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Developing a supply chain coordination model through a revised revenue sharing contract: Incorporating sales efforts and green quality considerations

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Abstract:

In response to the growing emphasis on supply chain sustainability and green business management, this study addresses the critical challenges faced by industries in balancing the costs and benefits of enhancing the green quality of products, pricing strategies, and sales efforts. As environmentally conscious consumers increasingly prefer products from sustainable supply chains, industries encounter the dilemma of investing in sustainable practices while maintaining profitability. This research identifies the need for effective decision optimization models and coordination mechanisms that incentivize supply chain members to collaborate towards sustainable production without compromising their financial goals. To this end, we propose a novel hybrid cost and revenue sharing (HCRS) contract designed to foster collaboration among supply chain partners and align their interests in promoting sustainability. By examining optimal decision-making conditions, coordination mechanisms, and profit-sharing arrangements, the study highlights how the implementation of the HCRS contract can lead to improved supply chain performance and profitability. Our findings indicate that an increase in customer sustainability consciousness (CSC) positively correlates with higher levels of product green quality and overall profit, illustrating the potential benefits of prioritizing sustainability in supply chain operations.

Key words:

Supply chain coordination, revenue-sharing contract, promotional effort, green quality.

1. Introduction

Air pollution, characterized by changes in the atmosphere due to chemical substances, dust, or biological factors, poses a significant threat to human health and ecosystems worldwide. The depletion of the stratospheric ozone layer, primarily caused by industrial activities, has underscored the need for urgent environmental action. As industries continue to expand at an unprecedented rate, the environmental landscape faces escalating challenges, necessitating innovative approaches to mitigate pollution and promote sustainability.

With a growing emphasis on environmental preservation and sustainable practices, the concept of green supply chain management has emerged as a strategic solution to address the environmental impact of industrialization. The imperative to reduce pollution and adopt eco-friendly practices has become paramount, driven not only by regulatory pressures but also by shifting consumer preferences towards sustainable products.

As global environmental challenges intensify, industries face mounting pressure not only from regulatory bodies but also from consumers who

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are increasingly demanding transparency and accountability in sustainability practices. This shift in consumer behavior necessitates a reevaluation of traditional supply chain strategies, which often prioritize cost and efficiency over environmental impact. The broader problem lies in the inherent conflict between short-term profitability and long-term sustainability goals, as businesses grapple with the financial implications of implementing green practices. Consequently, there is a critical need for integrated frameworks that not only address environmental concerns but also align with the economic objectives of supply chain members. By fostering collaboration and creating incentives for sustainable practices, businesses can navigate this complex landscape, ensuring that they meet consumer expectations while maintaining competitive advantage in a rapidly evolving market.

In this context, the importance of integrating environmental considerations into supply chain management becomes evident, particularly for countries like Iran undergoing rapid industrialization. By aligning supply chain operations with environmental objectives, businesses can enhance their operational efficiency, product quality, and environmental stewardship, paving the way for sustainable industrial development.

Moreover, as businesses strive to balance environmental responsibilities with marketing imperatives, the role of advertising and pricing strategies in promoting sustainable growth cannot be overlooked. Effective marketing campaigns not only create customer awareness but also influence purchasing decisions, contributing to business sustainability and competitiveness. Pricing strategies that reflect the value of environmental activities and advertising efforts are crucial in maintaining profitability while advancing environmental goals within the supply chain.

This research delves into the interplay between environmental concerns, marketing dynamics, and pricing strategies within the context of green supply chain management. Focusing on the manufacturer's pivotal role in environmental stewardship and product quality enhancement, we develop a decision optimization model to guide sustainable decision-making processes and drive eco-friendly product development.

The structure of this paper is as follows: Section 2 provides a comprehensive literature review. Section 3 presents the assumptions, notations and model employed in this study. Section 4 presents numerical examples and sensitivity analysis to further elucidate our findings. Conclusions and future perspectives are presented in Section 5.

