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RECONSTRUCTION OF S. MARGHERITA PROJECT OF 1685 AS DESIGNED BY AGOSTINO BARELLI

RECONSTRUCCIÓN DEL PROYECTO DE S. MARGHERITA DE 1685 TAL Y COMO FUE DISEÑADO POR AGOSTINO BARELLI

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Highlights:

- This study presents a structured methodology for the hypothetical digital 3D reconstruction of unbuilt or demolished buildings.
- The critical digital model (CDM) of the lost church of S. Margherita in Bologna designed by Agostino Barelli in 1685 was built.
- The reconstructive methodology is exhaustive, easily reproducible and transparent, and the 3D model is built and published in a way that is interoperable and accessible.

Abstract:

This paper presents a structured three-dimensional (3D) reconstruction methodology of architectural heritage adopted and implemented in the context of the CoVHer (Computer-based Visualisation of architectural Heritage) Erasmus+ project. The methodology consists of a multi-step process for hypothetically reconstructing never-built or demolished architectural heritage from the past in the form of 3D digital models. This reconstruction methodology was tested over the years with professionals, scholars and laypersons, on several case studies in the context of international workshops, museum exhibitions, VR dissemination, and it was also tested with students at the architectural drawing course at the University of Bologna. This last experimentation was particularly important because fostered us to systematise its steps and make it more easily sharable and applicable while not compromising quality and robustness. The methodological steps that we are going to address and discuss in this paper are: a) data acquisition, b) critical evaluation of historical and architectural sources, c) 2D digital redrawing of graphic material, d) construction of the 3D model, e) visualisation, f) uncertainty assessment and communication, g) documentation, and h) publication with a particular focus on interoperability and accessibility. These steps are explained in detail in order to be applicable to similar case studies and foster reproducibility, comparability, accessibility, transparency, and interoperability of the digital reconstruction. These are the key principles already recommended by the FAIR principles (Findable, Accessible, Interoperable, Reusable), the Seville Principles, and the London Charter, among others. The methodology, despite being tested on various case studies and fields, has been proven to be particularly effective for never-built or demolished architectural heritage with known authors. This paper presents the case study of the reconstruction of the unbuilt Church of S. Margherita in Bologna, designed by Agostino Barelli in 1685. This exemplary case study covers all aspects of our reconstruction methodology.

Keywords: hypothetical reconstruction; standard methodology; digital cultural heritage; documentation; 3D reconstruction

Resumen:

Este artículo presenta una metodología de reconstrucción tridimensional (3D) estructurada del patrimonio arquitectónico, adoptada e implementada en el marco del proyecto Erasmus+ CoVHer (‘*Computer-based Visualisation of architectural Heritage*’). La metodología consiste en un proceso en múltiples etapas que permite reconstruir hipotéticamente, en forma de modelos digitales 3D, el patrimonio arquitectónico nunca construido o demolido del pasado. Esta metodología de reconstrucción se ha probado a lo largo de los años con profesionales, académicos y el público general en varios estudios de caso en el contexto de talleres internacionales, exposiciones en museos, difusión en realidad virtual (VR), y también con estudiantes del curso de dibujo arquitectónico de la Universidad de Bolonia. Esta última experimentación fue particularmente importante porque nos permitió sistematizar los pasos y hacerlos más fácilmente compartibles y aplicables, sin comprometer la calidad ni la robustez. Los pasos metodológicos que abordaremos y discutiremos en este artículo son: a) adquisición de datos, b) evaluación crítica de fuentes históricas y arquitectónicas, c) redibujado digital 2D del material gráfico, d) construcción del modelo 3D, e) visualización, f) evaluación y comunicación de la incertidumbre, g) documentación, y h) publicación con un enfoque particular en la interoperabilidad y accesibilidad. Estos pasos se explican detalladamente para que sean aplicables a casos de estudio similares y fomenten la reproducibilidad, comparabilidad,

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accesibilidad, transparencia e interoperabilidad de la reconstrucción digital. Estos son los principios clave recomendados bajo los principios FAIR (encontrables, accesibles, interoperables y reutilizables), los Principios de Sevilla y la Carta de Londres, entre otros. La metodología, aunque ha sido probada en varios estudios de caso y campos, ha demostrado ser particularmente efectiva en patrimonio arquitectónico nunca construido o demolido cuyos autores son conocidos. Este artículo presenta el estudio de caso de la reconstrucción de la Iglesia no construida de Santa Margherita en Bolonia, diseñada por Agostino Barelli en 1685. Este caso de estudio ejemplar abarca todos los aspectos de nuestra metodología de reconstrucción.

Palabras clave: reconstrucción hipotética; metodología estándar; patrimonio cultural digital; documentación; reconstrucción 3D

1. Introduction

For decades, hypothetical 3D digital reconstructions of archaeological heritage have been published (e.g., Dell'Unto et al., 2013; Lengyel & Toulouse, 2015; Ortiz-Cordero et al., 2018; Giovannini, 2020; Kuroczyński et al., 2022). Despite the relevant value of many of these efforts, the produced 3D models are rarely shared publicly and, even when they are available for download or consultation, they are hardly reusable because they lack proper documentation and are not prepared in a way that maximise reusability and compatibility with modern 3D modelling applications and workflows.

The case study presented in this paper intends to apply some general rules, for documenting the process of reconstruction of unbuilt or demolished architecture, as suggested in the CoVHer Erasmus+ project, (Computer-based Visualisation of Architectural Cultural Heritage, https://covher.eu/). It has been developed to show the practical use of the ideas and terms proposed in the project.

Hypothetical reconstruction is a multidisciplinary work by architects, historians, archaeologists, designers, computer scientists, etc. This case study has been explored from the point of view of architects, with the aim of documenting the creation of a Critical Digital Model (CDM) that may virtually reconstruct the original project of an author at a particular time or date. The term "critical" is borrowed from textual criticism (ecdotics), and it is used here for studying 3D hypothetical reconstructions rather than texts (Apollonio, Fallavollita, & Foschi, 2021).

The methodology is divided into various steps or phases:

- Data acquisition.
- Critical evaluation of historical and architectural sources.
- 2D digital redrawing of graphic material:
  - Geometric/Proportional analysis and selection of the units of measurement.
  - Vectorisation: redrawing plans, sections, and elevations at a specific scale of representation.
- Construction of the 3D model:
  - Semantic segmentation of meaningful components in the 3D model.
  - Generating a solid 3D representation without topological and geometrical errors.
- Visualisation.
- Uncertainty assessment and communication.
- Documentation.
- Publication (interoperability and accessibility).

2. The lost church of S. Margherita, data acquisition

The hypothetical reconstruction of Agostino Barelli's 1685 design of the S. Margherita church (Fig. 1) was based on the extensive historical data gathered studied and published by Costarelli in 2015. This section reports in synthesis Costarelli's study about the church's history, for a more extensive and in-depth analysis refer to the original text (Costarelli, 2015).

