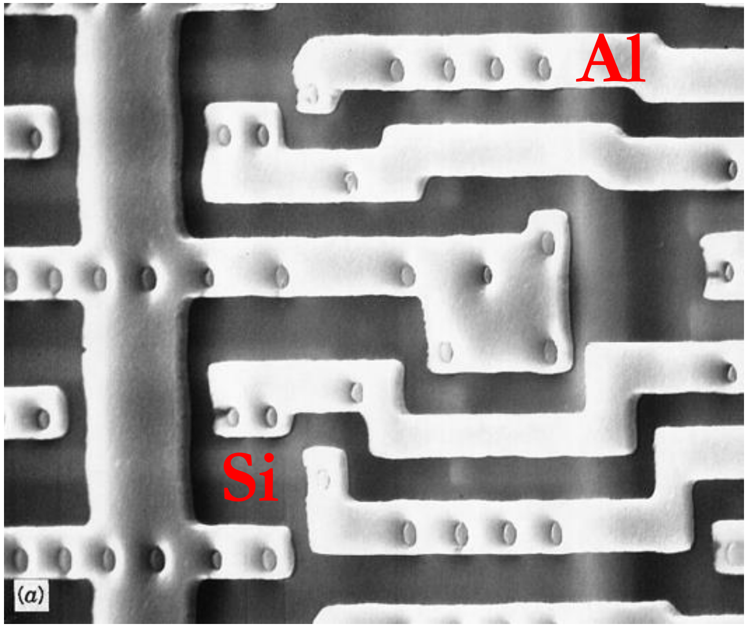
Cross-section SEM images of IC chips.

http://www.msscorps.com/archive/10%20nm%20technology%20node%20public(2).pdf

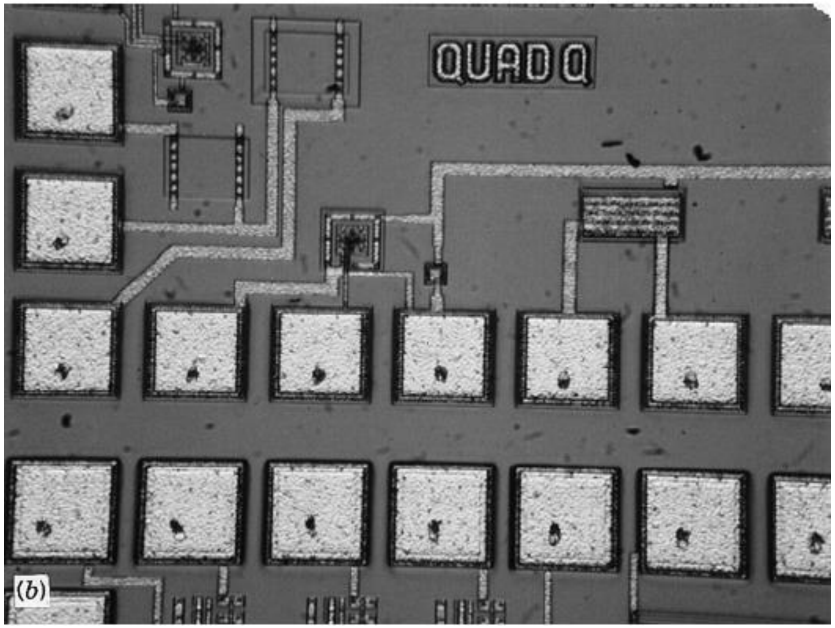
Integrated Circuit

- Scanning electron microscope (SEM) images of an IC

Microprocessing chip



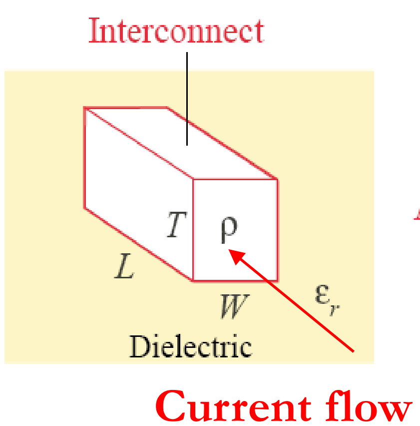
Testing Pad

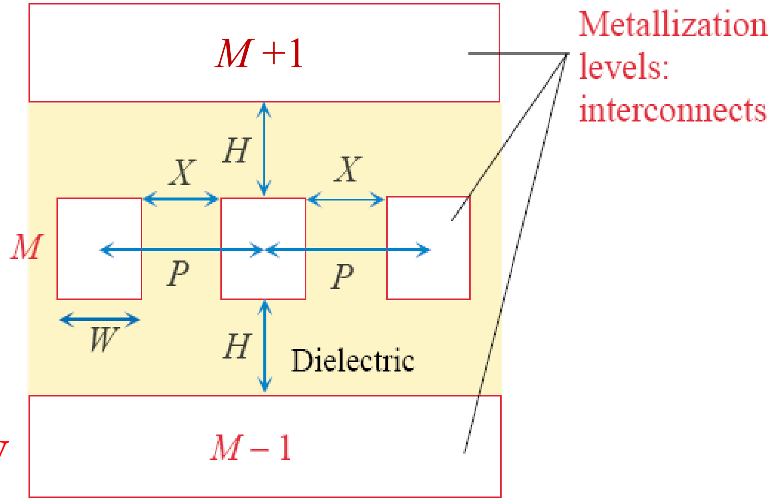


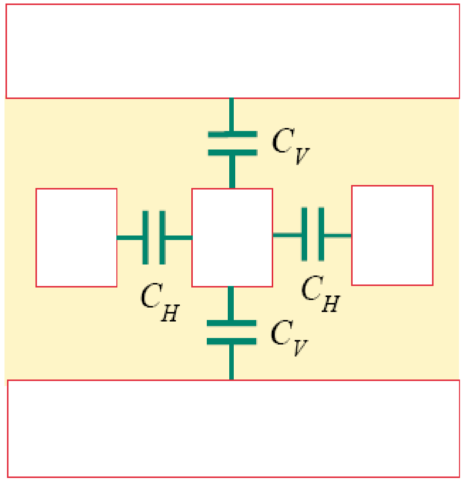
Adapted from Callister 7e

RC Time Delay

- RC delay, hinders the further increasing of speed in microelectronic integrated circuits. When the feature size becomes smaller and smaller to increase the clock speed, the RC delay plays an increasingly important role







$$C_H = \frac{\varepsilon_0 \varepsilon_r T L}{X} \qquad C_V = \frac{\varepsilon_0 \varepsilon_r W L}{H} \qquad C_{eff} = 2(C_H + C_V) = 2\ \varepsilon_0 \varepsilon_r L\left(\frac{T}{X} + \frac{W}{H}\right)$$

$$R = \rho \frac{L}{TW}$$

$$RC = C_{eff}\, x\, R = 2\ \varepsilon_0 \varepsilon_r \rho \left(\frac{L^2}{TW}\right)\left(\frac{T}{X} + \frac{W}{H}\right)$$

Example

EXAMPLE 2.28

MULTILEVEL INTERCONNECT *RC* TIME CONSTANT In a particular high-transistor-density IC where copper is used as the interconnect, one level of the multilevel interconnects has the following characteristics: pitch $P = 0.45\ \mu$m, $T = 0.36\ \mu$m, $A_R = 1.6$, $H = X$, and $\varepsilon_r \approx 3.6$. Find the effective capacitance per millimeter of interconnect length, and the *RC* delay time per L^2 as ps/mm^2 (as normally used in industry).

SOLUTION

Since $A_R = T/W$, $W = T/A_R = 0.36/1.6 = 0.225\ \mu$m. Further, from Figure 2.37b, $P = W + X$, so that $X = P - W = 0.45 - 0.225 = 0.225\ \mu$m. $H = X = 0.225\ \mu$m. Thus, Equation 2.61 for $L = 1$ mm $= 10^{-3}$ m gives

$$C_{\text{eff}} = 2\varepsilon_o\varepsilon_r L\left(\frac{T}{X} + \frac{W}{H}\right) = 2(8.85 \times 10^{-12})(3.6)(10^{-3})\left[\frac{0.36}{0.225} + \frac{0.225}{0.225}\right] = 0.17\ \text{pF}$$

which is about 0.2 pF per millimeter of interconnect. The *RC* time constant per L^2 is

$$\frac{RC}{L^2} = 2\varepsilon_o\varepsilon_r\rho\left(\frac{1}{TW}\right)\left(\frac{T}{X} + \frac{W}{H}\right) = 2\varepsilon_o\varepsilon_r\rho\left(\frac{1}{WX} + \frac{1}{TH}\right)$$

$$= 2(8.85 \times 10^{-12})(3.6)(17 \times 10^{-9})$$

$$\left[\frac{1}{(0.225 \times 10^{-6})(0.225 \times 10^{-6})} + \frac{1}{(0.36 \times 10^{-6})(0.225 \times 10^{-6})}\right]$$

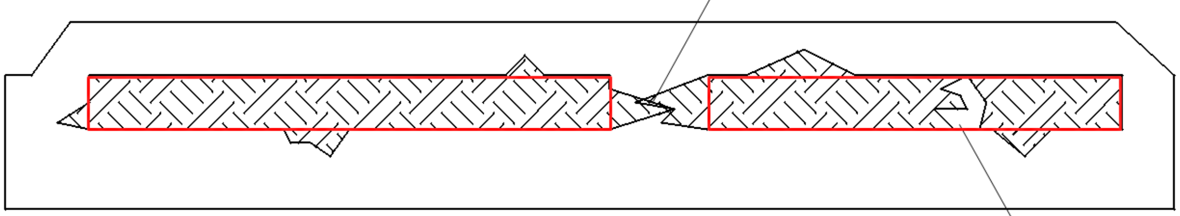
$$= 3.4 \times 10^{-5}\ \text{s m}^{-2} \qquad \text{or} \qquad 34\ \text{ps mm}^{-2}$$

Reduction in RC Delay

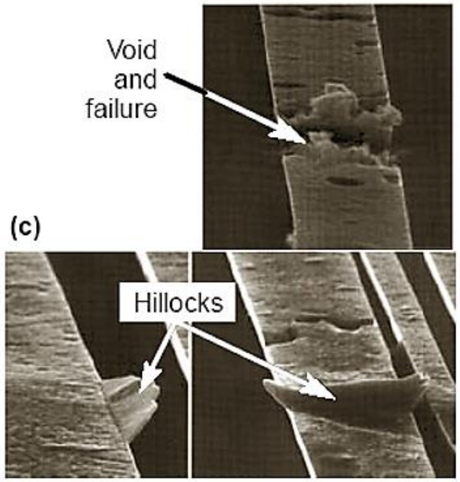
- Copper(Cu) is the ideal interconnect metal to replace of Al:

1. Reduction in resistivity by 40% (Cu:1.68 vs. Al:2.65 μΩ-cm)
2. Reduction in power consumption: $P=I^2R$
3. Cu has superior resistance to electromigration: the gradual movement of Al atoms due to the momentum transfer from the electrons, causing voids, thinning of metal line, open circuit, and hillocks.

Hillocks short-circuit two metal lines



Void in metal line

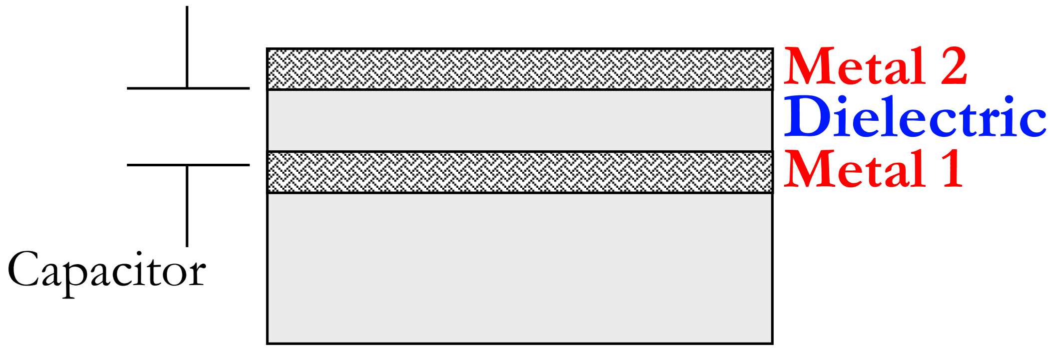


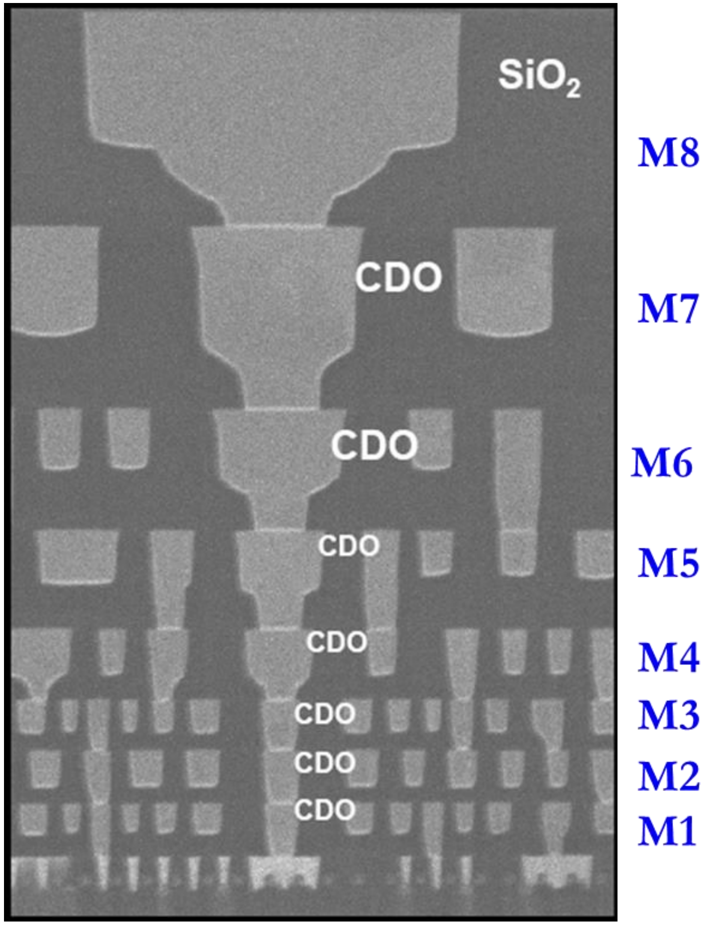
Reduction in RC Delay by Low-k Dielectric Material

- Parallel-plate capacitor:

$$C = \frac{\varepsilon_r \varepsilon_0}{T_{ox}} A$$

ε_0: permittivity of free space~8.85×10^{-12} (F/m); ε_r: dielectric constant of material (κ)





1P9M

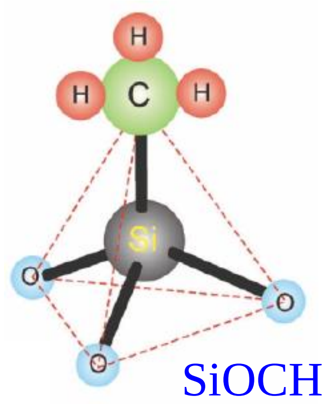
From :Intel

- Low-k dielectric reduces capacitance between metal layers.
- RC time delay can be reduced.