2. Literature review

2.1. Supply chain coordination

Ghosh et al. (2020) conducted a study on supply chain coordination using contracts, focusing on the different nature of demand. They developed coordination models that incorporated demand dependent on product price and utilized buyback contracts. Hou et al. (2022) developed a coordination model involving uncertain demand that was based on retail price and manufacturer discount, optimizing and coordinating wholesale and retail price decisions through revenue sharing and discount contracts. Wang et al. (2021) examined the possibility of coordinating a supply chain with supplier competition for price and brand-dependent demand, involving a retailer and two suppliers and they employed revenue sharing contract. Chungsuk (2022) developed a coordination model for wholesale price, production rate, retail price, and order quantity decisions. He found that employing revenue sharing contract leads to improvement in supply chain performance. Fakhrzad et al. (2022) developed a model based on a buyback contract with demand dependent on quality and sales effort but they found that the contract alone could not extend coordination in the supply chain under such conditions. However, by presenting a modified buyback contract, they established coordination in the supply chain. Huang and Ip (2019) developed a buyback contract modeled based on the Stackelberg game to optimize ordering and pricing decision variables in a humanitarian supply chain, providing both members with higher competitive advantages using this contract. Zhang and Yousaf (2020) proposed an extension to a two-part tariff contract that incorporates government intervention through taxes or subsidies to optimize green technology investment decisions and customer preferences. Their findings suggest that this contract can optimize the entire supply chain and facilitate green improvement. Additionally, planned government intervention can enhance supply chain performance and promote sustainable goals. Guo et al. (2020) developed a model that considers demand dependent on green quality, utilizing a wholesale price contract to coordinate greening determination and ordering decisions.

Swami and Shah (2013) developed a coordinating model based on a two-part tariff contract, allowing supply chain members to optimize variables related to green quality and price. Their research investigated the impact of supply chain coordination on increasing profit levels for all parties involved. Liu et al. (2021) extended three game models of the supply chain, including basic, unit trade credit, and trade credit cooperation models, in order to coordinate with stochastic demand and optimize decisions related to retail price, wholesale price, and order quantity. They developed a revenue sharing contract to facilitate decision coordination and utilized the index (CVaR) as a risk assessment criterion in their models. Alamdar et al. (2018) conducted research on coordinating contracts with sales effort-dependent demand, which proposed a novel coordination mechanism to promote the performance of a closed-loop supply chain (CLSC) in both a decentralized and centralized system. The presented contract effectively coordinates decentralized CLSCs and is useful from the perspective of the consumer and the environment. Heydari and Asl-Najafi (2021) investigated a revised sales rebate (RSR) contract that included an inventory penalty as a punitive approach as well as the approach of sales rebate mechanism. This contract aimed to coordinate the supply chain's member decisions. Jabarzare and Rasti-Barzoki (2020) developed a model for a bi-level supply chain considering price and quality of the product. The contract established revenue sharing between the manufacturer and a packaging company. The results show that adopting a cooperative approach in such conditions increases customer satisfaction and ultimately increases the profit of the supply chain. Li et al. (2018) developed a model based on a revenue sharing contract in a supply chain, where they coordinated pricing and carbon emission reduction decisions. Zhang et al. (2020) designed a two-channel closed-loop supply chain to coordinate with the revenue sharing contract, which encouraged the retailer to recycle the waste product. The results show that the retailer benefits from the collection and recycling of the product by employing the revenue sharing contract, and the supplier can reduce its production costs by reproducing from recycled materials. Xu et al. (2022) optimized the pricing and sales effort decision variables by developing the revenue sharing contract. In this research, a dual-channel supply chain consisting of a retailer with limited capital and a supplier is introduced while the retailer can also provide financial credit. Sabbaghnia et al. (2021) developed a model to extend coordination the decision variables of pricing and carbon reduction. In their research, the wholesale price was one of the decision variables, which is not typically the case in most revenue sharing contracts. The members of the supply chain make decisions based on Stackelberg's game, and the demand depends on the efforts made to reduce carbon. Dehghan-Bonari et al. (2021) developed a green supply chain coordination model with two suppliers and one retailer. One of the suppliers is a product manufacturer that takes environmental considerations into account. The research proposes a coordination mechanism for pricing and ordering decisions in the supply chain using revenue sharing and option contracts. The results show that the green supplier is more inclined to the option contract, while the retailer would rather the revenue sharing contract than the option contract. He et al. (2020) developed models to investigate the omnichannel strategy on the pricing and profit of a dual-channel supply chain and the environment. The research examines the effects of the omnichannel strategy on the pricing and the chain's performance while considering environmental factors. Li et al. (2018) developed a supply chain coordination model with a revenue sharing contract considering an omni-channel structure. The coordination developed in this research takes place between offline and online channels, and the supply chain management must decide how to divide the inventory between the both channels and the procedure of distributing the profit between members. Ma and Li (2019) developed a buyback contract for sales effort-dependent demand situations. The research shows that the buyback contract is ineffective in coordinating the supply chain. However, by proposing a combined contract of the buyback contract and the sales revenue-penalty contract, coordination in the supply chain can be obtained under these conditions. Taleizadeh et al. (2019) developed a coordination model with demand dependent on price, quality, and product returns in a CLSC. The model optimized the above variables, and the results show that the buyback mechanism improves the performance of members in this situation. Devangan et al. (2013) developed a model based on a repurchase agreement, where the retailer's inventory level affects demand. The research uses the simulation method to solve the model and presents its numerical results.