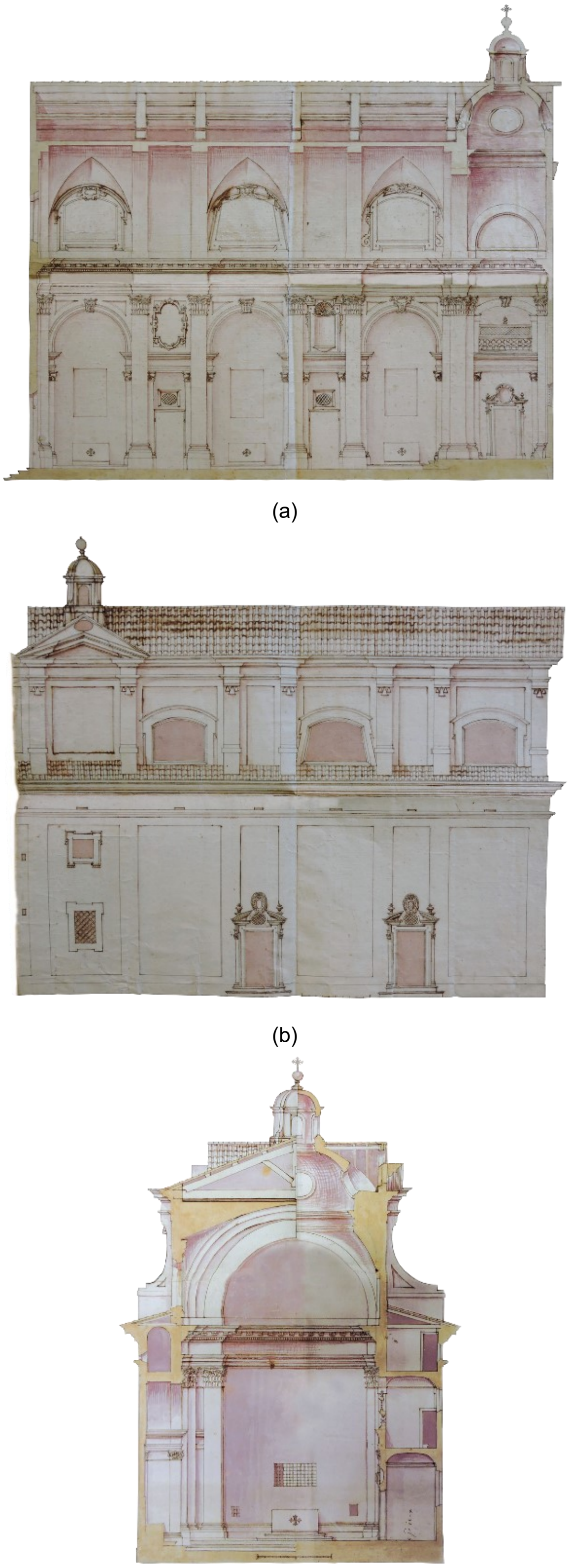
The first piece of evidence about the benedictine monastery of S. Margherita located in Gangaiolo di Val d'Aposa, No. 1442-1443, Bologna (today at the crossroad between via S. Margherita and via de Griffoni) dates to the 12th century, although the precise year of foundation is unknown.

The first bell tower was built in 1384. Around 1520, the religious building became a cloistered convent, which allowed increasing the number of nuns living there. Renovation and expansion works can be dated at the end of the 16th century and continued until the end of the following century. In 1598, the Sant'Antonio da Padova's oratory was annexed to the convent, and some neighbourhood houses a couple of decades later (1622).

The church building is today completely lost, and actual remains have been identified by Costarelli as the old building of the oratory of Sant'Antonio. An architectural draft by Agostino Barelli dated around 1685, is the most extensively documented version we have of the original building of S. Margherita (Fig. 1). However, the real building was probably never built according to this design. The original project had six chapels, but posterior documents show that only four were constructed. The original design was probably widely modified due to financial difficulties already started during the Great Plague of 1630 causing a diffuse crisis in the building sector, reaching its peak in 1700. Furthermore, some documents testify that the nuns had difficult relations with Barelli, the architect, due to their habit of subcontracting construction work. Another document showing the plan and section for the same church, datable only one or two years later, probably drawn by Barelli or his son, presents fewer ornaments, this would reinforce the hypothesis of economic difficulties.

A century later (1782-1790), new altars were designed by Angelo Venturoli (Venturoli, 1782, 1792) for a more modest church building. However, this updated project probably never saw the light of day due to the financial problems with the Curia and the Napoleonic suppressions of 1798. In 1806, the church was officially closed and was sold to non-religious owners, who introduced new modifications, until its demolition at an uncertain date before 1949, when a residential complex was built in the same area by a local real estate company.

Our CDM of the church of S. Margherita aims to hypothetically replicate it as close as possible to Barelli's 1685 design. The more modest version, designed a couple of years later by Barelli, and enriched with Venturoli's altars, has also been digitally modelled for comparison purposes.



(a)

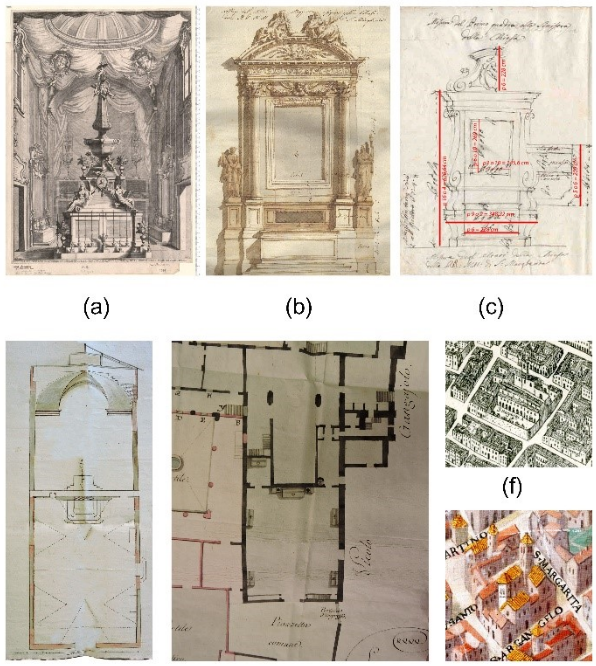
(b)

(c)

Figure 1: Primary sources for the CDM of the 1685 design of S. Margherita by Agostino Barelli (?): a) (Barelli, ca. 1685 a); b) (Barelli, ca. 1685 b); and c) (Barelli, ca. 1685c).

To rebuild the lost Church of S. Margherita as it was designed but never realised by Agostino Barelli in 1685, we have investigated the building's history, looking for as many primary sources as possible. The main primary graphical sources are three original drawings by the architect, dated 1685: a) the longitudinal section (Barelli, ca. 1685a), b) the lateral façade (Barelli, ca. 1685b), and c) the transverse section (Barelli, ca. 1685c) (Fig. 1).

Several additional sources describing S. Margherita have also been discovered (Fig. 2), and they refer to modifications plans at different periods, depicting alternative architectural configurations. We have archived them for use in later stages of our project, either to produce variants or to fill gaps in the primary sources.