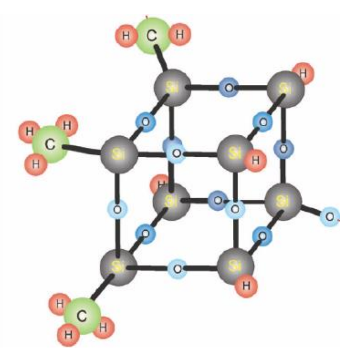
Reduction in RC Delay

- Parallel-plate capacitor: $C = \frac{\varepsilon_r \varepsilon_0}{T_{ox}} A$
- ε_0 is permittivity of free space~8.85 x 10^{-12} (F/m); ε_r is dielectric constant of material (κ).
- "Air" is the reference with "κ=1". SiO_2 κ is 3.9.

Potential low-*k* Dielectric	Dielectric Constant (*k*)	Gap Fill (μm)	Cure Temp. (°C)	Remarks
FSG (silicon oxyfluoride, Si_xOF_y)	3.4 – 4.1	<0.35	No issue	FSG has almost the same *k*-value as SiO_2 and reliability concern that fluorine will attack and corrode tantalum barrier metal.
HSQ (hydrogen silsesquioxane)	2.9	<0.10	350 – 450	Silicon-based resin polymer available in solution as Fox (Flowable Oxide) for spin-on coating application. May require surface passivation to reduce moisture absorption. Cure is done in nitrogen.
Nanoporous silica	1.3 – 2.5	<0.25	400	Inorganic material with tunable dielectric constant that relies on pore density. Increased porosity reduces mechanical integrity – porous material must withstand polishing, etching and heat treatments without degradation.
Poly(arylene) ether (PAE)	2.6 – 2.8	<0.15	375 – 425	Spin-on aromatic polymer with excellent adhesion and ability to be polished with CMP.
a-CF (fluorinated amorphous carbon or FLAC)[1]	2.8	<0.18	250 – 350	Leading candidate for CVD deposition with high density plasma CVD (HDPCVD) to produce film with good thermal stability and adhesion.
Parylene AF4 (aliphatic tetrafluorinated poly-p-xylylene)	2.5	<0.18	420 – 450	CVD film that meets adhesion and via resistance requirements with need to maintain gas delivery system at 200°C to control parylene precursor flow rate.

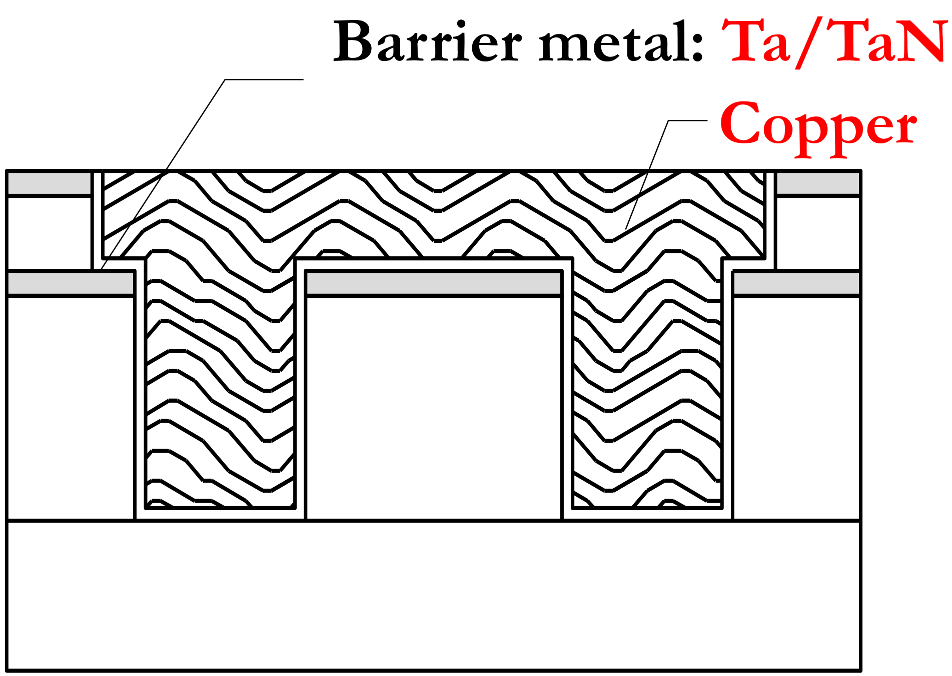


SiOCH


silsesquioxane (SSQ)

Barrier Layer for Copper Interconnect

- A barrier metal is a thin layer metal that prevents intermixing of the materials above and below the barrier.
- Cu barrier layer must function as an adhesion promoter and effective diffusion barrier.



Requirements for Copper Barrier Metal

1. Prevent copper diffusion.
2. Low film resistivity.
3. Good adhesion to both dielectric material and copper.
4. Compatible with chemical-mechanical polish (CMP).
5. Metal layer is continuous and conformal with good step coverage and deposition in high aspect ratio gaps.
6. Minimal thickness to allow the copper to occupy the maximum cross-sectional area.

Outline

- Electrical Conduction
- **Mobility Concept**
- Electrical Properties of Conductor
- Electrical Properties of Semiconductor
- Temperature Dependence of Electrical Conductivity

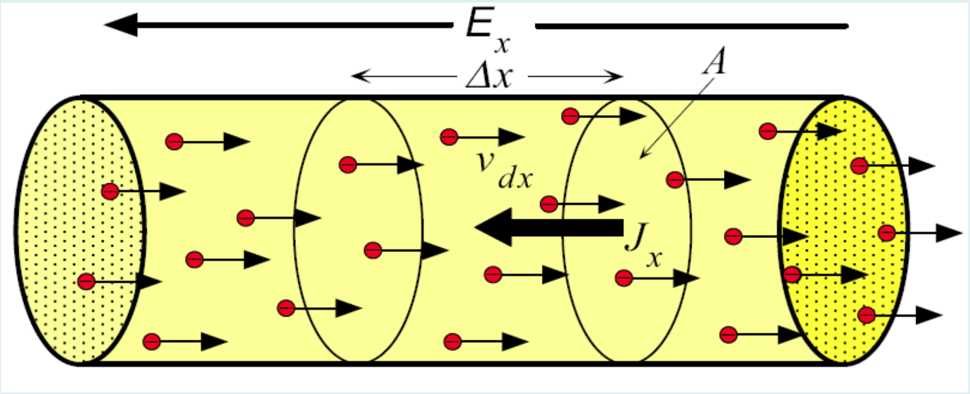
Charge Carriers

- Two charge carrying mechanisms:
  - ✓ Electron – negative charge
  - ✓ Hole – equal & opposite positive charge
- Net electron motion created in opposite direction to the electric field
  - ✓ Electronic current, $I = \left(\frac{dQ}{dt}\right)$, flow of charge

Current Density

Metals and Conduction by Electrons

$$J = \frac{\Delta q}{A \Delta t}$$



Net amount of charge flowing across an unit area per unit time

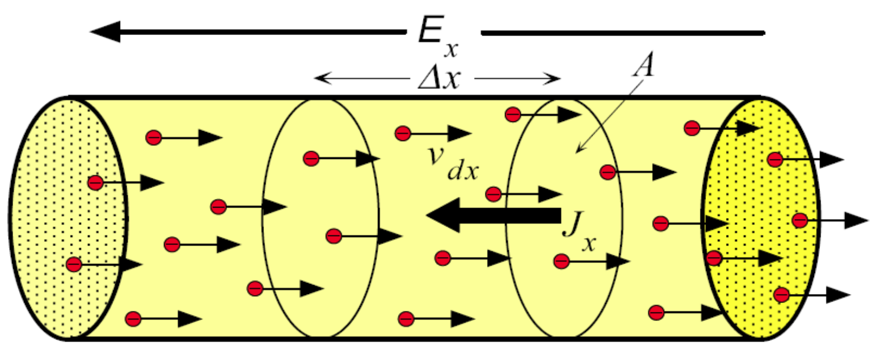
- Average velocity of electrons in the x direction at time t: $v_{dx}(t)$
  - ✓ Known as the drift velocity
  - ✓ Instantaneous velocity averaged over many e-

Current Density

$$J = \frac{\Delta q}{A \Delta t}$$

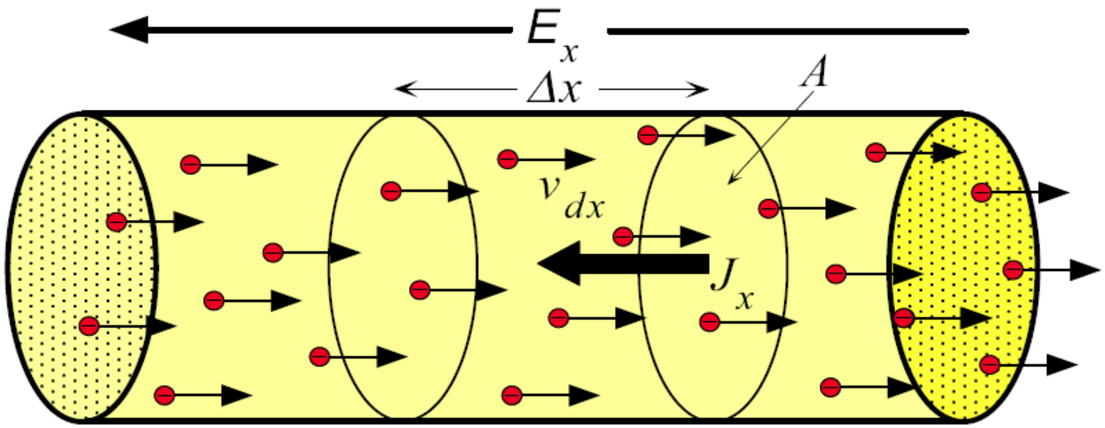
- Now, suppose that n is the number of e- per unit volume in a conductor (n = N/V)
- In a given amount of time, Δt, electrons can move a certain distance, Δx:

$$\Delta x = v_{dx} \Delta t$$



- All e- within the distance Δ*x* pass through A; thus n(A Δ*x*) is the total number of e- crossing A in time Δ*t*
- The total charge Δ*q* crossing the area A is: *en* A Δ*x*

Current Density



- Drift of electrons in a conductor in the presence of an applied electric field. Electrons drift with an average velocity v_{dx} in the x-direction. (E_x is the electric field.)

$$J_x = \frac{\Delta q}{A\,\Delta t} = \frac{e n A v_{dx}\,\Delta t}{A\,\Delta t} = e n v_{dx}$$

Definition of Drift Velocity

Average velocity of e- in the x direction at time, t:

$$v_{dx} = \frac{1}{N}\left[v_{x1} + v_{x2} + v_{x3} + \cdots + v_{xN}\right]$$

v_{dx} = drift velocity in x direction, N = number of conduction electrons,
v_{xi} = x direction velocity of ith electron

v_{dx}: the instantaneous velocity in the x direction averaged over many electrons

Current Density and Drift Velocity

$$\boxed{J_x = e\,n\,v_{dx} = \sigma E}$$

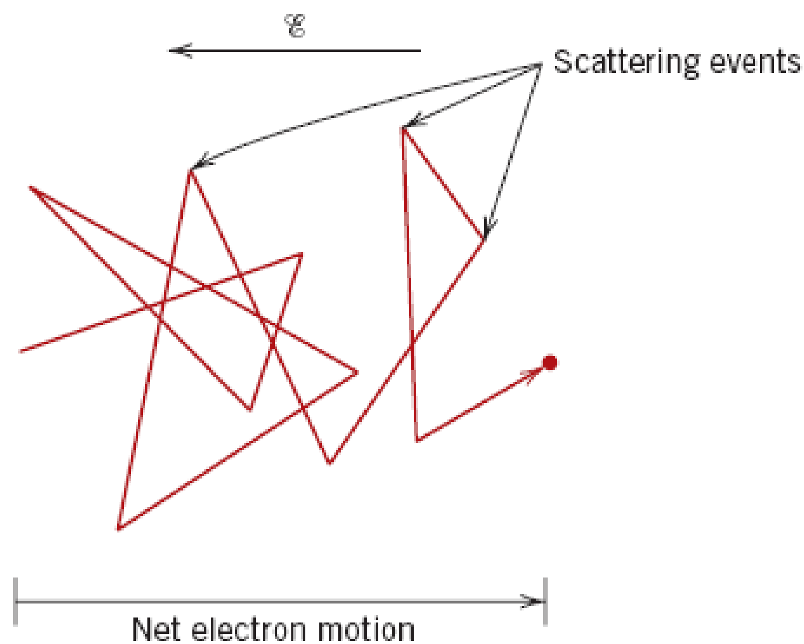
J_x = current density in the x direction, e = electronic charge, n = electron concentration, v_{dx} = drift velocity

Electron Movement

- When an electric field (E) is applied, free electrons accelerate in the opposite direction to the electric field.

Scattering of Electrons

- Frictional forces counter this acceleration, which are a result of scattering :
  - ✓ Electron loses kinetic energy
  - ✓ Electron changes direction of motion
  - ✓ Resistance to passage of electric current



Adapted from Callister 7e

Scattering of Electrons

- Scattering electrons by imperfections in the crystal lattice including:
  - ✓Impurity atoms
  - ✓Interstitial atoms
  - ✓Vacancies
  - ✓Dislocations
  - ✓Grain boundaries
  - ✓Thermal vibrations

Phonon Scattering (Lattice scattering)

- These scattering events constitute a "frictional force" that causes the velocity to maintain a constant mean value V_d.

- Phonons are lattice vibrations (atoms randomly vibrate about their position @ T>0K). Charged carriers collide with vibrating atoms and are scattered.

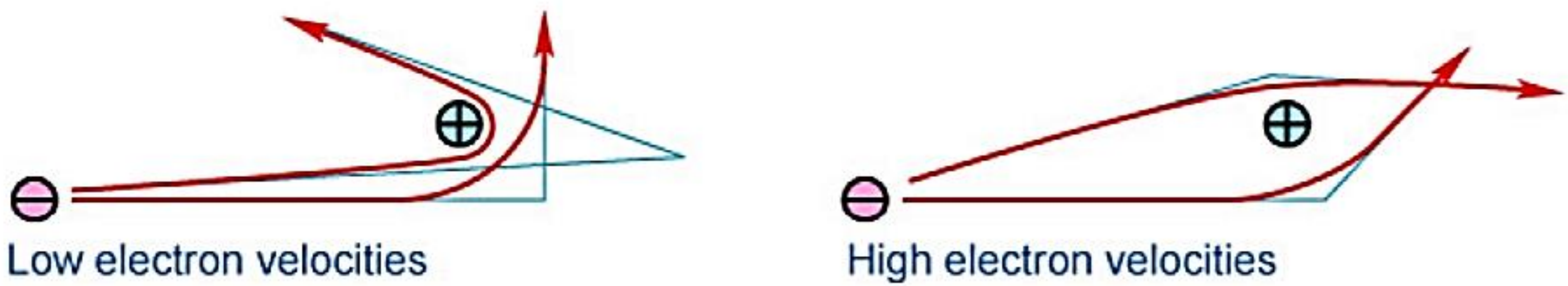
1-D 

- Mobility due to lattice scattering (as temp increases vibration of atoms also increases):

$$\boxed{\mu_L \propto T^{-3/2}}$$

Impurity Scattering

- Scattering due to coulomb interaction between electrons/holes and ionized impurities.