2.2. Research Gap and Objectives

Based on the literature review and the analysis presented in Table 1, a significant research gap is identified in the domain of supply chain coordination with contracts. Despite the extensive body of

Table 1. Literature review.

Author	Price	Demand pattern: Sales effort	Demand pattern: Green quality effort	Contract
(Dehghan-Bonari et al., 2021)		√		Buyback
(Chungsuk, 2022)	√			Revenue sharing
(Xu et al., 2022)	√	√		Revenue sharing
(Hou et al., 2022)	√			Revenue sharing and discount
(Sabbaghnia et al., 2021)	√			Revenue sharing
(Heydari and Asl-Najafi, 2021)		√		Revised sales rebate
(Liu et al., 2021)	√			Revenue sharing
(Wang et al., 2021)	√			Revenue sharing
(Ghosh et al., 2020)	√			Buyback
(Zhang and Yousaf, 2020)			√	Two-part tariff
(Li et al., 2018)	√		√	Revenue sharing
(Guo et al., 2020)	√			Wholesale price
(Jabarzare and Rasti-Barzoki, 2020)			√	Revenue sharing
(Alamdar et al., 2018)	√			A novel contract
(Zhang et al., 2020)	√	√		Revenue sharing
This paper	√	√	√	Hybrid cost and revenue sharing

research in this area, there is a notable absence of studies focusing on the development of a hybrid cost and revenue sharing contract that effectively coordinates sales efforts, pricing strategies, and green product quality decisions within the supply chain. Therefore, to address this research gap and contribute to the field, this study aims to develop a comprehensive model that optimizes all three decisions concomitantly, thereby achieving effective supply chain coordination.

The objectives and contributions of the present study to the modeling for supply chains are outlined as follows:

- Optimization of sales effort, price, and green quality jointly to explore their impacts on the supply chain dynamics.
- Designing a novel contract that facilitates supply chain coordination, presenting a “win-win” strategy beneficial for both members. Additionally, an acceptance interval is introduced for assessing the contract parameter.
- Ensuring that the design of the contract allows for the measurability of the effectiveness of different parameters through the acceptance interval.
- Joint optimization and coordination of pricing, sales effort, and green quality decisions, followed by a comparative analysis of outcomes between collaborative and decentralized structural approaches.

3. Model descriptions

In decentralized supply chains, suppliers and manufacturers collaborate with intermediaries like retailers to distribute products to end consumers. However, each entity within the supply chain has its own profit-maximizing objectives, leading to varied motivations among the companies involved. As a result, meticulous attention is required when formulating supply chain agreements. This study operates under the premise that the supplier proposes the contract terms, but both the supplier and retailer engage in activities that impact consumer demand.

The supplier makes investments in product development, encompassing technology enhancements and sourcing raw materials, with the aim of enhancing environmentally friendly practices (referred to as green quality) and driving up demand. Yet, these eco-friendly advancements entail costs borne solely by the supplier, potentially prompting an increase in wholesale prices to offset these expenditures. For instance, adopting sustainable materials for packaging or utilizing stone paper for book printing can help mitigate deforestation. In the manufacturing sector, the use of biodegradable components in detergents helps combat environmental degradation caused by harmful substances. Similarly, retailers can bolster demand through promotional initiatives, such as televised advertising, expanding product shelf space, offering customer incentives, and other endeavors that stimulate consumer interest.

The assumptions of the proposed problem are as follow:

- The final demand for the product is influenced by various factors, including promotional efforts ($D=a-bp+ue+to$). This is partly similar to what was presented by Heydani and Asl-Najafi (2021).
- The retailer's decision variables are sales effort and price, while the supplier's decision variable is the product's green quality. The demand increases with respect to sales effort and green quality, but decreases with respect to price.
- Shortages are not permitted.
- The potential market size is known with certainty.
- The HCRS contract is used to coordinate supply chain members. This contract is designed in such a way that the members do the global optimization of the decision variables and as a result, the total profit of the supply chain under the coordinated structure is equal to the total profit of the supply chain under the centralized structure. Also, by providing the contract acceptance interval, a win-win situation is provided for the parties.
- Sales occur within a single period, necessitating optimization of all decisions for one sales season.