(a) (b) (c)

(d) (e) (f) (g)

Figure 2: Additional secondary sources: a): print (Monogrammist AB. GB, 1759); b) Drawing (Venturoli, 1782); c) Drawing (Venturoli, 1792); d) Plan and section (Barelli, ca. 1686–87); e) Plan (Tubertini, 1807); f) map (de' Gnudi, 1702); g) map (Tibaldi, Sabatini, & others, 1575). These sources were also published and discussed in (Costarelli, 2015).

3. Critical evaluation of historical and architectural sources

The primary sources have been critically analysed and evaluated regarding the following aspects:

- Level of Detail (LoD), accuracy, and readability.
- Degree of deformation.
- Lacking elements and inconsistencies.

The early evaluation of the sources has been useful in detecting eventual elements and poorly documented details. We have relied on later sources to compensate for the eventual shortcomings of these primary sources.

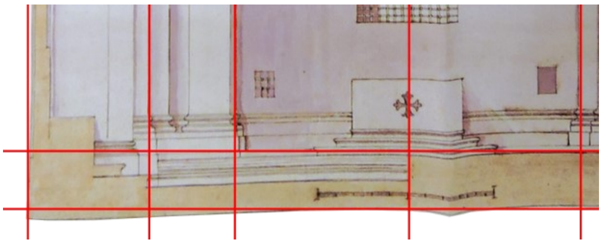
The maximum LoD that can be reached in the 3D model based on the available primary sources must be established in the early stages. The LoD represented by Barelli in his 1685 plans is consistent with a modern scale of 1:100. That means we have enough information to draw accurately walls and structures, but not enough information to reproduce ornaments with accuracy. The information retrievable from the primary sources suggests the clear presence of some frames and the use of certain architectural orders typical of the classical language. Even if the architectural order and the overall placement and layout of cornices and other ornaments are recognisable, it is not possible to redraw the mouldings with a high level of detail because the drawn lines are too thick to reliably represent small details. Thus, further information about those tiny details was retrieved from the treaty by Andrea Palladio "*I quattro libri dell'Architettura*" (Palladio, 1570), a secondary source that allowed to digitally replicate mouldings, cornices and classical order (Ionic and Corinthian).

It is not always possible to avoid deformations during the data acquisition phase, especially if the documents present intrinsic deformations caused by the ageing of their physical support (e.g., paper, wood), when they are precious documents and cannot be manipulated much to avoid damaging them, or when they are hardly accessible.

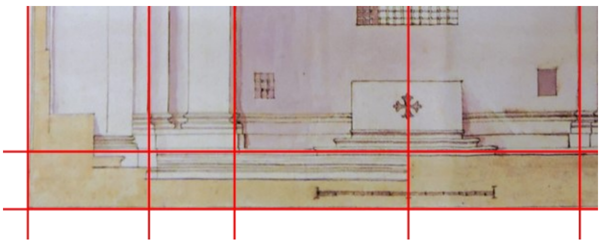
In general, in the case of paper documents, it would be preferable to digitise the documents while trying to minimise:

- perspective convergence/foreshortening: this problem can be minimised by keeping the camera projection plane perfectly parallel to the document;
- paper bending: this problem can be minimised by applying some weights at the corner of the document, or by placing a transparent panel on top of the document, before digitisation.

The direct digitisation of the original draws by Barelli presents noticeable deformation (Fig. 3a). To solve this problem, digitised documents have been post-processed in a photo-editing application (Adobe Photoshop 2022) to minimise stretches, bending, and perspective foreshortening (Fig. 3b). Document retouching has been restricted to its absolute minimum, and in all cases, proportions of the original project have been strictly maintained.



(a)



(b)

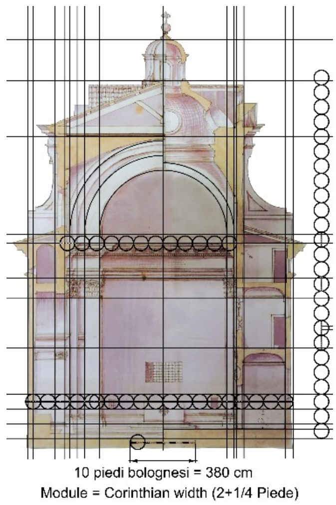
Figure 3: (a): Deformed original acquisition; (b) post-processed rectified document through a photo editing software. The rectification was performed critically in order to equalise the units of the graduate scale affected by the deformations (adapted from Barelli, ca. 1685c).

In cases with more severe deformations or strong perspective foreshortening, more advanced methodologies can be applied (Mikolajewska, 2023). After the rectification, it is important to keep the original un-rectified documents within reach to consult them in the event of any doubt.

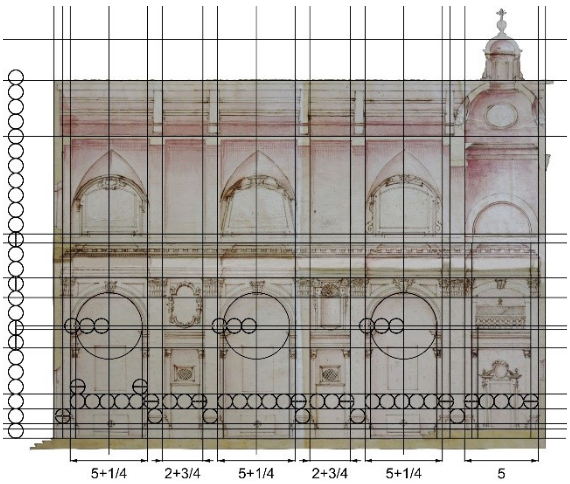
The preliminary evaluation of missing elements and inconsistencies in the reference sources intends to identify as many flaws as possible by consulting and cross-referencing available materials, and, if necessary, searching for additional secondary sources to fill those gaps. Inconsistencies by the original author and blind spots are almost always present in any architectural project and have also been identified in this project. Some blind spots could only be filled by deduction or by completing geometry based on its symmetry with some present elements, or by geometric completion, while others required the use of new secondary sources.

4. 2D digital redrawing of graphic material

4.1. Geometric/proportional analysis and selection of the units of measurement



(a)



(b)

Figure 4: Proportional analysis on the (a) transverse section (Barelli, ca. 1685c) and the (b) longitudinal section (Barelli, ca. 1685a) using the width of the Corinthian pilaster as the base module.

A preliminary geometrical analysis of the drawings has been carried out, based on the method presented in Apollonio, Fallavollita, & Foschi (2024a), which has allowed for estimating the historical units of measurement and the compositive rules plausibly used by the author for the proportioning and dimensioning of the building (Fig. 4). The plan of the ground floor is usually used for estimating the proportions of the whole building. However, given the absence of an original drawing of the Church ground floor, the longitudinal and transverse sections, as it was originally designed by Barelli in 1685, were used instead.

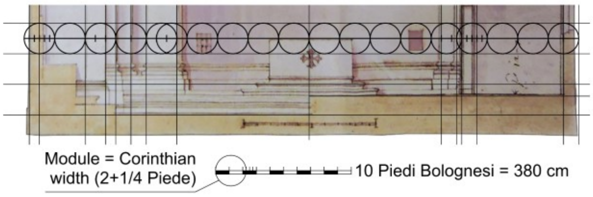


Figure 5: Base module (Corinthian diameter) compared to the unit of measurement (Piede Bolognese) (Barelli, ca. 1685c).