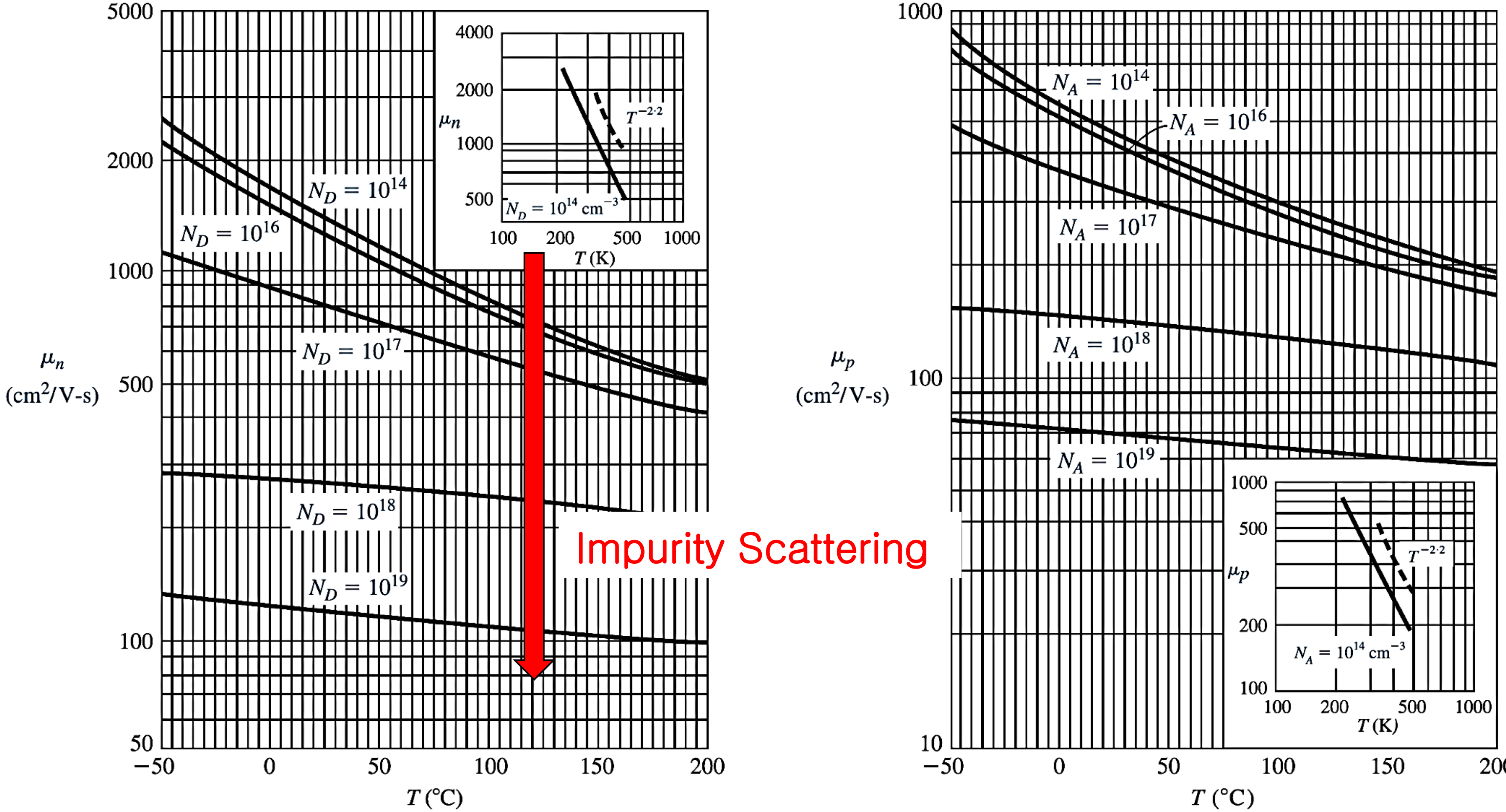
- **T** increases ➔ thermal velocity v_{th} increases, so less time spent for scattering.
- $\mathbf{N_I}$ increases the scattering chance increases.

$$\mu_I \propto \frac{T^{+3/2}}{N_I}$$

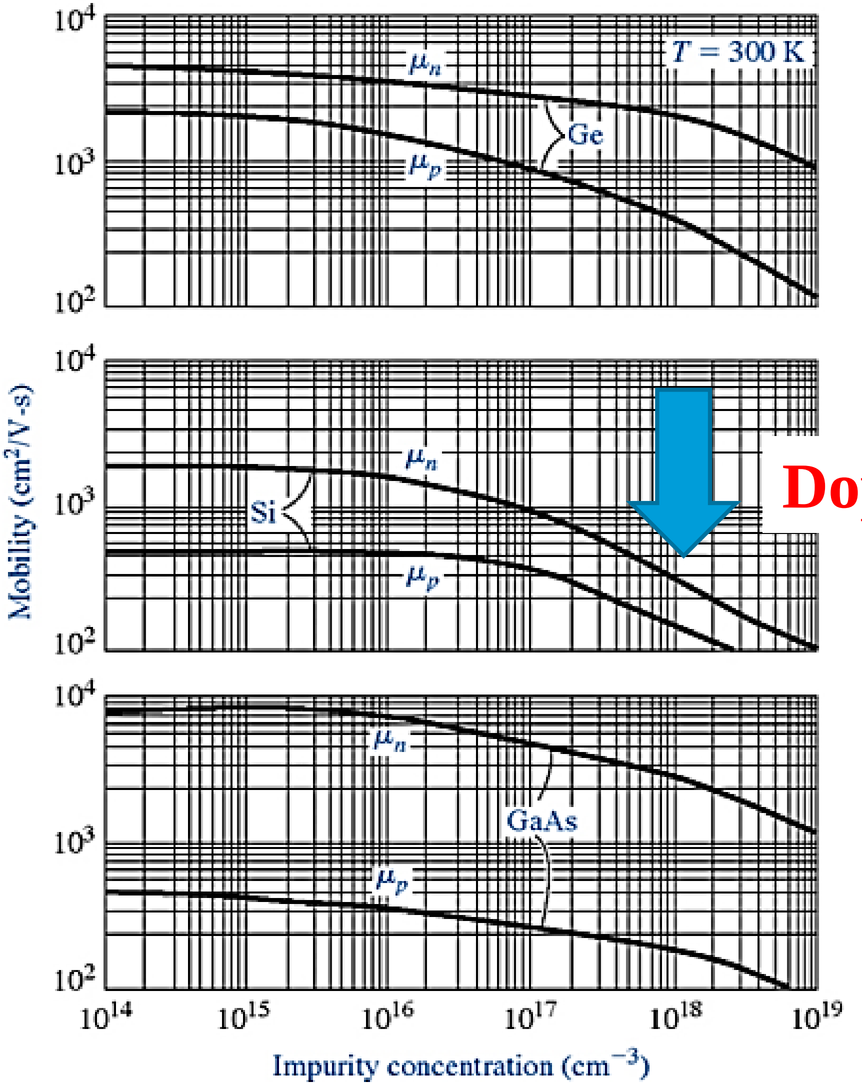
$$N_I = N_d^+ + N_a^-$$

Temp. Dependence of Mobility

- (a) Electron and (b) Hole mobility in Si vs. T at different doping concentrations. (Inserts show dependence for almost intrinsic Si)



Mobility vs. Impurity Concentration



Doping level in modern planar devices

- High doping is required to achieve the target threshold voltage (V_t) even though it reduces mobility.
- Electron mobility is always larger than hole mobility.

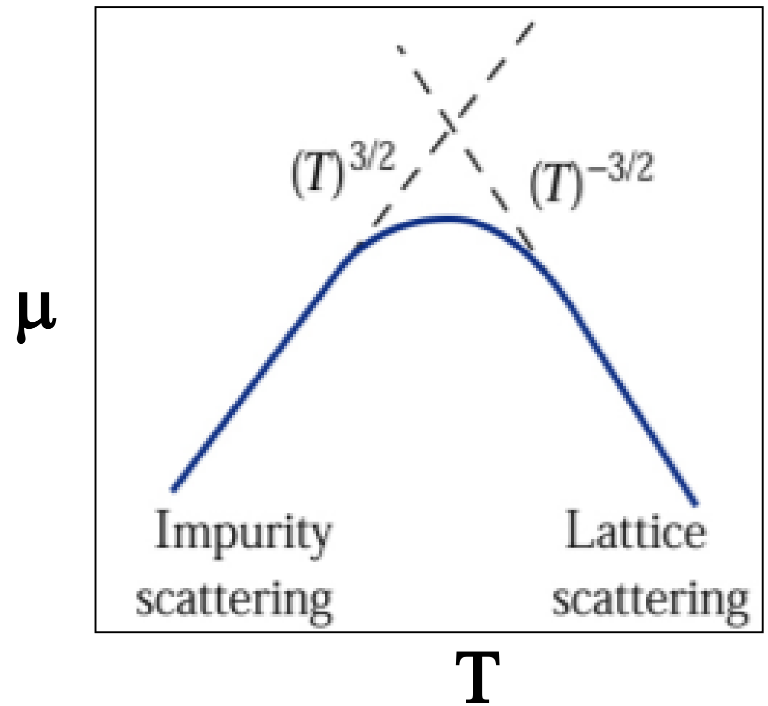
Mobility Effects

- Total number of collisions in *dt*:

$$\frac{dt}{\tau} = \frac{dt}{\tau_I} + \frac{dt}{\tau_L}$$

$$\mu_n = \frac{e\tau_{cn}}{m_n^*}$$

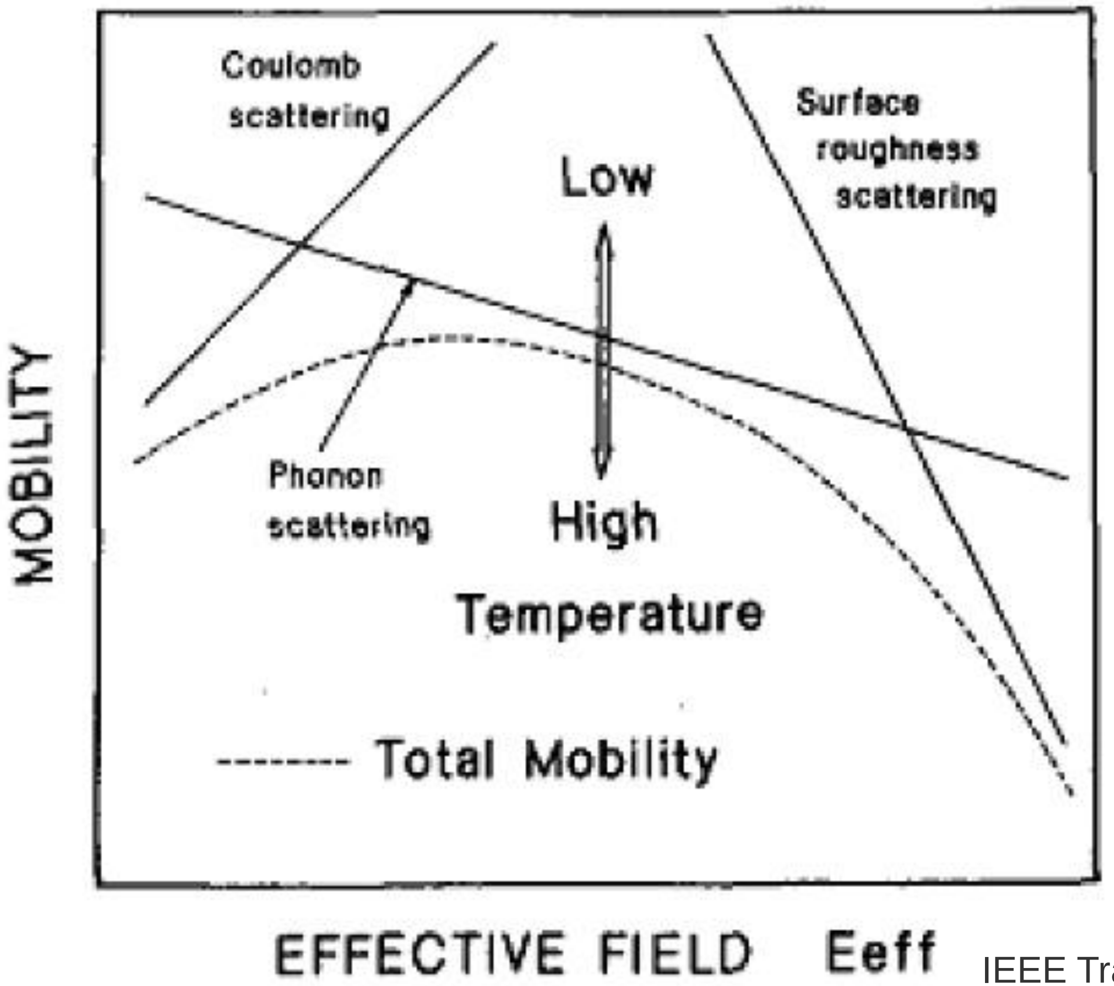
$$\boxed{\frac{1}{\mu} = \frac{1}{\mu_I} + \frac{1}{\mu_L}}$$



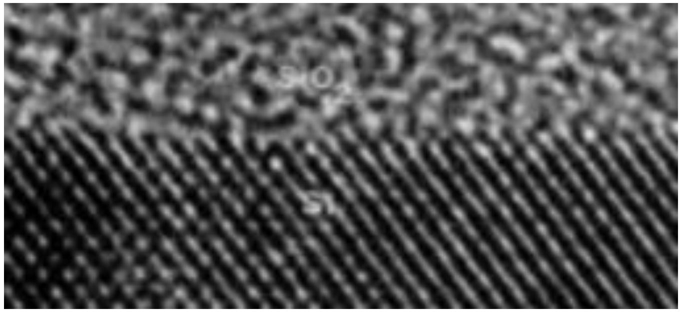
τ_I: average time between two collisions with dopant atoms