Notations

Decision variables:

e_c	Sales effort level by retailer in centralized structure
p_c	Retail price in centralized structure
o_c	Green quality level by supplier in centralized structure
e_{dec}	Sales effort level by retailer in decentralized structure
p_{dec}	Retail price in decentralized structure
o_{dec}	Green quality level by supplier in decentralized structure
e_{co}	Sales effort level by retailer in coordinated structure
p_{co}	Retail price in coordinated structure
o_{co}	Green quality level by supplier in coordinated structure

Parameters:

w	Wholesale price
n	Cost coefficient of sales effort
E	Cost coefficient of green quality
t	Customer sustainability consciousness (CSC) level
u	Sales effort elasticity coefficient in demand
a	Potential market size
φ	Hybrid cost and revenue sharing contract parameter
c	Cost per unit
π_r^{dec}	Retailer's profit in decentralized structure
π_s^{dec}	Supplier's profit in decentralized structure
π_r^{co}	Retailer's profit in coordinated structure
π_s^{co}	Supplier's profit in coordinated structure
π_T	Total supply chain's profit

3.1. Centralized structure

In a centralized structure, the decision variables are optimized together to maximize the whole supply chain profit. This structure integrates the supply chain members and enables them to act as a unified entity. The profit function of the supply chain in the centralized structure can be expressed as follows:

$$Max\ \pi_T=(p-c)(a-bp+ue+to)-n\frac{e^2}{2}-E\frac{o^2}{2} \tag{1}$$

S.T.:

$$p,c,a,b,u,e,t,o,n,E \geq 0 \tag{2}$$

Taking the first derivative of the above function with respect to the variables e, p and o, we have:

$$\frac{\partial \pi_T}{\partial e}=c-en+pu \tag{3}$$

$$\frac{\partial \pi_T}{\partial p} = bc + (a - bp + eu + ot) - bp \tag{4}$$

$$\frac{\partial \pi_T}{\partial o} = -\ t(c - w) + pt + Eo - tw \tag{5}$$

By setting the above relations equal to zero and solving the system of three equations - three unknowns, we determine the optimal values of the decision variables in this system:

$$e_c^* = -\frac{u\left(Ea\ -\ Ebc\right)}{nt^2 +\ Eu^2 -\ 2Ebn} \tag{6}$$

$$p_c^* = \frac{cnt^2\ +\ Ecu^2\ -\ Ean\ -\ Ebcn}{nt^2\ +\ Eu^2\ -\ 2Ebn} \tag{7}$$

$$o_c^* = -\frac{t\left(an\ -\ bcn\right)}{nt^2\ +\ Eu^2\ -\ 2Ebn} \tag{8}$$

In order to check the optimality conditions of the values obtained for the decision variables, we obtain the Hessian matrix of the profit function of the entire supply chain with respect to the three decision variables as follows:

$$Hessian\left(\pi_T^c; e, p, o\right) = \begin{bmatrix} \frac{\partial \pi_T^2}{\partial e^2} & \frac{\partial \pi_T^2}{\partial e \partial p} & \frac{\partial \pi_T^2}{\partial e \partial o} \\ \frac{\partial \pi_T^2}{\partial p \partial e} & \frac{\partial \pi_T^2}{\partial p^2} & \frac{\partial \pi_T^2}{\partial p \partial o} \\ \frac{\partial \pi_T^2}{\partial o \partial e} & \frac{\partial \pi_T^2}{\partial o \partial p} & \frac{\partial \pi_T^2}{\partial o^2} \end{bmatrix} =$$

$$= \begin{bmatrix} -n & u & 0 \\ u & -2b & t \\ 0 & t & -E \end{bmatrix} \tag{9}$$

By checking the sign of the minor determinants of the above Hessian matrix, we will have:

$$\left|A_{11}\right| = a_{11} = -\,n < 0 \tag{10}$$

$$\left|A_{22}\right| = \begin{vmatrix} -n & u \\ u & -2b \end{vmatrix} = 2nb - u^2 > 0 \rightarrow 2nb > u^2 \tag{11}$$

$$\left|A_{33}\right| = \begin{vmatrix} -n & u & 0 \\ u & -2b & t \\ 0 & t & -E \end{vmatrix} =$$
$$= nt^2 + Eu^2 - 2Ebn < 0 \rightarrow nt^2 + Eu^2 < 2Ebn \tag{12}$$

If the profit function of the entire supply chain is concave with respect to the above three decision variables, the calculated Hessian matrix must be negative definite and this will be possible if the odd minors are negative and the even minors are positive. Therefore, in a summary we will have:

- $-n < 0$,
- $2nb > u^2$,
- $nt^2 + Eu^2 < 2Ebn.$

Then the optimal solution (e_c^*, p_c^*, o_c^*) is an extremum of the maximum type for the profit function of the entire supply chain. Considering that n is always positive, the first condition is always true, but the second and third conditions are true under specific conditions.