The original drawings have an explicit graphical scale composed of ten units (Fig. 5), however, its unit of measurement is not explicitly written. Thus it was estimated by comparison with different projects by the same author for the same church, dated one year later, where the unit of reference was explicitly written. The assumed historical unit of measurement was the *Piede Bolognese* equivalent to ca. 38 cm (Ministero di Agricoltura, Industria e Commercio, 1877, p. 115). In order to avoid scaling errors, a scale based on this measure has been verified by measuring known elements such as the rise and tread of the steps of a stair or the height and widths of the doors. Although this verification method does not always guarantee correct scaling, it is an effective method to detect errors.

The reference image was imported into a Computer Aided Design (CAD) application (McNeel Rhinoceros 8) and properly scaled in centimetres. Despite working on a model scaled in centimetres, the 3D model was proportioned using a grid based on the original historical unit of measurement: in this case, the *Piede Bolognese* divided into 12 *once*. Using the metric system (or the imperial/customary system) as the reference unit for the software, while employing a grid based on the original historical units of measurement to proportion the model, may initially complicate the redesign process. Nevertheless, this approach is preferable because software packages provide presets for modern measurement systems which guarantee a proper setting of the absolute tolerance, and using a grid based on the historical units guarantees a proportioning as close as possible to the one designed by the original author.

Using the metric (or imperial/customary) system consistently between different reconstructive projects will also contribute to simplifying comparison even when dealing with different periods, authors, and geographical areas. Employing a modern measurement system also facilitates the process of dimension checking during the design workflow. Conversely, by using archaic measurement systems the software would deal with unconventional fractional values which would complicate and slow down the assessment of the actual dimensions of the various architectural elements.

4.2. Vectorisation: redrawing of plans, sections, and elevations at a specific scale of representation

The next step was the vectorisation of the drawing. This operation was not performed by automatic tools, or by free hand tracing of the sources. It was a critical 2D redrawing of the main lines following the rules and modularity studied in the previous steps (Fig. 6).

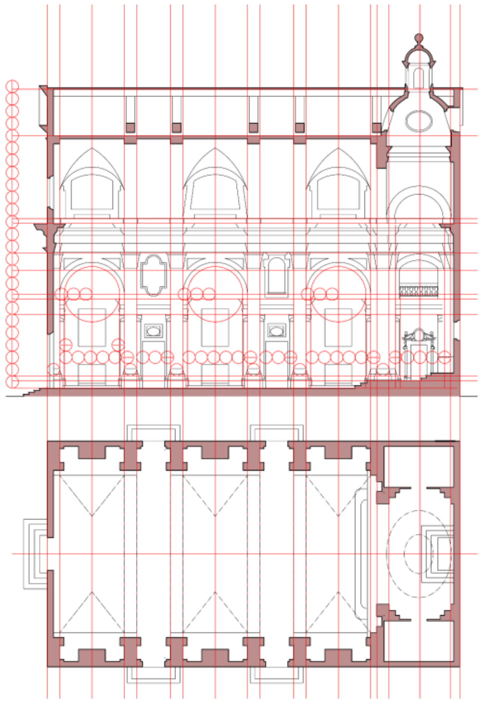
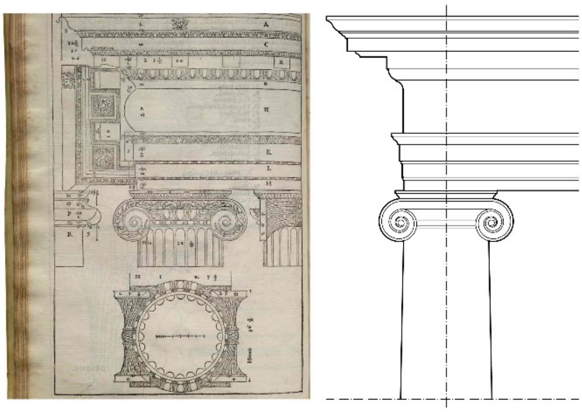


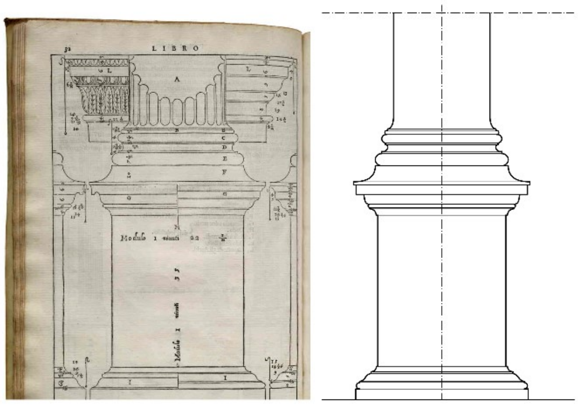
Figure 6: Rectified and modularised critical vectorisation of the longitudinal section and plan (in Rhinoceros by McNeel).

This approach is crucial for improving transparency and reproducibility and simplifying the subsequent 3D modelling phase. Eventual missing views have been inferred based on other available sources. For example, the plan was derived by cross-referencing the two sections and the lateral façade. On the contrary, the front façade, which was also missing, was left unsolved since the available elements from the primary sources were too lacking and no reliable secondary sources were ever found.

Some complex details, such as the classical orders and the cornices, were not readable in the original primary sources. Consequently, they have been drawn as separate files based on secondary sources with higher detail (Fig. 7). Some parts of the preliminary 2D vectorisation might not match the final 3D reconstruction because some critical points are more easily solvable in 3D. Foreseeing all complex spots in advance in the preparatory drawings is not always easy. Therefore, more accurate 2D orthogonal views matching the 3D model are extracted after the finalisation of the reconstruction.



(a)



(b)

Figure 7: Critical vectorisation of the Ionic order, based on Andrea Palladio's 4 books of architecture (Palladio, 1570): a) Ionic capital and entablature (p. 36); b) Ionic base, plinth, and cornices (p. 32).

5. Construction of the 3D model

5.1. Semantic segmentation of meaningful components in the 3D model

The building has been geometrically modelled in 3D NURBS-based (Non-Rational Uniform B-Spline) CAD, starting from the 2D generative curves drawn in the previous phase. In this context, this mathematical representation method, which creates continuous smooth surfaces, is preferable to the widespread polygonal discrete method because it enables more advanced analysis possibilities and better control over the geometry. Each 3D element has been modelled as an independent watertight closed manifold volume without intersections (e.g. walls and floors are not single surfaces, but are closed independent parallelepipeds, etc.). This is relevant to keep the model easy to reuse, edit, and interrogate. Each 3D element is organised in a list of clearly named layers to help keep the workflow smooth (Fig. 8).

It may be useful to name/tag each element to help future queries. However, this last step has not been carried out in this case because it was not needed for the scope of the reconstruction. This process of organising knowledge by segmenting the model and assigning names to each part is known as semantic segmentation (or semantic organisation), and the process of adding further information and/or properties to each part is called semantic enrichment. This approach forces the rational digital replica of the building and helps the critical analysis of the resulting architectural model.