τ_L: average time between two collisions with “vibrating" lattice

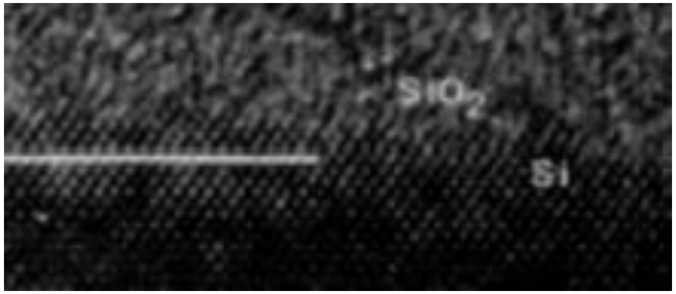
Si MOSFET Universal Mobility



Roughness ~0.1 nm



Roughness ~0.2 nm



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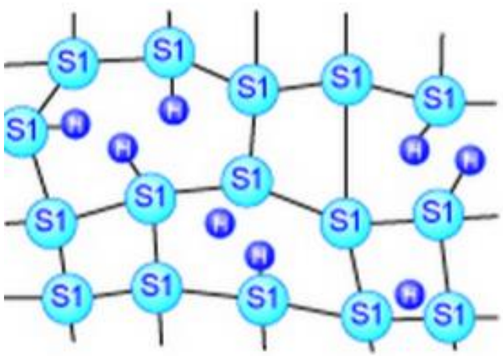
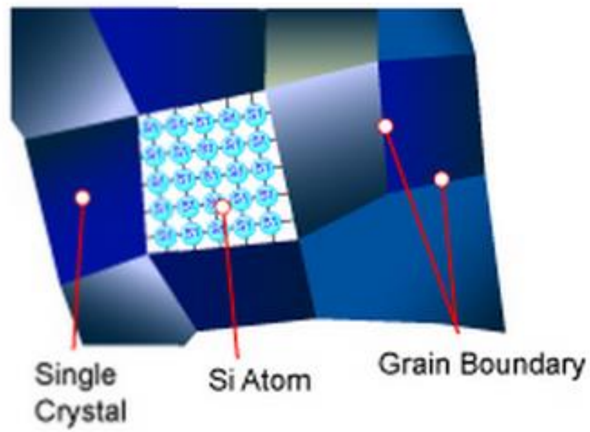
Types of Solids

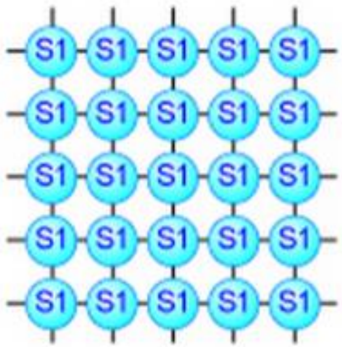
Amorphous Sillcon



Poly-silicon



Single Crystal Sillcon



Mobility: Low

Mobility: Medium

Mobility: High

Drift Velocity

- These scattering events constitute a "frictional force" that causes the velocity to maintain a constant mean value V_d.
- **Drift velocity:**
  - ✓ Average electron velocity in the opposite direction of the electric field

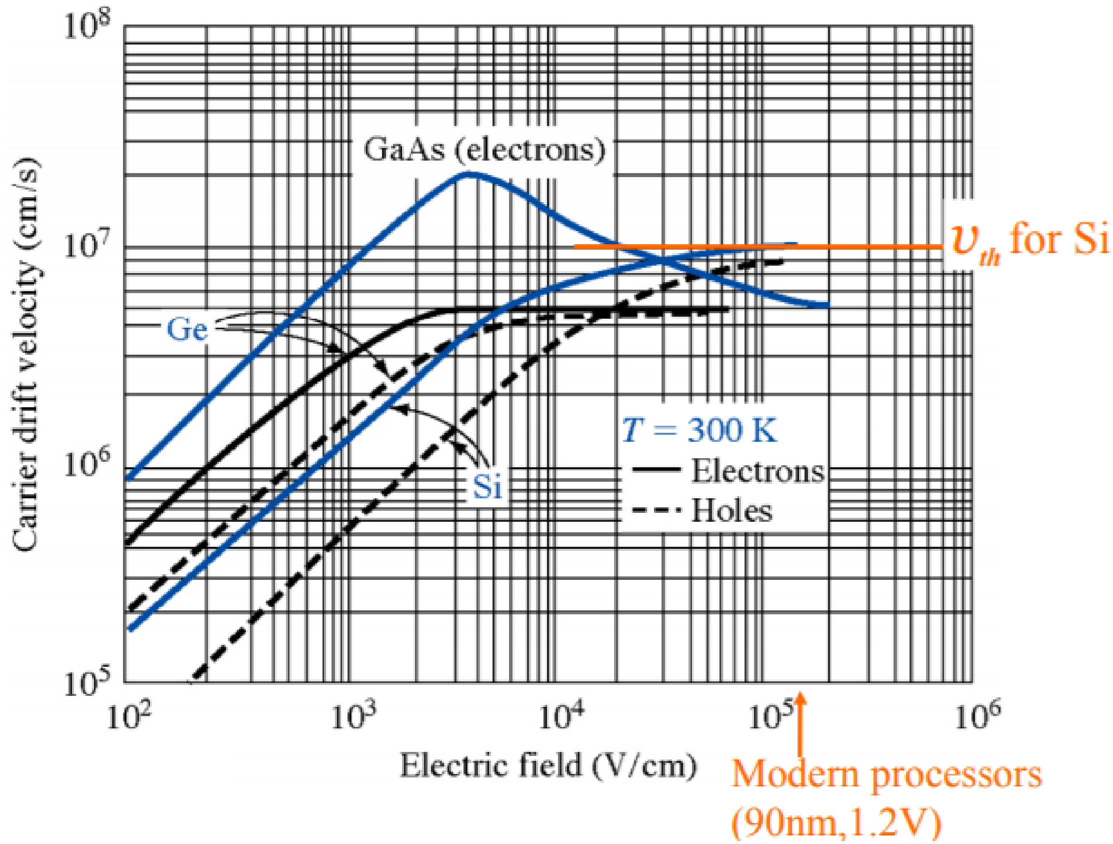
$$\boldsymbol{V_d = \mu_e E}$$

  - ✓ μ_e : electron mobility (m^2/V-s)
  - ✓ E : electric field (V/m)

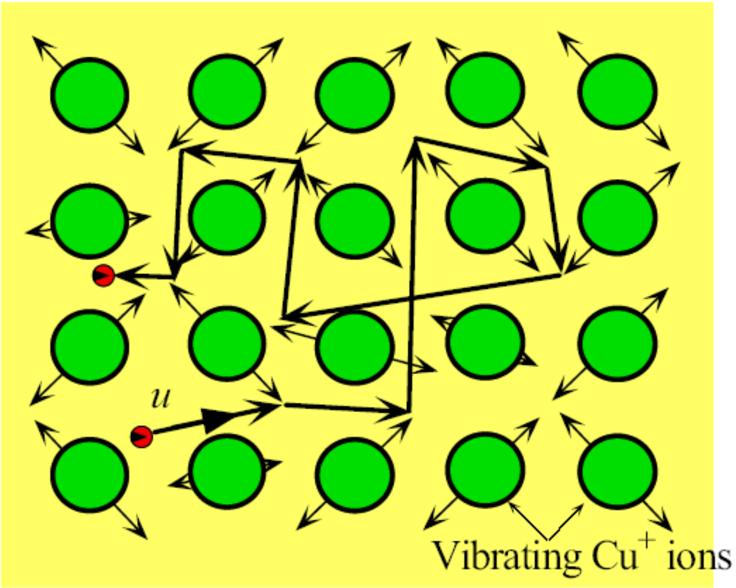
Velocity Saturation

- At high electric fields, the average velocity of carriers is NOT proportional to the field; it saturates at ~10^7 cm/s for both electrons and holes.

$$\boxed{\upsilon_d = \mu E}$$



Thermal Velocity w/o E-field



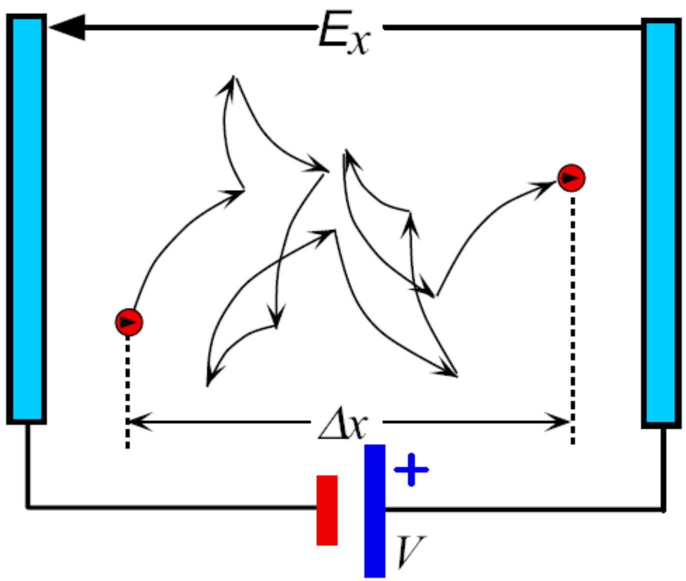
- **Random thermal velocity, V_{th}:**
  - ✓ The average velocity between the collisions

$$\frac{1}{2} m^* v_{th}^2 = \frac{3}{2} KT = 6.2245 \times 10^{-21} J$$

Typical value of $v_{th} \sim 10^7$ cm/s

- A conduction electron in the electron gas moves about randomly in a metal being frequently and randomly scattered by thermal vibrations of the atoms. In the absence of an applied field there is no net drift (displacement) in any direction.

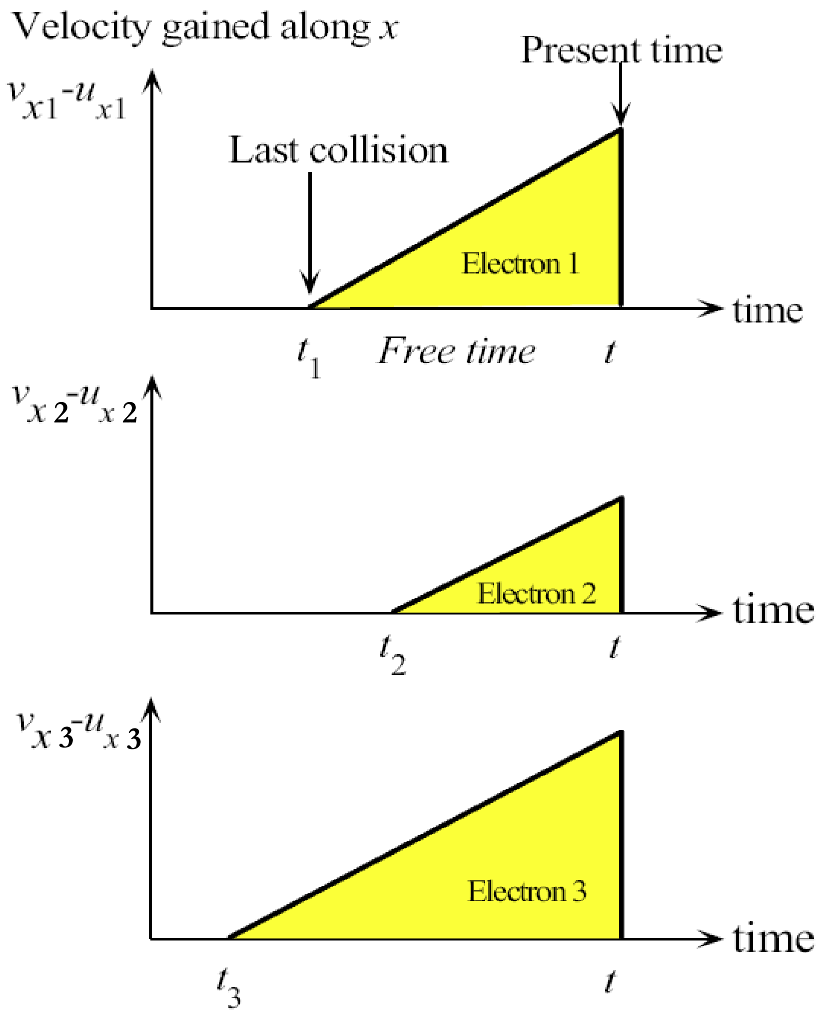
Drift Velocity in E-field



$$\frac{\Delta x}{\Delta t} = v_{dx}$$

- In an applied field, E_x, there is a net drift along the x-direction, which is superimposed on the random motion of one electron. After many scattering events the electron has been displaced by a net distance, ▲x, from its initial position toward the positive terminal

Drift Velocity in E-field



Velocity gained in the *x* direction at time t from the electric field for three electrons.

$$v_{xi} = u_{xi} + \frac{e\mathcal{E}_x}{m_e}(t - t_i)$$

Immediately after a collision at time t_i, e- drifts again for a time t → $(t-t_i)$ where the e- can accelerate in this time interval

u_{xi}: velocity of e- in the x direction just after collision

$\frac{eE_x}{m_e}$: acceleration of the electron

However, this is for just one e-. We need average velocity for all electrons in the x direction

$$\tau = \overline{(t - t_i)}$$

The average free time for N electrons between collisions known as the **mean free time**

Drift Mobility

$$v_{xi} = u_{xi} + \frac{e\mathcal{E}_x}{m_e}(t - t_i)$$

plug in: $\tau = \overline{(t - t_i)}$

$$v_{dx} = \frac{1}{N}(v_{x1} + v_{x2} \ldots \ldots + v_{xN}) = \frac{eE_x}{m_e}\overline{(t - t_i)} = \frac{eE_x}{m_e}\tau$$

$$v_{dx} = \mu_d E_x$$

Drift Mobility and Mean Free Time

$$\boxed{\mu_d = \frac{e\tau}{m_e^*}}$$

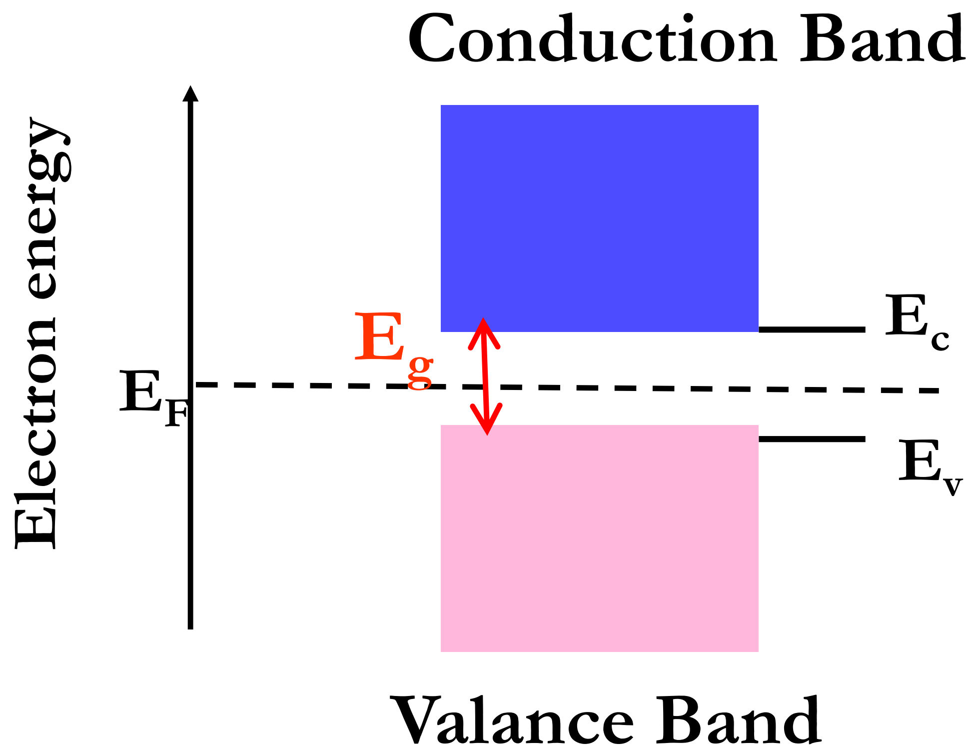
- Relaxation time, τ, is directly related to scattering of electrons: lattice vibrations, crystal imperfections, impurities, etc.

e = electronic charge, τ = mean scattering time (mean time between collisions) = relaxation time= mean free time, m^*_e =effective mass of an electron

Outline

- Electrical Conduction
- Mobility Concept
- **Electrical Properties of Conductor**
- Electrical Properties of Semiconductor
- Temperature Dependence of Electrical Conductivity

Definition of Bands



Fermi energy (E_F) :
energy of the highest filled state at 0 K

- **Valence band** (filled): highest occupied energy levels, E_v
- **Conduction band** (empty):lowest unoccupied energy levels, E_C

The Fermi Energy

- Remember "Pauli Exclusion Principle" that only two electrons (spin up, spin down) can occupy a given "state" defined by quantum numbers n, l, m_l and m_s.