3.2. Decentralized structure

In this structure, the retailer and the supplier perform some activities to promote the level of demand. Exerting sales effort by the retailer and performing activities to improve the green quality level of the product by the supplier, they seek to promote the level of demand and, consequently, their sales.

The retailer's profit function in this case is formulated as follows:

$$Max\,\pi_r^{dec} = \left(p - w\right)\left(a - bp + ue + to\right) - n\frac{e^2}{2} \tag{13}$$

S.T.:

$$p, c, a, b, u, e, t, o, n, E \geq 0 \tag{14}$$

And the supplier’s profit function is formulated as follows:

$$Max\,\pi_s^{dec} = (w - c)\left(a - bp + ue + to\right) - E\frac{o^2}{2} \tag{15}$$

S.T.:

$$p, c, a, b, u, e, t, o, n, E \geq 0 \tag{16}$$

Taking the first derivative of the profit function of the retailer with respect to the variables e and p and the profit function of the supplier with respect to o, we have:

$$\frac{\partial \pi_r^{dec}}{\partial e} = u\left(p\ -\ w\right)\ -\ en \tag{17}$$

$$\frac{\partial \pi_r^{dec}}{\partial p} = a - b\,p + e\,u + o\,t - b\,(p - w) \tag{18}$$

$$\frac{\partial \pi_s^{dec}}{\partial o} = -\,Eo - t\,(c - w) \tag{19}$$

By setting the above relations equal to zero and solving the system of three equations - three unknowns, we determine the optimal values of the decision variables in this system:

$$e_{dec}^{*} = \frac{u\left(t^2(w - c) + E(a - b\,w)\right)}{E\left(2bn - u^2\right)} \tag{20}$$

$$p_{dec}^{*} = \frac{n\,t^2(w - c) + Ea\,n + E\,w\,(b\,n - u^2)}{E\left(2b\,n - u^2\right)} \tag{21}$$

$$o_{dec}^{*} = \frac{t\,(w - c)}{E} \tag{22}$$

In order to check the optimal conditions of the values obtained for the decision variables, we calculate the Hessian matrix of the retailer's profit function with respect to the e and p variables, as well as the second derivative of the supplier's profit function with respect to the o variable as follows:

$$Hessian\left(\pi_r^{dec}; e, p\right) = \begin{bmatrix} \frac{\partial^2 \pi_r^{dec}}{\partial e^2} & \frac{\partial^2 \pi_r^{dec}}{\partial e\,\partial p} \\ \frac{\partial^2 \pi_r^{dec}}{\partial p\,\partial e} & \frac{\partial^2 \pi_r^{dec}}{\partial p^2} \end{bmatrix} = \begin{bmatrix} -n & u \\ u & -2b \end{bmatrix} \tag{23}$$

$$\left|A_{11}\right| = a_{11} = -\,n < 0 \tag{24}$$

$$\left|A_{22}\right| = \begin{vmatrix} -n & u \\ u & -2b \end{vmatrix} = 2n\,b - u^2 > 0 \rightarrow 2n\,b > u^2 \tag{25}$$

If the retailer's profit function is concave with respect to the two decision variables e and p, the calculated Hessian matrix must be negative definite and this will be possible if the odd minors are negative and the even minors are positive. Therefore, in a summary we will have:

- $-n < 0$,
- $2n\,b > u^2$

then the obtained optimal solution is an extremum of the maximum type for the retailer's profit function. Considering that n is always positive, the first condition is always true, but the second condition is true under specific conditions.

In order to check the optimality of O^*_{dec}, we obtain the second derivative of the supplier's profit function with respect to o as follows:

$$\frac{\partial^2 \pi_s^{dec}}{\partial o^2} = -\,E < 0 \tag{26}$$

As a result, optimal conditions are always true.

Theorem 1: The changes of p^*_{dec}, e^*_{dec} and O^*_{dec} are incremental with respect to *CSC* if $2nb>u^2$

Proof: By taking derivatives of p^*_{dec}, e^*_{dec} and O^*_{dec} with respect to *CSC*, it is proven p^*_{dec}, e^*_{dec} and O^*_{dec} are increasing with respect to *CSC* if $2nb>u^2$. The mentioned derivatives are presented in the Appendix 1.