The organisation of the diverse architectural elements and layers is usually performed simultaneously with the construction of the model itself. However, this operation, like all creative activities, is not always linear and simple and the precise identification of some parts is not always possible from the beginning of the process, but can be adjusted at a later stage. The complex elements replicated using secondary sources (e.g., the classical orders, the cornices, etc.) have been modelled independently and imported as discrete blocks instances later, keeping the scene lighter. Recurring elements have been converted into discrete block instances to decrease the computer file size and simplify eventual future editing. When symmetries occur, it is always preferable to model only one half and mirror it as the last step.

5.2. Generating a solid 3D representation without topological and geometrical errors

During the modelling phase, particular attention was dedicated to creating a model free of topological or geometric error. Furthermore, at the end of the modelling phase, the 3D model was double-checked and cleaned up from eventual duplicate geometries, bad objects, exploded poly-surfaces, single surfaces, and intersecting volumes that might have been overlooked or forgotten in previous steps. It was also important to check, reorganise, and clean up the layer list to make the model easier for later research and interrogation. For instance, the generative curves were moved into inactive layers, the names and colours were revisited, etc. The semantic organisation of the architectural elements should not necessarily correspond to the layers. For example, the column layer can contain the base, the shaft, and the capital. The rational choice of semantic organisation can vary according to the needs of scholars.

In synthesis the 3D model has been developed paying particular attention to the following formal and technical aspects:

- Only manifold watertight solids.
- No intersections and self-intersections.
- No duplicate geometries.
- Accurate snapping.
- Accurate tangents and curvature at connection points.
- Discrete block instances developed for repeated elements.
- Proper use of layers and grouping.
- Appropriate modularity analysis (based on Corinthian diameter).
- Appropriate units of measurement ("*Piede Bolognese*" converted into cm).
- 1:1 scale.
- Appropriate LoD (comparable to 1:50 scale).

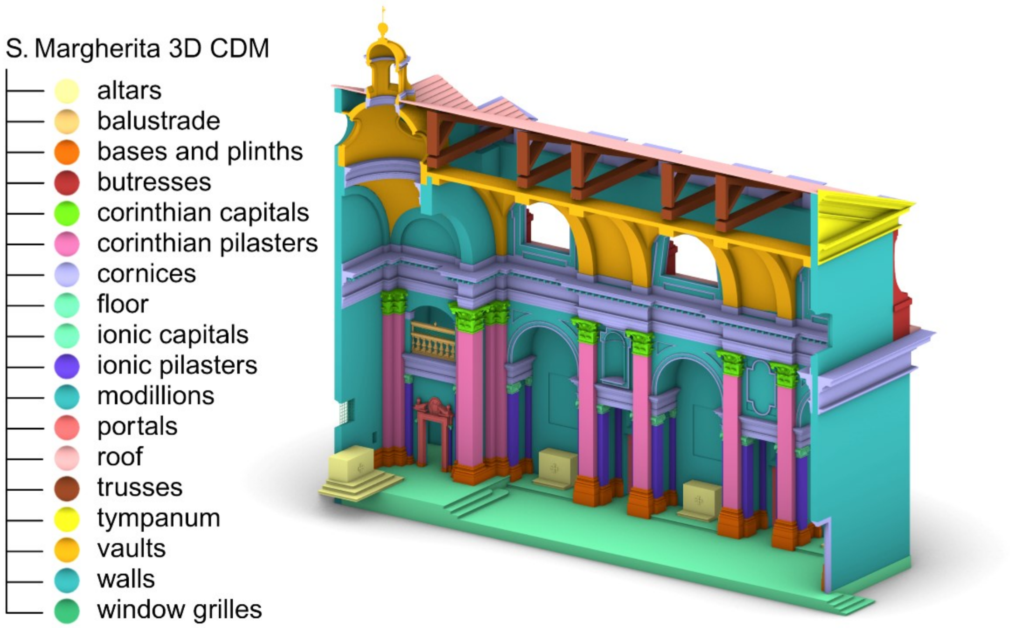


Figure 8: Semantically segmented 3D model of S. Margherita.

Figure 9: S. Margherita CDM as designed by Barelli in 1685. Central perspective view at the human height of the main aisle rendered with a white mono-material, and with physically plausible lights.

6. Visualisation

Visualisation can be interactive, dynamic, or static, and enables the communication and investigation of the final digital model in different ways. In the case of S. Margherita, the visualisation consisted of the production of static raster mages.

The finished NURBS 3D model was converted into a 3D geometric mesh and exported, with the ".obj" exchange file format, to several rendering applications (Blender, Autodesk 3DMax and Maxton Cinema 4D) to produce renderings from various points of view (Figs. 9 and 10). The NURBS to mesh conversion was performed layer by layer (in McNeel Rhinoceros), checking at each step that each element was tessellated with the suitable number of polygons depending on its uses. At the conversion stage, the maximum difference between the NURBS surface and the derived mesh was set to 1 mm, and the minimum edge length was set to 2 mm. The tessellation density is adaptive proportionally to the curvature. Since the NURBS model has been modelled with watertight manifold solids, its derivate mesh is suitable for 3D printing. The final 3D NURBS model was slightly inconsistent with the preliminary preparatory 2D vectorised drawing. Consequently, updated 2D drawings were recalculated by re-projecting the edges of the NURBS solids on given planes from orthogonal views (this step would have been much harder to carry out in case of mesh models).

(a) (b)

Figure 10: S. Margherita CDM as designed by Barelli in 1685. Perspective details of S. Margherita church: a) elliptical sail vault in the presbytery; b) lateral internal façade of the main nave.

The 2D re-projections were later overlapped with some of the shaded renderings (with Adobe Photoshop). These operations have improved the readability of architectural elements (Fig. 11), avoiding the fact that a single mono-material shader hides sometimes the edges and can delete some details from the view.

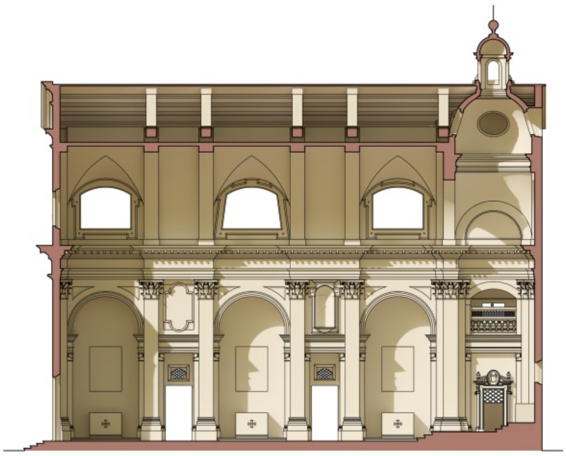


Figure 11: Longitudinal section of S. Margherita. The edges of the 3D model were projected on a plane and overlapped with the rendered orthographic view to enhance the architectural details and ornaments.