- At 0 K, electrons pack into the lowest energy states and build up a "Fermi sea" of electron energy states. The Fermi energy is the surface of that sea at absolute zero where no electrons have enough energy to rise above the surface.

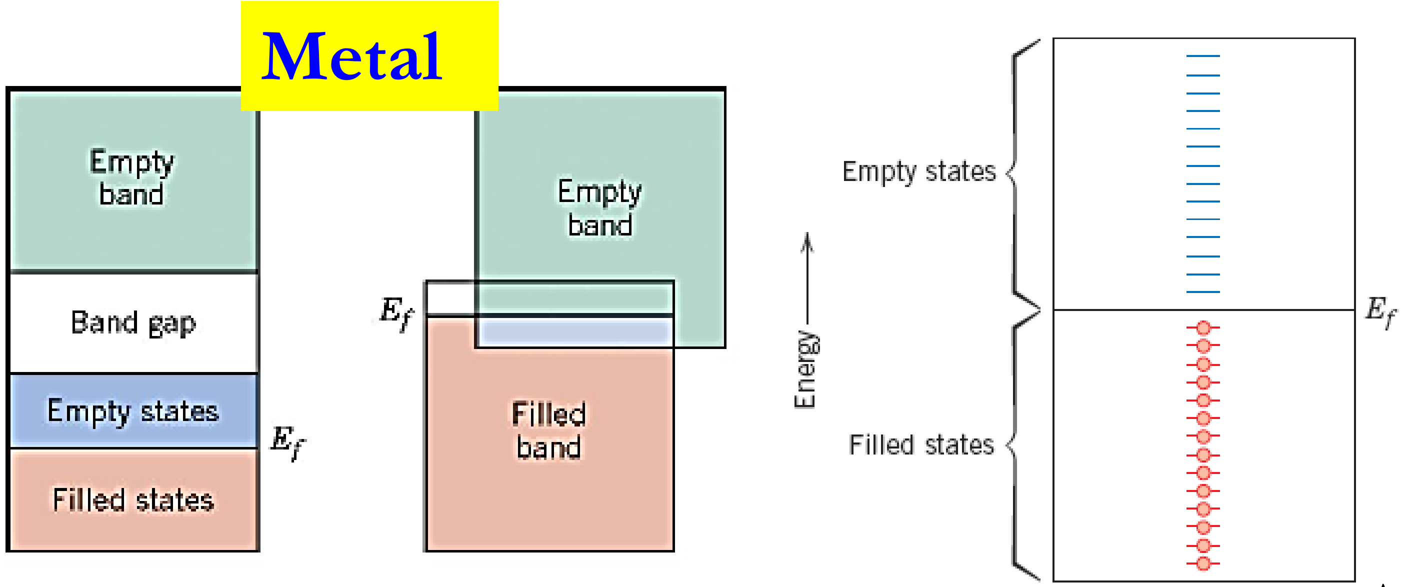
- At 0 K, all electron states below Fermi energy (E_F) are filled, and all electron states above E_F are vacant.

Conduction

- Only electrons with energy greater than E_F can conduct current:
  - Electrons need to scatter into empty energy states above the Fermi energy
  - These electrons participating in the conduction are called **free electrons**.
- When an electron is promoted above the E_F it leaves behind **a hole** (empty electron state)
  - A hole can also move and thus conduct current: it acts as a "positive" electron ($+1.6 \times 10^{-19}$ C)
  - Holes can and do exist in metals, but are more important in semiconductors and insulators

Conduction in Metals

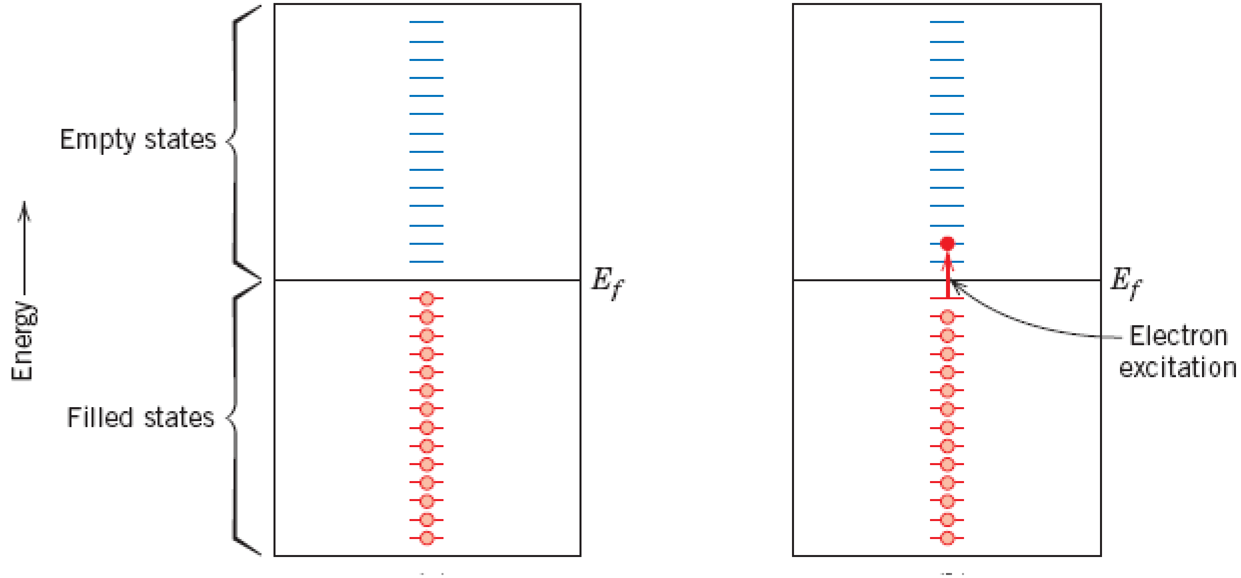
- If a band is partially full or bands overlap ➔ Metal
- Electrons are excited into one of the empty and available energy states above E_f
- Fermi energy (E_f) : energy of the highest filled state at 0 K.



Adapted from Callister 7e

Conduction in Metals

- There are vacant energy states adjacent to the highest filled state at E_f. Thus, very little energy is required to promote electrons to empty states
  - ✓ Electric field (E) provides sufficient energy



Adapted from Callister 7e

Conductivity: Conductors (Metals)

- **Electrical conductivity depends on how many free electrons there are and how easily they move.**
  - ✓ dependence on electron **concentration** and **mobility**
- **The conductivity** expressed as:

$$\boldsymbol{\sigma = \frac{1}{\rho} = n.q.\mu_e}$$

- ***n*** is the number of **free** electrons per unit volume (e.g., per cubic meter)
- ***q*** is the charge (1.6 x 10^{-19} C)

Conductivity Comparison

(m) = Metal (s) = Semicon	Mobility (RT) μ ($m^2V^{-1}s^{-1}$)	Carrier Density N_e (m^{-3})
Na (m)	0.0053	2.6×10^{28}
Ag (m)	0.0057	5.9×10^{28}
Al (m)	0.0013	1.8×10^{29}
Si (s)	(μ_e) 0.15	(n_i) 1.5×10^{10} cm^{-3}
GaAs (s)	(μ_e) 0.85	(n_i) 1.8×10^{6} cm^{-3}
InSb (s)	(μ_e) 8.00	

semiconductor

$$\sigma = \frac{1}{\rho} = n.q.\mu_e$$

$$\sigma_{metal} \gg \sigma_{semiconductor}$$

Example

- The electron drift mobility in silver has been measured to be 56 $cm^2V^{-1}s^{-1}$ at 27 °C. The atomic mass and density of Ag are given as 107.87 g/mol and 10.5 g/cm^3, respectively. Assuming that each Ag atom contributes one conduction electron, calculate the resistivity of Ag at 27 °C.
- ***Answer:***

Atomic concentration n_{at} is

$$n_{at} = \frac{dN_A}{M_{at}} = \frac{(10.50\times10^3 \text{ kg m}^{-3})(6.022\times10^{23} \text{ mol}^{-1})}{(107.87\times10^{-3} \text{ kg mol}^{-1})} = 5.862 \times 10^{28} \text{ m}^{-3}$$

If we assume there is one conduction electron per Ag atom, the concentration of conduction electrons (n) is 5.862×10^{28} m^{-3}, and the conductivity is therefore:

$$\sigma = en\mu_d = (1.602 \times 10^{-19} \text{ C})(5.862 \times 10^{28} \text{ m}^{-3})(56 \times 10^{-4} \text{ m}^2 \text{ V}^{-1}\text{s}^{-1}) = 5.259 \times 10^7\ \Omega^{-1} \text{ m}^{-1}$$

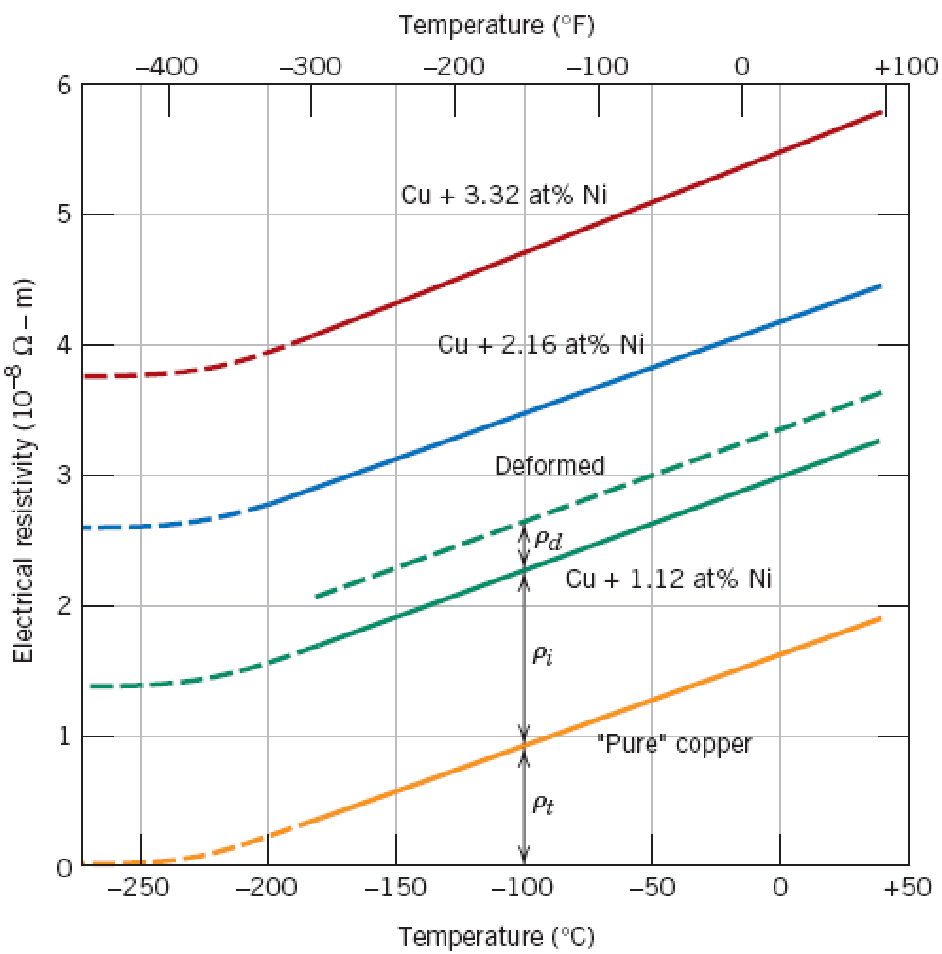
and the resistivity, $\rho = 1/\sigma = 19.0$ nΩ m

Electrical Resistivity of Conductors

- Conductors (metals) have high conductivity ➔ contain large numbers of free electrons that have been excited into empty states above the Fermi energy (E_f)

- The resistivity of a conductor depends on:
  - ✓ Temperature (T)
  - ✓ Composition
  - ✓ Degree of cold work

Matthiessen's Rule



- Matthiessen's Rule:

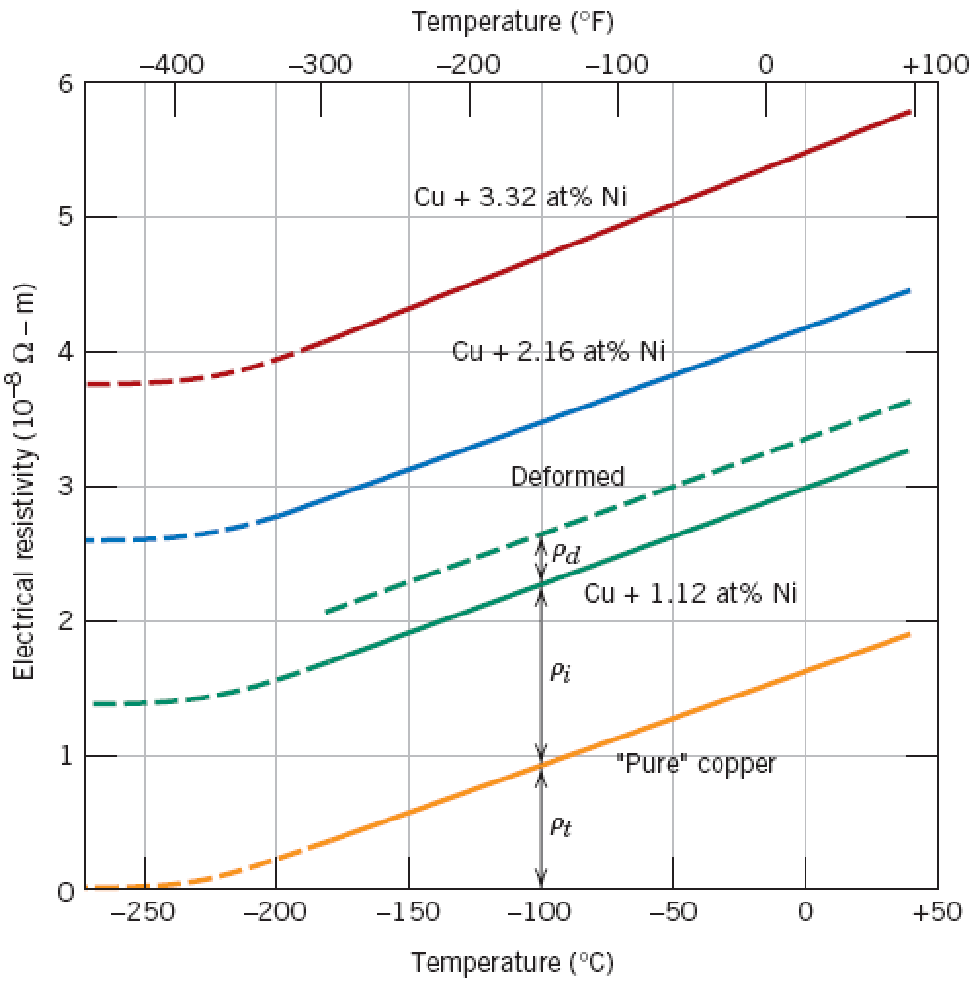
Total resistivity of a metal is the sum of the all factor contributions:

$$\boldsymbol{\rho_{total} = \rho_t + \rho_i + \rho_d}$$

t: thermal
i: impurity
d: deformation

Adapted from J. O. Linde, *Ann. Physik*, **5,** 219 (1932); and C. A.Wert and R. M. Thomson, *Physics of Solids*, 2nd edition, McGraw-Hill Book Company, New York, 1970.