3.3. Supply chain coordination using the HCRSC

Based on (Cachon, 2003), revenue sharing contracts are developed in order to coordinate the supply chain by developing appropriate conditions and mechanisms for revenue sharing among supply chain members. In this contract, which is proposed by the supplier to the retailer, the supplier participates in a part of the retailer's revenue in exchange for reducing the wholesale price. In this article, by presenting a modified and hybrid contract, called the hybrid cost and revenue sharing contract, the development of a suitable mechanism for profit sharing among the members has been discussed. To achieve such coordination, the contract must specify in very clear terms how the profit will be earned, measured, and distributed. Therefore, the profit function of the retailer under the hybrid cost and revenue sharing contract will be as follows:

$$Max\ \pi_r^{co} = (\varphi p - w)(a - bp + ue + to) - \varphi\, n\,\frac{e^2}{2} - \varphi\, E\,\frac{o^2}{2} \tag{27}$$

S.T.:

$$0 \le \varphi \le 1 \tag{28}$$

$$p, c, a, b, u, e, t, o, n, E \ge 0 \tag{29}$$

And the supplier's profit function is formulated as follows:

$$Max\ \pi_s^{co} = (1 - \varphi)p\left(a - bp + u\,e + t\,o\right) + \\ + (w - c)\left(a - bp + u\,e + t\,o\right) - \\ - (1 - \varphi)n\,\frac{e^2}{2} - (1 - \varphi)E\,\frac{o^2}{2} \tag{30}$$

S.T.:

$$0 \leq \varphi \leq 1 \quad (31)$$

$$p, c, a, b, u, e, t, o, n, E \geq 0 \quad (32)$$

φ is the parameter of the HCRSC and actually determines the retailer's share of the total profit of the supply chain in the coordinated structure. In the following, more explanations about this parameter will be provided. In this contract, the supplier undertakes to determine the selling price of his product as $w=\varphi c$ and in exchange for this discount, the retailer gives part of her income to the supplier at the end of the sales period. In fact, in this contract, both members of the supply chain share their revenues and costs. Taking the first derivative of the profit function of the retailer with respect to the variables e and p and the profit function of the supplier with respect to o, we have:

$$\frac{\partial \pi_r^{co}}{\partial e} = p\varphi u - en\varphi - uw \quad (33)$$

$$\frac{\partial \pi_r^{co}}{\partial p} = bw + \varphi(a - bp + eu + ot) - bp\varphi \quad (34)$$

$$\frac{\partial \pi_s^{co}}{\partial o} = Eo(\varphi - 1) - pt(\varphi - 1) - t(c - w) \quad (35)$$

By setting the above relations equal to zero and solving the system of three equations - three unknowns, we determine the optimal values of the decision variables in this system:

$$e_{co}^* = -\frac{u\begin{pmatrix} t^2w - c\varphi t^2 + Ea\varphi - Ebw \\ - Ea\varphi^2 + Eb\varphi w \end{pmatrix}}{\varphi^2(1-\varphi)(nt^2 + Eu^2 - 2Ebn)} \quad (36)$$

$$p_{co}^* = \frac{\begin{pmatrix} E\varphi u^2w - Ean\varphi^2 \\ - Ebn\varphi w - E\varphi^2u^2w \\ + cn\varphi^2t^2 - n\varphi^2t^2w \\ + Ean\varphi^3 + Ebn\varphi^2w \end{pmatrix}}{\varphi\begin{pmatrix} n\varphi t^2 + E\varphi u^2 - E\varphi^2u^2 \\ - n\varphi^2t^2 - 2Ebn\varphi + 2Eb\varphi^2 \end{pmatrix}} \quad (37)$$

$$o_{co}^* = -\frac{t\begin{pmatrix} c\varphi^2u^2 - \varphi u^2w + an\varphi^2 \\ + bn\varphi w - an\varphi^3 \\ - 2bcn\varphi^2 + bn\varphi^2w \end{pmatrix}}{\varphi\begin{pmatrix} n\varphi t^2 + E\varphi u^2 - E\varphi^2u^2 \\ - n\varphi^2t^2 - 2Ebn\varphi + 2Ebn\varphi^2 \end{pmatrix}} \quad (38)$$

In order to check the optimal conditions of the values obtained for the decision variables, we calculate the Hessian matrix of the retailer's profit function with respect to the e and p variables, as well as the second derivative of the supplier's profit function with respect to the o variable as follows:

$$Hessian(\pi_r^{co}; e, p) = \begin{bmatrix} \frac{\partial^2 \pi_r^{co}}{\partial e^2} & \frac{\partial^2 \pi_r^{co}}{\partial e \partial p} \\ \frac{\partial^2 \pi_r^{co}}{\partial p \partial e} & \frac{\partial^2 \pi_r^{co}}{\partial p^2} \end{bmatrix} = \begin{bmatrix} -\varphi n & \varphi u \\ \varphi u & -\varphi 2b \end{bmatrix} \quad (39)$$

$$|A_{11}| = a_{11} = -\varphi n < 0 \quad (40)$$

$$|A_{22}| = \begin{vmatrix} -\varphi n & \varphi u \\ \varphi u & -\varphi 2b \end{vmatrix}$$
$$= 2\varphi^2 nb - \varphi^2 u^2 > 0 \to 2nb > u^2 \quad (41)$$