7. Uncertainty assessment and communication

Visualisation is useful not only for showing the features of the model from different points of view but also for investigating its shape or dimensions. It can also be a useful tool for expressing other information, such as the kind, authorship, or quality of the sources used. It may contribute to estimating the uncertainty of each part of the reconstruction, too. For the visualisation of S. Margherita's, a false-colour visualisation of the 3D model was also generated (Fig. 12).

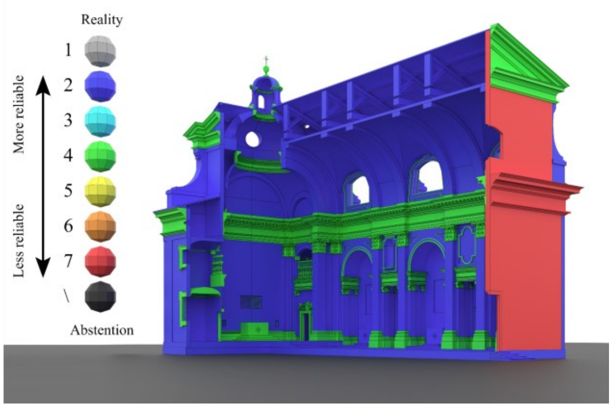


Figure 12: Uncertainty visualisation of S. Margherita based on the availability of information and detail in original sources. Refer to Table 1 for the meaning of each colour.

In this representation, each colour refers to a specific level of a particular scale of uncertainty. In general, these scales help to simplify the communication and understanding of the relationships between the used sources and the reconstructed elements through visual cues. The topic of evaluating and communicating the uncertainty of hypothetical reconstructions is highly debated by scholars (Battis-Schinker, 2023; Rodríguez-Moreno, 2024; Zhang, Zou, & Xiao, 2023; Mekheimar, 2023; Collina & Fabbri, 2024). Different approaches and different methods have been suggested over the years, the most popular of which imply the use of reference scales (or matrices). Cazzaro (2023) offers a convenient and extensive state of the art on the assessment of uncertainty in replicated digital models of heritage architecture. In this case, the original scale developed by us is based on the authorship and quality of the sources and it is divided into seven levels. This scale focuses specifically on unbuilt architecture and has been developed to minimise ambiguities and overlaps between different levels to make its application simpler and more objective (Table 1).

Table 1: Textual definitions of each of the 7 (+1) levels of the scale of uncertainty. This scale is a synthesis of the scale presented in (Apollonio, Fallavollita, Foschi, & Smurra, 2024b)

	Descriptions
1	The analysed feature of the 3D model is derived mainly from good-quality, REALITY-BASED DATA which reach the target LoD.
2	Reliable conjecture, based mainly on clear and accurate DIRECT/PRIMARY SOURCES which reach the target LoD. When REALITY-BASED DATA are unavailable, available but unusable, or not reaching the target LoD.
3	Conjecture, based mainly on INDIRECT/SECONDARY SOURCES, by the SAME AUTHOR/S, which reach the target LoD, or logic deduction/selection of variants. When DIRECT/PRIMARY SOURCES ARE AVAILABLE, but minimally unclear, damaged, inconsistent, inaccurate, or not reaching the target LoD.

4	Conjecture, based mainly on INDIRECT/SECONDARY sources by DIFFERENT AUTHOR/S (or unknown authors) which reach the target LoD. When DIRECT/PRIMARY SOURCES ARE AVAILABLE, but minimally unclear, damaged, inconsistent, inaccurate, or not reaching the target LoD.
5	Conjecture, based mainly on INDIRECT/SECONDARY SOURCES by the SAME AUTHOR/S which reach the target LoD. When DIRECT/PRIMARY SOURCES ARE NOT AVAILABLE or unusable.
6	Conjecture, based mainly on INDIRECT/SECONDARY sources by DIFFERENT AUTHOR/S (or unknown authors) which reach the target LoD. When DIRECT/PRIMARY SOURCES ARE NOT AVAILABLE or unusable.
7	Conjecture, based mainly on personal knowledge due to missing or UNREFERENCED SOURCES.
/	Not relevant, not considered, left unsolved, missing data, and missing conjecture (does not count for the calculation of the average uncertainty).

The selected colours are widely spaced in the colour spectrum to make them easily recognisable, also when applied on a shaded model. Univocal numbers were also assigned to each level to eventually calculate the average uncertainty with a single number if needed. Such synthetic averaged calculation can be useful when comparing different numerous 3D reconstructions.

This uncertainty scale has been developed with different levels of granularity (Fig. 13), meaning that it can be reduced to 5, or to 3 more general levels, enabling in this way its application to simpler cases. The definition of each level of the scale (Table 1) has been specifically developed to minimise subjectivity. In this way, its application will allow for more consistent and user-independent results.

Some authors have suggested calculating the averaged measure of holistic uncertainty in mathematical terms (Nicolucci & Hermon, 2010). In S. Margherita's model, we have applied the methodology developed by (Foschi, Fallavollita, & Apollonio, 2024). This method was conceived to simplify the comparison of alternative reconstructions.

The calculation of the average uncertainty is carried out by weighting the uncertainty values assigned to each element by their individual volumes, based on the model semantic segmentation. In this reconstruction of S. Margherita, the average uncertainty weighted on the volume of the individual architectonic elements (AU_V), is 29%. If needed, it is also possible to multiply each volume by an additional relevance (weight) factor. This variation of the formula can be useful in cases where certain elements are considered more important than others. In this latter case, it is crucial to clearly indicate the used relevance factors and for which elements. For example, for Corinthian and Ionic capitals and the relative entablatures and cornices, we used a relevance factor equal to 5, and the result of the average uncertainty weighted on the volume and relevance of the elements (AU_VR) is 37%. This latter value is more user-dependent than the previous one, but it is more informative. As far as the relevance factors applied are clearly declared, and despite being more subjective, the result is still transparent and reproducible.

These results have been automatically obtained from the model's geometry through an algorithmic approach, which allowed the calculation of the volumes, assigning the given weights, and applying the relative formulas automatically. Of course, these values by themselves are too synthetic and must be always considered as complementary and not exclusive tools to evaluate the uncertainty of a reconstruction. The applications of the AU_V and AU_VR formulas were performed in Grasshopper (for McNeel Rhinoceros) and in a custom-made open-source plugin for Blender available for download at the following link (https://github.com/rikkarlo/Blender-Uncertainty-Calculator).