1. Influence of Temperature



$$\rho_{total} = \rho_t + \rho_i + \rho_d$$

$$\rho_t = \rho_0[1 + \alpha_0(T - T_0)\,]$$

α_0 : **t**emperature **c**oefficient of **r**esistivity, **TCR** (the change in resistance as a function of the temperature)

1. Influence of Temperature

- Resistivity of Metals:

$$\sigma = \frac{1}{\rho} = n.q.\mu_e$$

- As T $\uparrow$, n $\uparrow$

- But, why $\boldsymbol{\rho}$ increases when T increasing ??
- As T $\uparrow$, μ_e $\downarrow$ ➔ $\boldsymbol{\rho}$ increases when T increasing

- For metal, mobility (μ_e) dominates when we consider the temperature effect.

2. Influence of Impurities

- **Nordheim's Rule for Solid Solutions**

Additions of impurities that form a solid solution:

$$\boldsymbol{\rho_i = A\, C_i(1 - C_i)} \qquad \textit{(increases } \rho\textit{)}$$

ρ_i = resisitivity due to scattering of electrons from impurities

A: Nordheim coefficient, composition-independent constant, a function of both the impurity and host metals

C_i: impurity concentration in atomic fraction (at.%/100)

Nordheim Coefficient, *C*

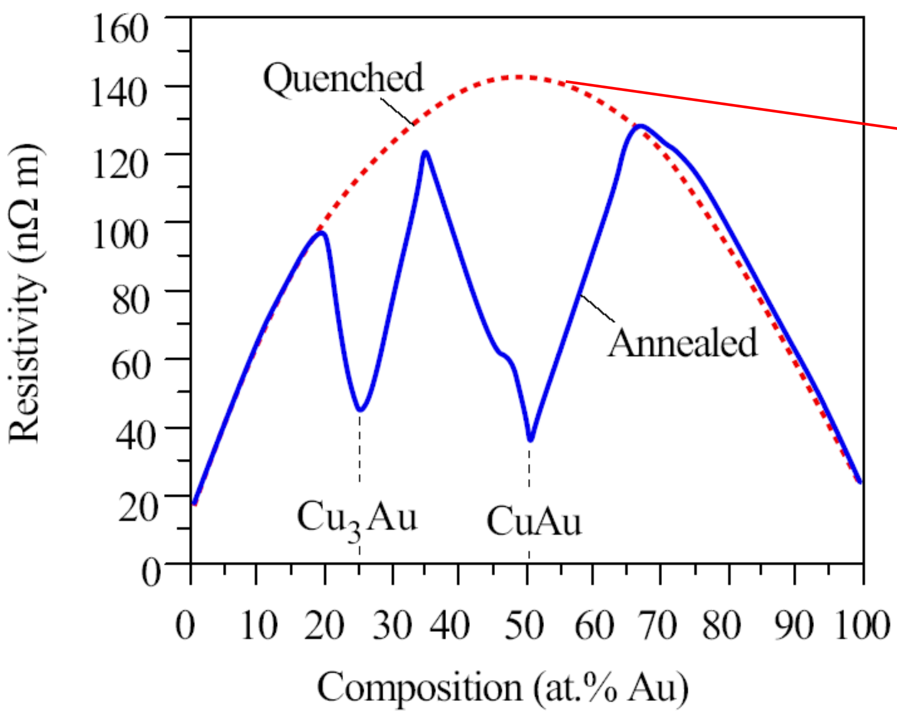
Table 2.3 Nordheim coefficient *C* (at 20 °C) for dilute alloys obtained from $\rho_I = CX$ and $X < 1$ at.%*

Solute in Solvent (element in matrix)	C (nΩ m)	Maximum Solubility at 25 °C (at.%)
Au in Cu matrix	5500	100
Mn in Cu matrix	2900	24
Ni in Cu matrix	1200	100
Sn in Cu matrix	2900	0.6
Zn in Cu matrix	300	30
Cu in Au matrix	450	100
Mn in Au matrix	2410	25
Ni in Au matrix	790	100
Sn in Au matrix	3360	5
Zn in Au matrix	950	15

*NOTE: For many isomorphous alloys *C* may be different at higher concentrations; that is, it may depend on the composition of the alloy.

SOURCES: D.G. Fink and D. Christiansen, eds., *Electronics Engineers' Handbook*, 2nd ed., New York, McGraw-Hill, 1982. J. K. Stanley, *Electrical and Magnetic Properties of Metals*, Metals Park, OH, American Society for Metals, 1963. Solubility data from M. Hansen and K. Anderko, *Constitution of Binary Alloys*, 2nd ed., New York, McGraw-Hill, 1985.

Resistivity vs. Composition (Cu-Au alloys)



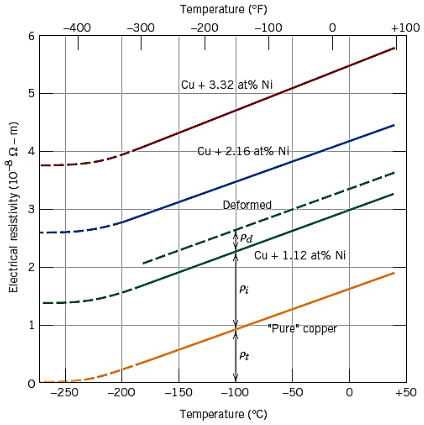
$$\rho_i = A\,C_i(1 - C_i)$$

- The quenched sample is obtained by quenching the liquid, and the Cu and Au atoms are randomly mixed. The resistivity obeys the Nordheim's rule.

- When the quenched sample is annealed or the liquid is slowly cooled, certain compositions (Cu_3Au and CuAu) result in an ordered crystalline structure in which the Cu and Au atoms are positioned in an ordered fashion in the crystal and the scattering effect is reduced.

3. Influence of Deformation

- Cold working (CW) or deformation a metal results in higher concentration of dislocations and therefore increase the residual resistivity.
- Cold-worked samples have a shift-up curve by ρ_{cw} that depends on the extent of cold working. Usually, its influence is much weaker than that of increasing temperature or the presence of impurities.

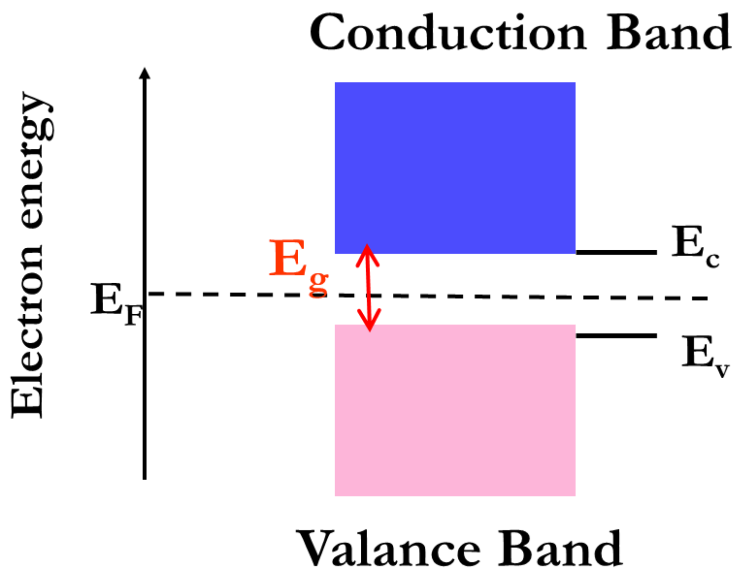


Outline

- Electrical Conduction
- Mobility Concept
- Electrical Properties of Conductor
- **Electrical Properties of Semiconductor**
- Temperature Dependence of Electrical Conductivity

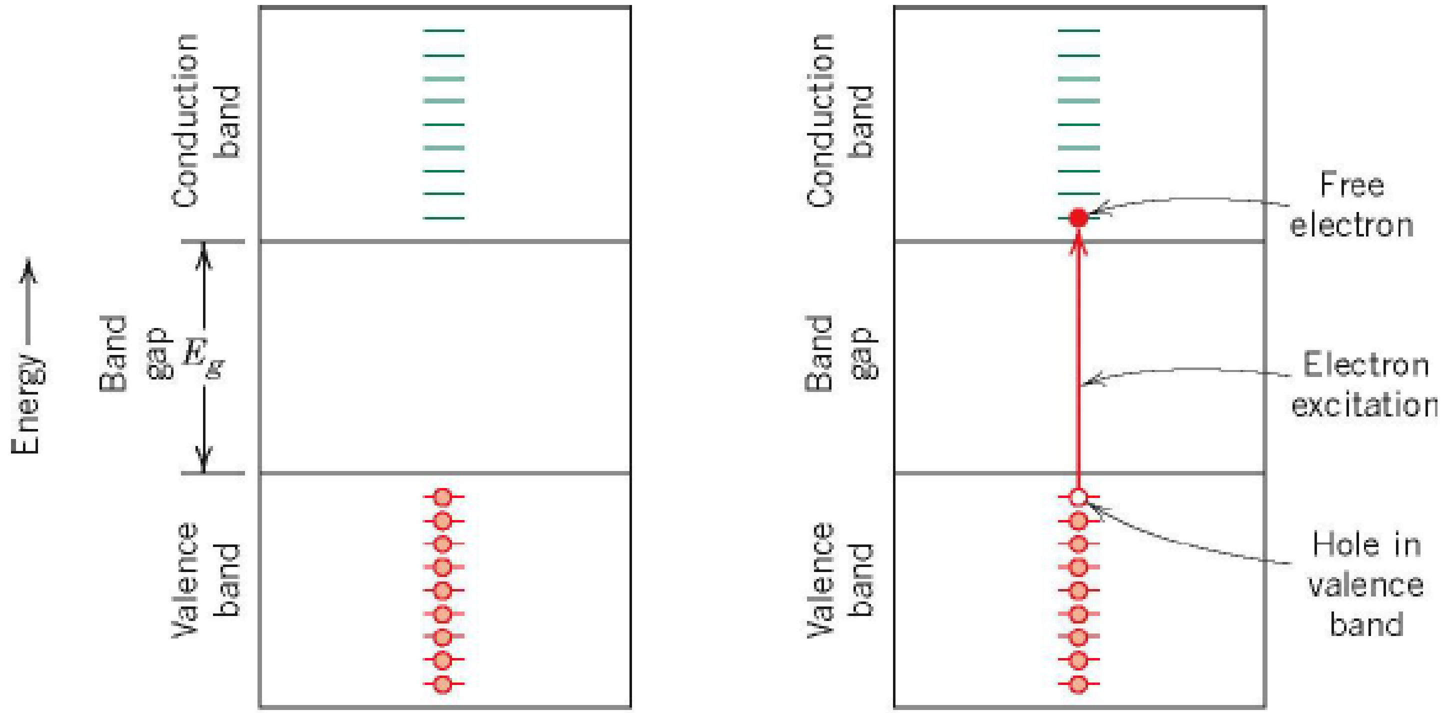
Conduction in Semiconductors

- No available empty state adjacent to filled valence band
  - ✓ Thermal promotion of electrons across the band gap (Eg)
  - ✓ Eg > 3eV means the gap is practically insurmountable and the material is an insulator.



Conduction in Semiconductors

- Number of excited electrons into the conduction band depends on bandgap (Eg) and temperature (T)
- At a given T, as Eg increases, the σ decreases.