If the retailer's profit function is concave with respect to the two decision variables e and p, the calculated Hessian matrix must be negative definite and this will be possible if the odd minors are negative and the even minors are positive. Therefore, in a summary we will have:

- $-\varphi n < 0$,
- $2nb > u^2$.

Then the obtained optimal solution is a maximum point for the retailer's profit function. Considering that n is always positive, the first condition is always true, but the second condition is true under certain conditions.

In order to check the optimality of o_{co}^*, we calculate the second derivative of the supplier's profit function with respect to o as follows:

$$\frac{\partial^2 \pi_s^{co}}{\partial o^2} = -\varphi E < 0 \quad (42)$$

As a result, optimal conditions are always true.

In order to make the analysis easy, we summarize and compare the obtained models in a.

3.3.1. Conditions of members participation in the contract

In order to accept the contract by the parties, the contract parameter (φ) should be adjusted so that both members of the supply chain are willing to participate in the contract. For this purpose,

Table 2. Summary of the optimal effort levels under different scenarios.

	Centralized Structure	Decentralized structure	Coordinated Structure
e	$-\frac{u(Ea-Ebc)}{nt^2+Eu^2-2Ebn}$	$\frac{u\left(t^2(w-c)+E(a-bw)\right)}{E\left(2bn-u^2\right)}$	$-\frac{u\begin{pmatrix} t^2w-c\varphi t^2+Ea\varphi \\ -Ebw-Ea\varphi^2+Eb\varphi w\end{pmatrix}}{\varphi^2(1-\varphi)(nt^2+Eu^2-2Ebn)}$
p	$\frac{cnt^2+Ecu^2-Ean-Ebcn}{nt^2+Eu^2-2Ebn}$	$\frac{\begin{pmatrix} nt^2w-cnt^2+Ean \\ -Eu^2w+Ebnw\end{pmatrix}}{E\left(2bn-u^2\right)}$	$\frac{\begin{pmatrix} E\varphi u^2w-Ean\varphi^2-Ebn\varphi w-E\varphi^2u^2w \\ +cn\varphi^2t^2-n\varphi^2t^2w+Ean\varphi^3+Ebn\varphi^2w\end{pmatrix}}{\varphi\begin{pmatrix} n\varphi t^2+E\varphi u^2-E\varphi^2u^2 \\ -n\varphi^2t^2-2Ebn\varphi+2Eb\varphi^2\end{pmatrix}}$
o	$-\frac{t(an-bcn)}{nt^2+Eu^2-2Ebn}$	$\frac{t(w-c)}{E}$	$-\frac{t\begin{pmatrix} c\varphi^2u^2-\varphi u^2w+an\varphi^2+bn\varphi w \\ -an\varphi^3-2bcn\varphi^2+bn\varphi^2w\end{pmatrix}}{\varphi\begin{pmatrix} n\varphi t^2+E\varphi u^2-E\varphi^2u^2 \\ -n\varphi^2t^2-2Ebn\varphi+2Ebn\varphi^2\end{pmatrix}}$

the profit of both members after participating in the contract should be higher than their profit under the decentralized decision model. In other words $\pi_r^{co}(e_c^*, p_c^*, o_c^*) \geq \pi_r^{dec}(e_{dec}^*, p_{dec}^*, o_{dec}^*)$ and $\pi_s^{co}(e_c^*, p_c^*, o_c^*) \geq \pi_s^{dec}(e_{dec}^*, p_{dec}^*, o_{dec}^*)$. In this situation, the parties will be willing to accept the contract and make decisions under coordinated conditions. Therefore, the values of φ_{min} and φ_{max} should be determined in such a way that there is sufficient motivation for the parties to participate in the contract.