8. Documentation

One of the last but most important steps in the creation of S. Margherita's CDM (Apollonio et al., 2021), has been documenting the critical reconstruction process,

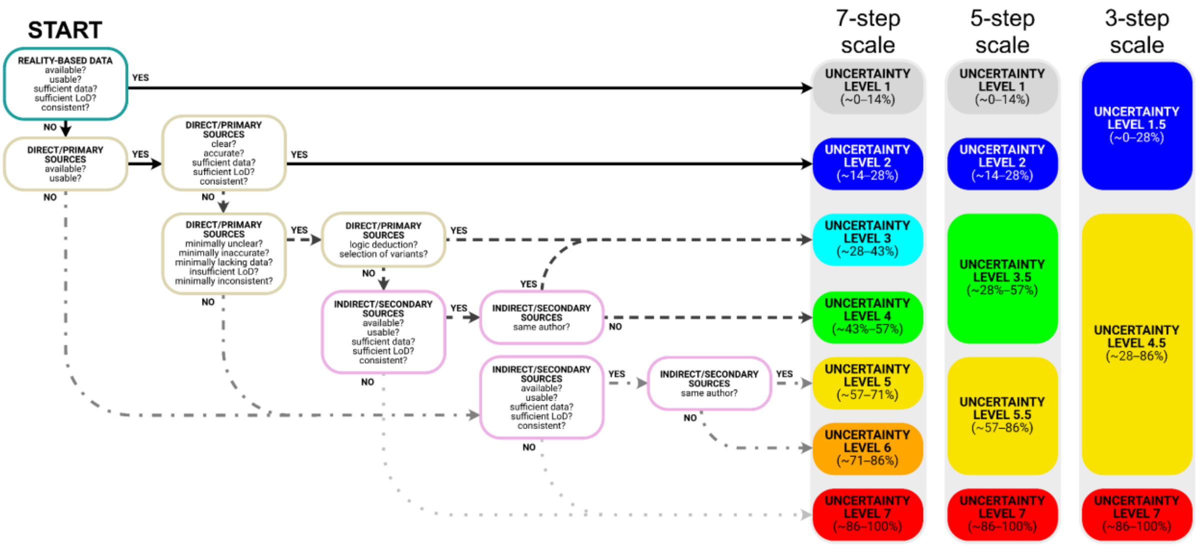


Figure 13: Flow chart of the uncertainty scale (Apollonio, Fallavollita, Foschi, & Smurra, 2024b).

organising all materials and sources, and carefully discussing all the reconstructing choices element by element. Criticism material has included other architectural drafts and variants of the same church designed by Agostino Barelli and Angelo Venturoli in later periods, which have also been modelled in 3D (Fig. 14).

To document a digital reconstruction, it is crucial to archive the raw data used, and information of the source such data come from (metadata). To allow future reuse of the model, we must keep informed the user of methodologies, techniques and all decisions, operations and every subjective choice, inference, logic deduction, or selection of variants. We should allow back-tracking of the reconstruction process, documenting all steps of the hypothetical reconstruction.

9. Publication (interoperability and accessibility)

If the hypothetical reconstruction is presented in an academic scientific context, the production and sharing of the proper documentation becomes mandatory. This is the only way to guarantee transparency and reproducibility which are core concepts of the scientific method. To foster accessibility, the 3D model should be shared openly with all the documentation attached.

S. Margherita CDM will be published in an online open repository which is going to be publicly available at the end of the project (refer to the CoVHer project official page for more updates, https://covher.eu/). To maximise interoperability, it is important to share the 3D models in open exchange file formats other than the native formats to minimise data loss due to data conversion.

Other than publishing scientific papers, for the publication of the in-depth documentation of the reconstructive process, it is possible to use also online open platforms such as IDOVIR (Grellert, Wacker, Bruschke, Stille, & Beck, 2023). In the case of S. Margherita, we did not use such type of documentation online platforms because the sources used are a few and the discussion of how they were used was thoroughly resolved in this paper, so the use of the platform would have been redundant. However, for more complex cases, such type platforms can help keep the documentation organised and simplify its communication.

10. Discussion and conclusions

The case study of S. Margherita described here is an example of the methodology settled and refined within the UE-Erasmus+ funded CoVHer project (https://covher.eu/), for the development and construction of a CDM of architectural heritage. It should be noted that hypothetical 3D reconstruction of demolished or never-existing architectural heritage is not a linear process. In fact, some of the phases of the methodology described do not occur in chronological order. However, in this paper, they were reported as such for a smoother reading. In practical cases, all stages of the reconstruction are often revisited and re-evaluated multiple times as new insights and inferences can emerge at later stages.

Hypothetical reconstruction is akin to the design process, including moments of intuition, conjecture, and reconsideration. The specific case study described here presents some of the most typical challenges to be faced by scholars in the fields of architecture and history.

Among the results of the CoVHer project, the drafting of a shared glossary with the definitions of terms and concepts used in hypothetical reconstructions is certainly one of the most relevant. In this study, we aimed to demonstrate how these terms and concepts are also useful in practice. Although the methodology was presented referring to a specific case study, it can be adapted and applied to other cases where the goal is the creation of a hypothetical 3D model of an architectural object designed by a certain author, in a given historical period, while aiming to make the process as transparent and reproducible as possible, and thus scientific.

Figure 14: Hypothetical variant of S. Margherita's church, embellished with the altars designed by Angelo Venturoli in 1782/92 in a more modest planimetric layout designed by Barelli between 1686–87 (Barelli, ca. 1686–87).

Concerning the flexibility of the methodology, in particular, if the scale of uncertainty presented here will turn out to be inadequate for specific niche scenarios, it can be adjusted, as the definitions of the various levels of the scale can be rewritten to better fit specific circumstances. As far as the definitions are clear and the overall approach remains the same, the results will still be comparable and reproducible.

In other words, the purpose of this paper was to describe the potentialities of the methodology, while accurately describing it step by step and highlighting its value beyond the case study illustrated.

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References

Apollonio, F. I., Fallavollita, F., & Foschi, R. (2021). The Critical Digital Model for the Study of Unbuilt Architecture. In F. Niebling, S. Münster, H. Messemer (Eds.), *Workshop on Research and Education in Urban History in the Age of Digital Libraries (*pp. 3–24). Cham: Springer International Publishing. https://dx.doi.org/10.1007/978-3-030-93186-5_1.

Apollonio, F.I., Fallavollita, F., & Foschi, R. (2024a). Some reflections on the module, the unit of measurement, and the 3D models of hypothetical reconstructions. In F. Bergamo, A. Calandriello, M. Ciammaichella, I. Friso, F. Gay, G. Liva, C. Monteleone (Eds.), *Misura / Dismisura | Measure / Out of Measure Ideare Conoscere Narrare | Devising Knowing Narrating* (pp. 756–764). Milano: FrancoAngeli. https://doi.org/10.3280/oa-1180-c509.

Apollonio, F. I., Fallavollita, F., Foschi, R., & Smurra, R. (2024b). Multi-feature uncertainty analysis for urban-scale hypothetical 3D reconstructions: Piazza delle Erbe case study. *Heritage, 7*(1), 476–498. https://doi.org/10.3390/heritage7010023.

Barelli, A. (presumed) (ca. 1685a). *Sezione longitudinale interna per il progetto della chiesa di S. Margherita*. Archivio di Stato, Demaniale S. Margherita, 52/3919, Bologna, Italy.

Barelli, A. (presumed) (ca. 1685b). *Prospetto laterale esterno per il progetto della chiesa di S. Margherita.* Archivio di Stato, Demaniale S. Margherita, 52/3919, Bologna, Italy.

Barelli, A. (presumed) (ca. 1685c). *Sezione della zona presbiteriale per il progetto della chiesa di S. Margherita*. Archivio di Stato, Demaniale S. Margherita, 57/3924, Bologna, Italy.