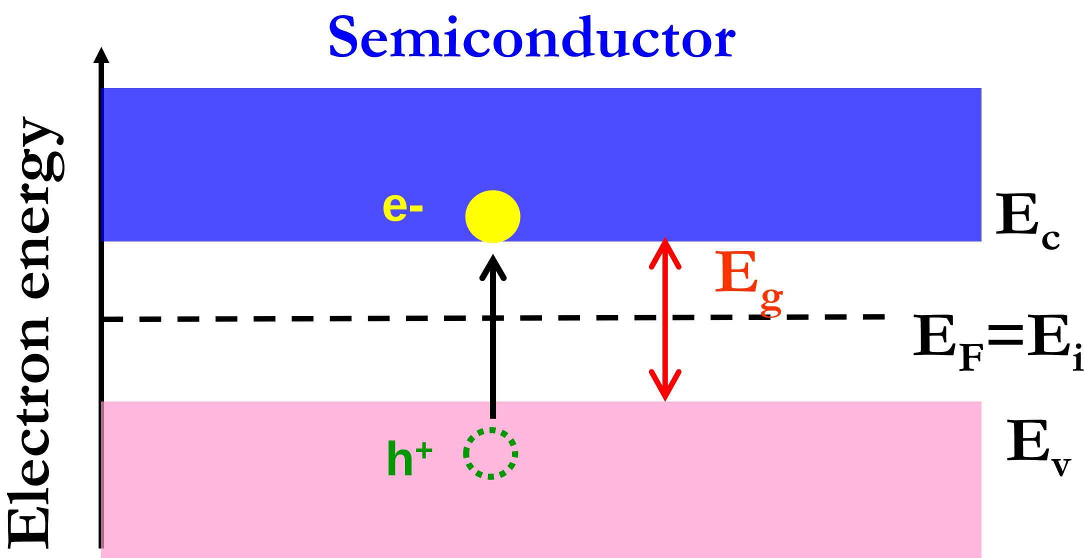


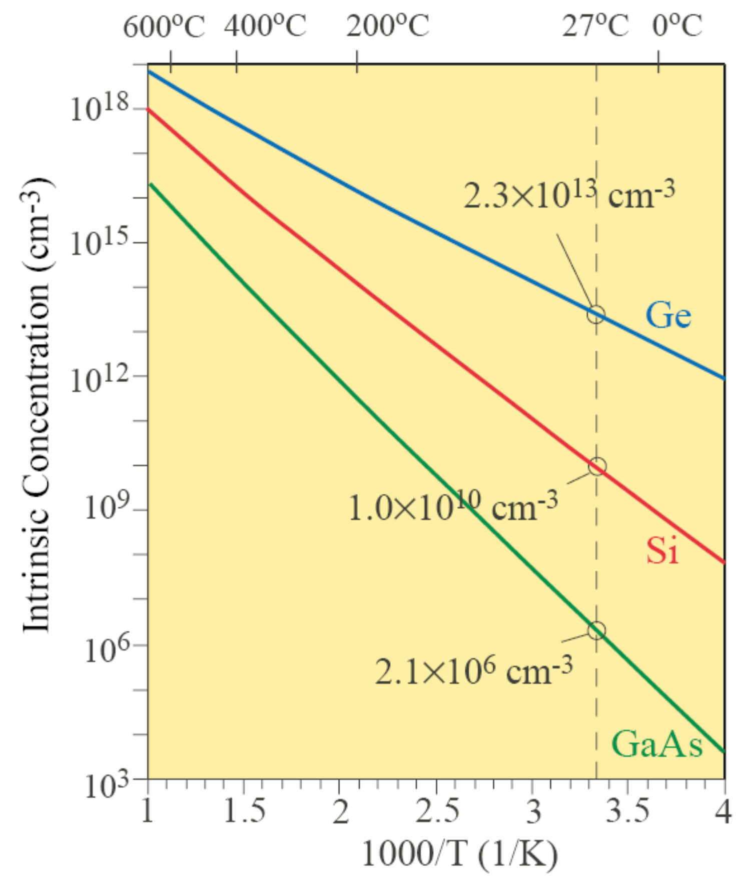
Thermal Excitation

- Intrinsic carrier concentration, n_i is the number of electrons in the conduction band.

$$n = n_o \exp\left(-\frac{E_g}{2kT}\right)$$

Intrinsic semiconductor

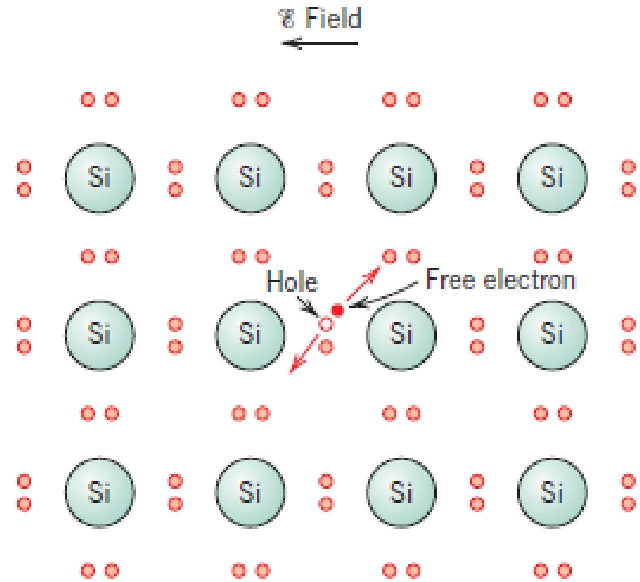




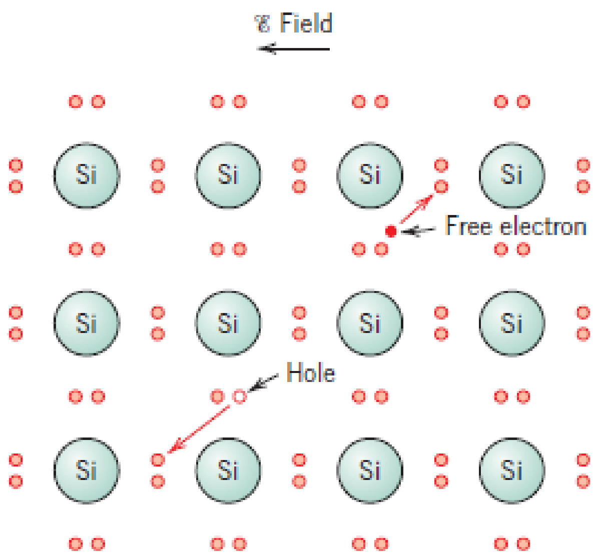
Intrinsic Conductivity

(a)

Excitation



(b)



(c)

Electrons and holes move in response to the external electric field (E)

Intrinsic Conductivity

- Two types of charge carriers:
  - ✓ Free electrons (n ➔ number of electrons/m^3)
  - ✓ Holes (p ➔ number of holes/m^3)
- Conductivity:

$$\boldsymbol{\sigma = n.q.\mu_e + \mathrm{p}.q.\mu_p}$$

- For intrinsic semiconductors, since the number of hole is the same as that of electron ($n=p$):

$$\boldsymbol{\sigma = n.q.(\mu_e + \mu_p)}$$

Example

- Calculate the number of electrons and holes for an intrinsic silicon (Si) at room temperature?

$$n = p = \frac{\sigma}{|e|(\mu_e + \mu_h)} = 1.33 \times 10^{16} / m^3$$

$n_i = p_i \sim$ **10^{10} cm^{-3}**

Material	*Band Gap (eV)*	*Electrical Conductivity* $[(\Omega\text{-}m)^{-1}]$	*Electron Mobility* $(m^2/V\text{-}s)$	*Hole Mobility* $(m^2/V\text{-}s)$
		Elemental		
Si	1.11	4×10^{-4}	0.14	0.05
Ge	0.67	2.2	0.38	0.18
		III–V Compounds		
GaP	2.25	—	0.03	0.015
GaAs	1.42	10^{-6}	0.85	0.04
InSb	0.17	2×10^4	7.7	0.07
		II–VI Compounds		
CdS	2.40	—	0.03	—
ZnTe	2.26	—	0.03	0.01

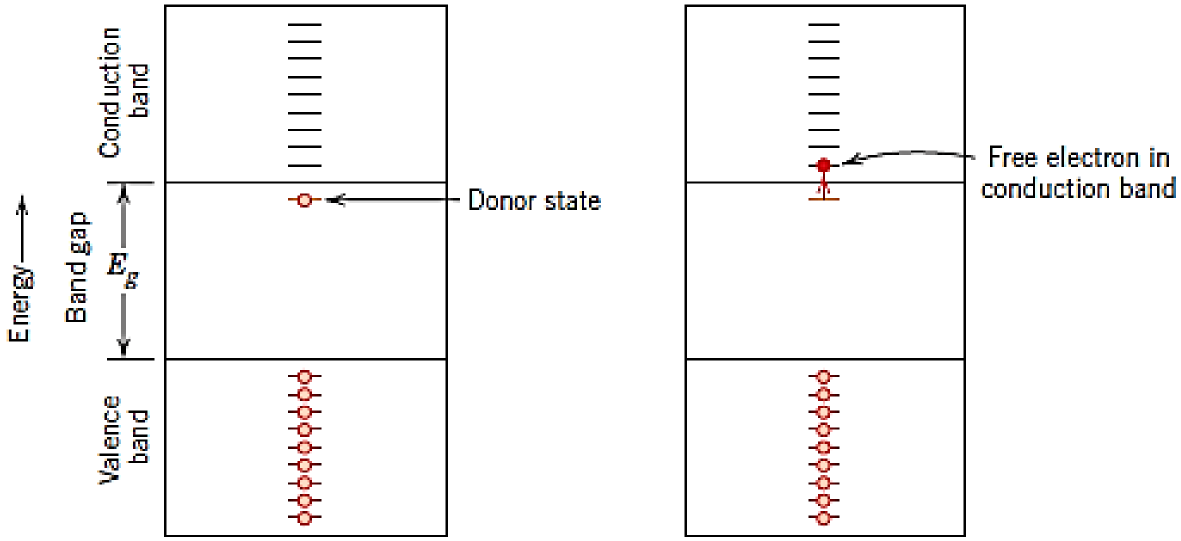
Why doping is important?

It changes the conductivity depends on purpose.

- Number of silicon (**Si)** atoms is ~ 10^{23} cm^{-3}.
- At room temperature, number of silicon (Si) intrinsic electrons/holes is ~ **10^{10} cm^{-3}**
- In industry, we use **extrinsic** Si with doping group IIIA or group VA elements of the order of **10^{18} ~ 10^{20} cm^{-3}**

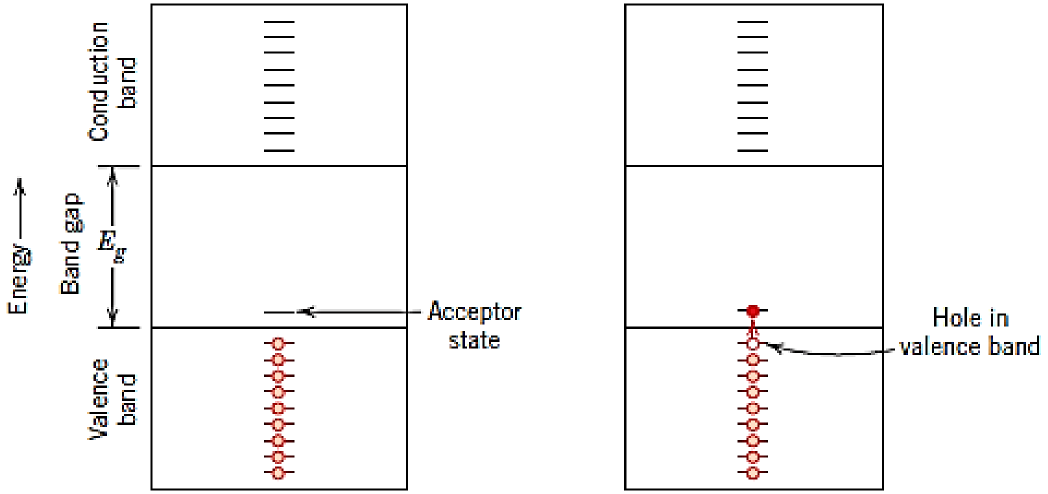
n-Type Semiconductor

- Excess electrons locate in a **donor state (E_D)** close to the top of the band gap.
- Each excitation donates a electron to the conduction band; This type of impurity is called a ***donor***.
- Conductivity: $\boxed{\boldsymbol{\sigma = n.q.\mu_e} + \mathrm{p}.\boldsymbol{q.\mu_p} \approx \boldsymbol{n.q.\mu_e}}$



p-Type Semiconductor

- Excess holes locate in a **acceptor state (E_A)** close to the bottom of the band gap. When an electron fills the acceptor state, a hole is left in the valence band.
- Each excitation accept a electron to the conduction band; This type of impurity is called a ***acceptor.***
- Conductivity: $\boxed{\sigma = n.q.\mu_e + \mathrm{p}.q.\mu_p \approx p.q.\mu_p}$

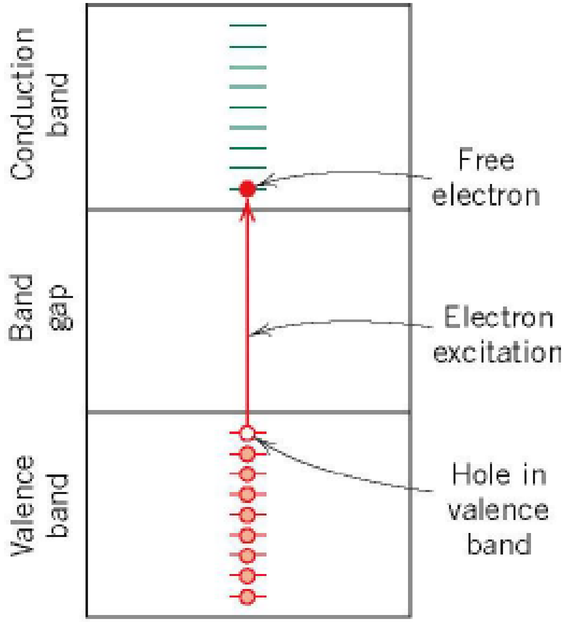


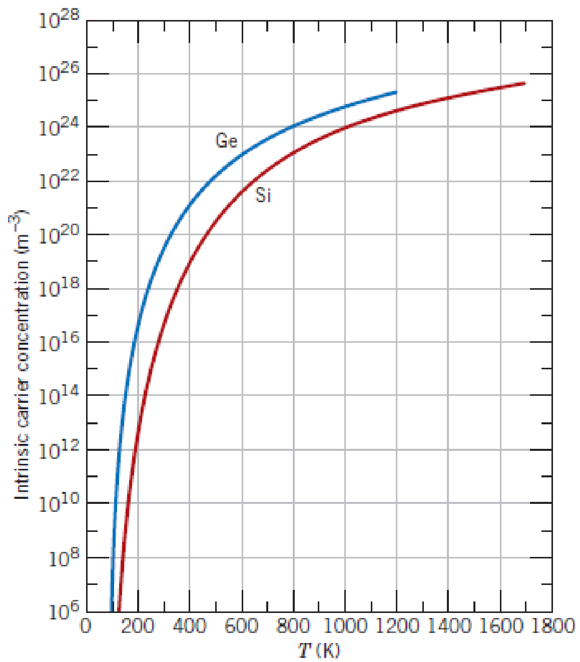
Outline

- Electrical Conduction
- Mobility Concept
- Electrical Properties of Conductor
- Electrical Properties of Semiconductor
- **Temperature Dependence of Electrical Conductivity**

Temp. Dependence of Carrier Conc.

- For **intrinsic** semiconductor, increasing in **temperature** (thermal energy) or decreasing in **bandgap (Eg)** allow more electrons excite to the conduction band→ carrier concentration increases





$$n = n_o \exp\left(-\frac{E_g}{2kT}\right)$$

Temp. Dependence of Carrier Conc.