Barelli, A. or N. (presumed) (ca. 1686-87). *Pianta e sezione S. Margherita*. Archivio di Stato, Demaniale S. Margherita, 57/3924, Bologna, Italy.

Battis-Schinker, E. (2023). *Assessing authenticity in heritage conservation: case study: architectural conservation in Bahrain* (Doctoral thesis, BTU Cottbus-Senftenberg, Germany). https://doi.org/10.26127/BTUOpen-6511

Cazzaro, I. (2023). *Digital 3D Reconstruction as a Research Environment in Art and Architecture History: Uncertainty Classification and Visualisation* (Doctoral thesis, Alma Mater Studiorum Università di Bologna, Italy). https://doi.org/10.48676/unibo/amsdottorato/10817

Collina, F., & Fabbri, F. (2024). A First Digital Reconstruction of the so-Called Armor of Theoderic. In M. Corsini, D. Ferdani, A. Kuijper, & H. Kutlu (Eds.), *Eurographics Workshop on Graphics and Cultural Heritage*. Eindhoven: The Eurographics Association. https://doi.org/10.2312/gch.20241264

Costarelli, A. (2015). La scomparsa chiesa parrocchiale di Santa Margherita a Bologna: arredo interno e vicende costruttive. In Comitato per Bologna Storica e Artistica (Eds.), *Strenna Storica Bolognese, anno LXV – 2015 (*pp. 103–130). Bologna: Patron Editore.

de' Gnudi, F. (1702). *Disegno dell'Alma citta' di Bologna ichnoscenografia (Part.)*. Bologna, Italy.

Dell'Unto, N., Leander, A. M., Dellepiane, M., Callieri, M., Ferdani, D., & Lindgren, S. (2013). Digital reconstruction and visualization in archaeology: Case-study drawn from the work of the Swedish Pompeii Project. In *2013 digital heritage international congress (DigitalHeritage)* IEEE (Vol. 1, pp. 621–628). Marseille, France. https://doi.org/10.1109/DigitalHeritage.2013.6743804

Foschi, R., Fallavollita, F., & Apollonio, F. I. (2024). Quantifying uncertainty in hypothetical 3D reconstruction—A user-independent methodology for the calculation of average uncertainty. *Heritage, 7*(8), 4440–4454. https://doi.org/10.3390/heritage7080209

Giovannini, E. C. (2020). Workflow for an evidence-based virtual reconstruction: the marbles of the ciborium of the early medieval Monte Sorbo church. *International Archives of the Photogrammetry, Remote Sensing and Spatial Information Sciences*, 43, 1395–1402. https://doi.org/10.5194/isprs-archives-XLIII-B2-2020-1395-2020.

Grellert, M., Wacker, M., Bruschke, J., Stille, W., & Beck, D. (2023). Documentation and Evaluation of Virtual Reconstructions*. International Conference on Metrology for Archaeology and Cultural Heritage (2023 IMEKO)*, (pp. 653–658). https://doi.org/10.21014/tc4-ARC-2023.122

Kuroczyński, P., Bajena, I., Große, P., Jara, K., & Wnęk, K. (2022). Digital reconstruction of the new synagogue in breslau: new approaches to object-oriented research. In *Workshop on Research and Education in Urban History in the Age of Digital Libraries* (pp. 25-45). Cham: Springer International Publishing. https://doi.org/10.1007/978-3-030-93186-5_2

Lengyel, D., & Toulouse, C. (2015). The consecution of uncertain knowledge, hypotheses and the design of abstraction. In *Proceedings of the 20th International Conference on Cultural Heritage and New Technologies (CHNT 2015), Vienna, Austria*.

Mikolajewska, S. (2023). Smart survey methodologies for the digitization of architectural drawings. The maps of some convents located in the city of Parma, drawn by Giuseppe Cocconcelli in 1811. In L. Inzerillo, & F. Acuto (Eds.), *Digital & Documentation. The new boundaries of digitization* (pp. 112–121). Pavia University Press.

Mekheimar, F. (2023). Virtualizing the uncertainty of digital archaeological reconstructions applications on the Egyptian labyrinth of Hawara (Doctoral thesis, Nottingham Trent University, United Kingdom).

Ministero di Agricoltura, Industria e Commercio (1877). *Tavole di ragguaglio dei pesi e delle misure già in uso nelle varie Provincie del Regno col sistema metrico decimale*. Roma: Stamperia Reale.

Monogrammist AB., GB. (1759). *Macchina Pel Funerale Di ... Gio: Batta: Stella Vescovo Di Rimino ...* Staatliche Museen zu Berlin, Kunstbibliothek, Berlin, Germany. Retrieved September 4, 2024, from: https://id.smb.museum/object/919099.

Nicolucci, F., & Hermon, S. (2010). A fuzzy logic approach to reliability in archaeological virtual reconstruction. In F. Nicolucci, S. Hermon (Eds.), *Beyond the Artifact. Digital Interpretation of the Past. Proceedings of CAA2004*. Budapest: Archaeolingua, (pp. 28–35).

Ortiz-Cordero, R., Pastor, E. L., & Fernández, R. E. H. (2018). Proposal for the improvement and modification in the scale of evidence for virtual reconstruction of the cultural heritage: A first approach in the mosque-cathedral and the fluvial landscape of Cordoba. *Journal of Cultural Heritage*, 30, 10-15. https://doi.org/10.1016/j.culher.2017.10.006

Palladio, A. (1570). *I Quattro Libri dell'Architettura.* Venezia: D. De Franceschi.

Rodríguez-Moreno, C. (2024). Depicting the Uncertain in Virtual Reconstructions of Architectural Heritage. In L. Hermida González, J. P. Xavier, I. Pernas Alonso, C. Losada Pérez (Eds.), *Graphic Horizons. EGA 2024. Springer Series in Design and Innovation*, (vol 44, pp. 120–127). Cham: Springer. https://doi.org/10.1007/978-3-031-57579-2_15

Tibaldi , P., Sabatini, L., & others (presumed) (1575). *Pianta prospettica di Bologna* (part.), affresco. Sala Bologna Palazzi Vaticani, Rome, Italy.

Tubertini, G. (1807). *Pianta della chiesa esterna di S. Margherita, Part. Dalla pianta generale del monastero*. Archivio di Stato Notarile, Betti Serafino, filza 662, Bologna, Italy.

Venturoli, A. (1782). *Progetto per l'altare Maggiore della chiesa di S. Margherita*. Archivio Collegio Venturoli, Album n° 1, Abbozzi di Architettura ed Ornati eseguiti, p. 47, Bologna, Italy.

Venturoli, A. (1792). *Progetto per il rifacimento degli altari laterali della chiesa di S. Margherita*. Archivio Collegio Venturoli, Perizia Cartone M, fasc. 5, Bologna, Italy.

Zhang, Z., Zou, Y., & Xiao, W. (2023). Exploration of a virtual restoration practice route for architectural heritage based on evidence-based design: a case study of the Bagong House. *Heritage Science, 11*(1), 35. https://doi.org/10.1186/s40494-023-00878-8