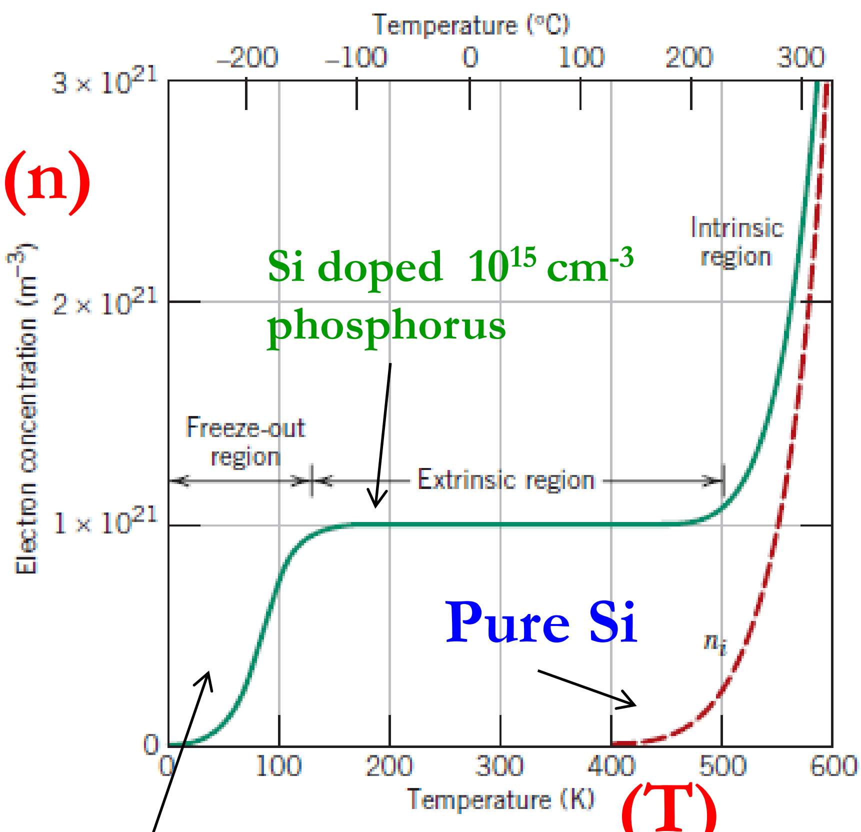


Table 5.2 Examples of donor and acceptor ionization energies (eV) in Si

Donors			Acceptors		
P	As	Sb	B	Al	Ga
0.045	0.054	0.039	0.045	0.057	0.072

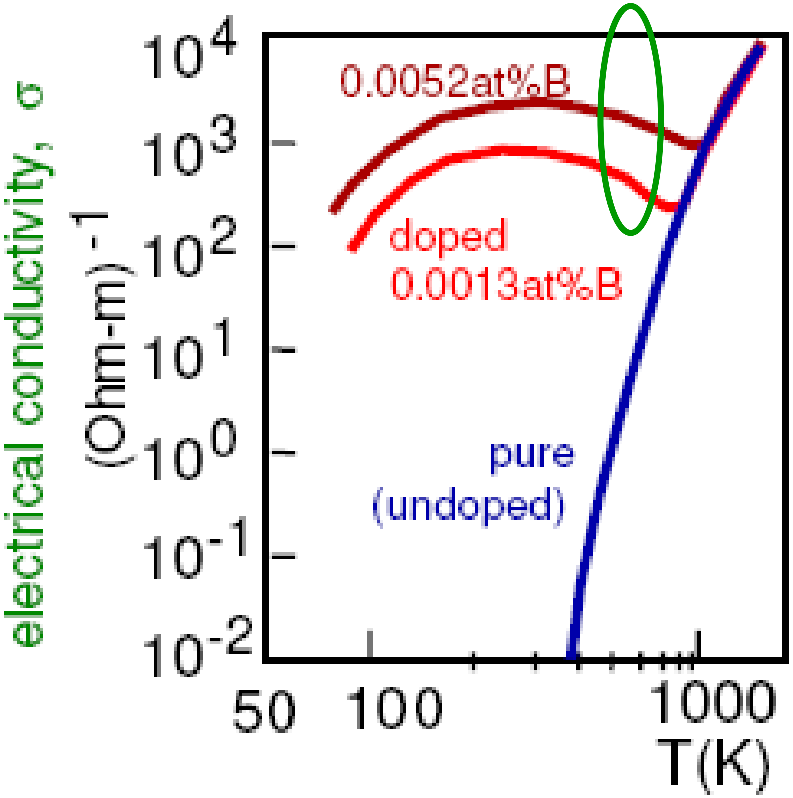
For extrinsic semiconductor:

- Low Temperature (<100K):
  - ✓ Extrinsic region: less dopant ionized **(Freeze-out region)**
- Medium Temp. (150~450K):
  - ✓ **Extrinsic region**: most dopant ionized
- High Temperature (>500K):
  - ✓ **Intrinsic region** dominates

$$n = n_o \exp\left(-\frac{E_g}{2kT}\right)$$

From S. M. Sze, *Semiconductor Devices, Physics and Technology*.

Temperature vs. Conductivity



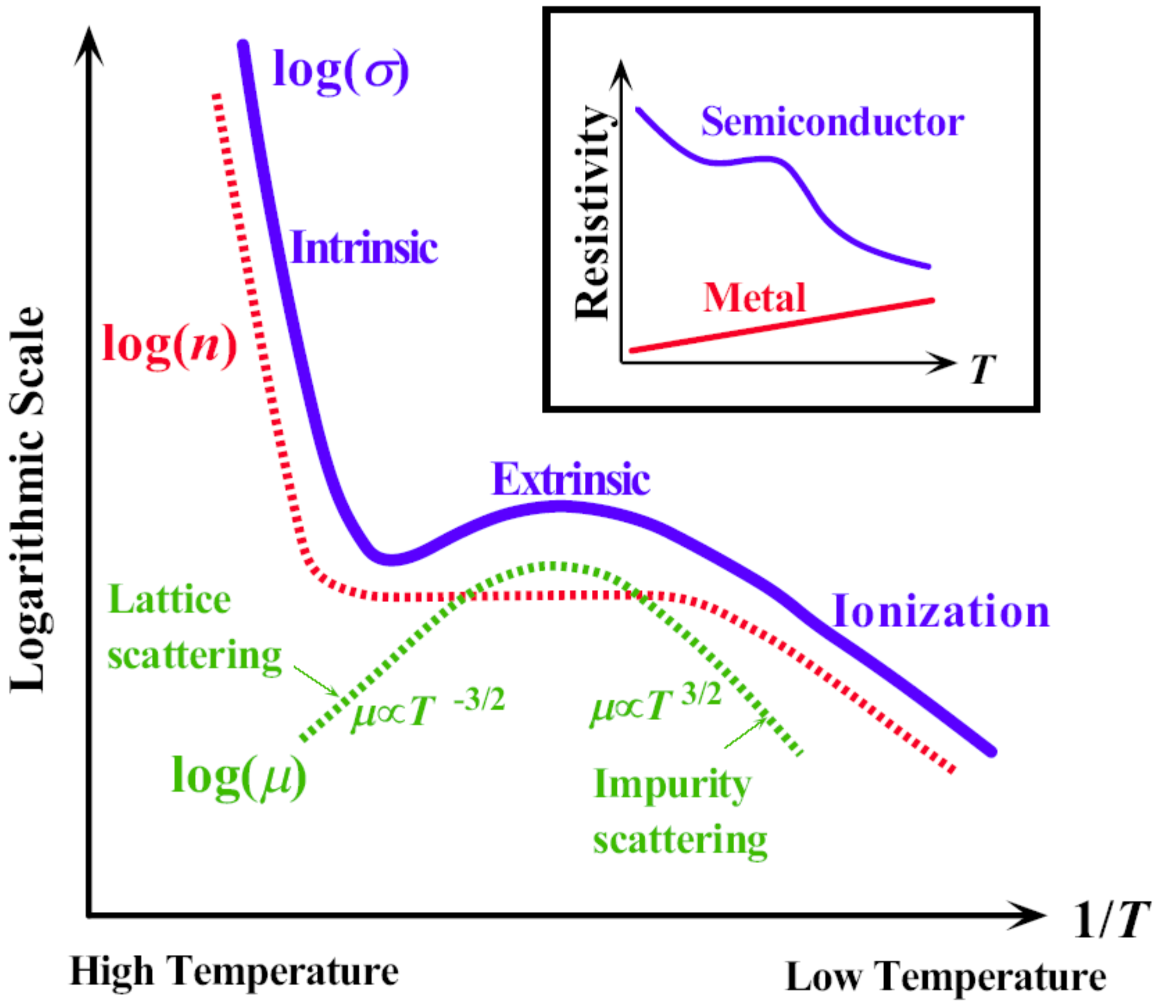
- Intrinsic Si:

Carrier concentration increase with temperature.

- Extrinsic Si:

- ✓ Carrier number is constant.
- ✓ Mobility decreases with temperature due to scattering.

Temperature vs. Conductivity



Schematic illustration of the temperature dependence of electrical conductivity for a doped (*n*-type) semiconductor.

Mobility vs. Impurity Concentration

- Carrier mobility is maximum up to impurity concentrations of 10^{20} m^{-3}
- Mobility decreases with increasing impurity.
- Electron mobility is always larger than hole mobility.

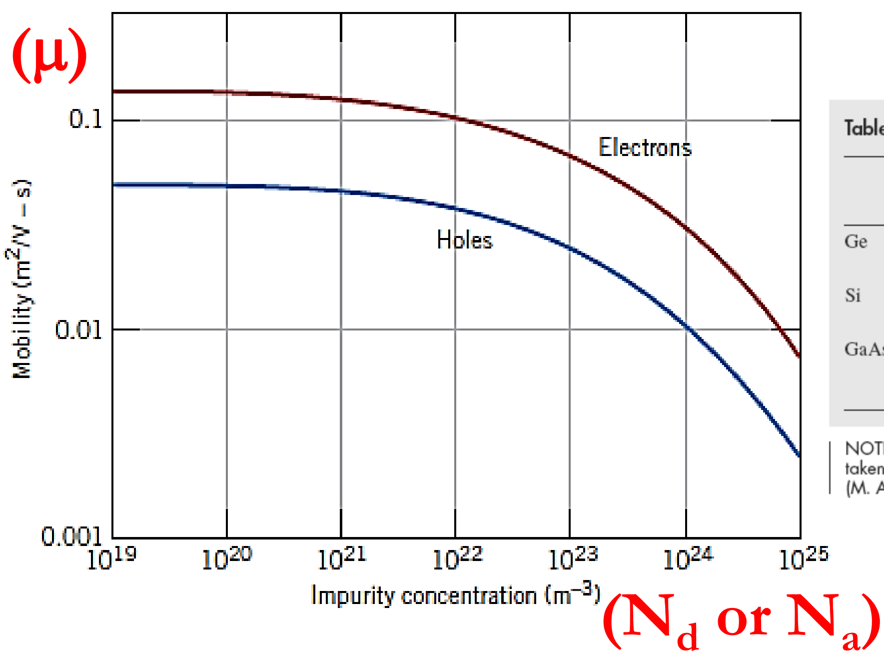


Table 5.1 Selected typical properties of Ge, Si, and GaAs at 300 K

	E_g (eV)	χ (eV)	N_c (cm^{-3})	N_v (cm^{-3})	n_i (cm^{-3})	μ_e ($cm^2\ V^{-1}\ s^{-1}$)	μ_h ($cm^2\ V^{-1}\ s^{-1}$)	m_e^*/m_e	m_h^*/m_e	ε_r
Ge	0.66	4.13	1.04×10^{19}	6.0×10^{18}	2.3×10^{13}	3900	1900	0.12*a* 0.56*b*	0.23*a* 0.40*b*	16
Si	1.10	4.01	2.8×10^{19}	1.2×10^{19}	1.0×10^{10}	1350	450	0.26*a* 1.08*b*	0.38*a* 0.60*b*	11.9
GaAs	1.42	4.07	4.7×10^{17}	7×10^{18}	2.1×10^{6}	8500	400	0.067*a,b*	0.40*a* 0.50*b*	13.1

NOTE: Effective mass related to conductivity (labeled *a*) is different than that for density of states (labeled *b*). In numerous textbooks, n_i is taken as 1.45×10^{10} cm^{-3} and is therefore the most widely used value of n_i for Si, though the correct value is actually 1.0×10^{10} cm^{-3}. (M. A. Green, *J. Appl. Phys.*, **67**, 2944, 1990.)

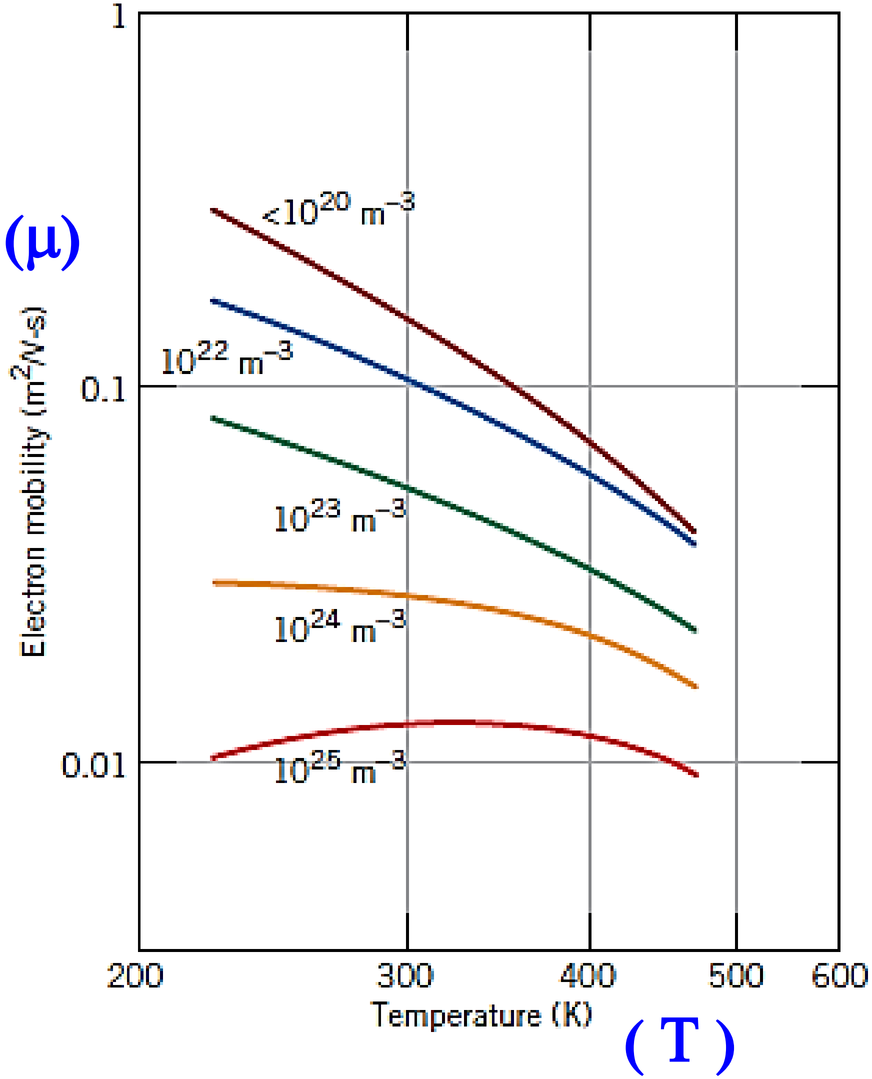
Mobility vs. Temperature

- Temperature enhances thermal scattering of carriers
- Concentration $< 10^{20}\ m^{-3}$:

Mobility is dependent on temperature.

- Higher conc. $> 10^{24}\ m^{-3}$:

Mobility is independent of temperature.



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