Hence, the condition of contract acceptance by the retailer is as follows:

$$\pi_r^{co}\left(e_c^*, p_c^*, o_c^*\right) \geq \pi_r^{dec}\left(e_{dec}^*, p_{dec}^*, o_{dec}^*\right) \tag{43}$$

Solving the above inequality, the value of φ_{min} is equal to:

$$\varphi_{min} = \frac{n\frac{(e_{dec}^*)^2}{2} - \left(p_{dec}^* - w_{dec}\right)B}{E\frac{(o_{co}^*)^2}{2} + n\frac{(e_{co}^*)^2}{2} - \left(p_{co}^* - c\right)A} \tag{44}$$

Such that

$$A = a - bp_{co}^* + ue_{co}^* + to_{co}^* \tag{45}$$

$$B = \left(a - bp_{dec}^* + ue_{dec}^* + to_{dec}^*\right) \tag{46}$$

Therefore, if we have $\varphi > \varphi_{min}$, the retailer will be willing to accept the contract.

In order to accept the contract by the supplier, φ_{max} should be determined in such a way that it is profitable for the supplier to accept the contract, in other words:

$$\pi_s^{co}\left(e_c^*, p_c^*, o_c^*\right) \geq \pi_s^{dec}\left(e_{dec}^*, p_{dec}^*, o_{dec}^*\right) \tag{47}$$

Solving the above inequality, the value of φ_{max} is equal to:

$$\varphi_{max} = -\frac{\begin{pmatrix} \left(c - w_{dec}\right)B - E\frac{(o_{co}^*)^2}{2} \\ +E\frac{(o_{dec}^*)^2}{2} - n\frac{(e_{co}^*)^2}{2} + \left(p_{co}^* - c\right)A \end{pmatrix}}{E\frac{(o_{co}^*)^2}{2} + n\frac{(e_{co}^*)^2}{2} - \left(p_{co}^* - c\right)A} \tag{48}$$

Such that

$$A = a - bp_{co}^* + ue_{co}^* + to_{co}^* \tag{49}$$

$$B = \left(a - bp_{dec}^* + ue_{dec}^* + to_{dec}^*\right) \tag{50}$$

Therefore, if we have $\varphi < \varphi_{max}$, the supplier will be willing to accept the contract.

3.3.2. *Examining the applicability of the contract; checking whether the interval $[\varphi_{min}, \varphi_{max}]$ is not zero*

To prove the applicability of the contract, it should be proved that the interval $[\varphi_{min}, \varphi_{max}]$ is not zero. In other words, there is always a φ such that $\varphi_{min}<\varphi<\varphi_{max}$ and the members of the supply chain are willing to accept the contract based on it.

Theorem 2: $\varphi_{min}>\varphi_{max}$.

Proof: The non-nullity check of the interval $[\varphi_{min}, \varphi_{max}]$ is presented in Appendix 2.

3.3.3. *3.3.3. Determining the value of the contract parameter φ and the profit-sharing strategy (Nash bargaining problem)*

If the contract parameter is set to φ_{max}, the total profit from the coordination will be obtained by the retailer, and if the contract parameter is set to φ_{min}, then the total profit from the coordination will be obtained by the supplier. In other words, the profit obtained from the establishment of coordination is distributed between two members in a specific ratio that depends on φ.

The retailer can only use the benefits of the contract if she adopts e^*_{co} and p^*_{co} values instead of e^*_{dec} and p^*_{dec}.

After the implementation of the contract, the profit functions of each member of the supply chain can be rewritten as follows:

$$\pi_r^{co} = (\varphi p - w)(a - bp + ue + to) - \varphi n\frac{e^2}{2} - \varphi E\frac{o^2}{2} \quad (55)$$

Putting $w=\varphi c$ as an agreement in the contract, we have:

$$\xrightarrow{w=\varphi c} \pi_r^{co} = \varphi(p-c)(a - bp + ue + to) - \varphi n\frac{e^2}{2} - \varphi E\frac{o^2}{2} \quad (56)$$

Therefore, the profit function of the retailer can be rewritten as a coefficient of the profit function of the entire supply chain:

$$\pi_r^{co} = \varphi \pi_T \quad (57)$$

And for the supplier's profit function we have:

$$\pi_s^{co} = (1-\varphi)p(a - bp + ue + to) + (w-c)(a - bp + ue + to) - (1-\varphi)n\frac{e^2}{2} - (1-\varphi)E\frac{o^2}{2}$$

$$\xrightarrow{w=\varphi c} \pi_s^{co} = (1-\varphi)p(a - bp + ue + to) + (1-\varphi)c(a - bp + ue + to) - (1-\varphi)n\frac{e^2}{2} - (1-\varphi)E\frac{o^2}{2} \quad (58)$$

Therefore, the profit function of the supplier can be rewritten as a coefficient of the profit function of the entire supply chain:

$$\pi_s^{co} = (1-\varphi)\pi_T \quad (59)$$

and we have:

$$\pi_r^{co} + \pi_s^{co} = \varphi \pi_T + (1-\varphi)\pi_T = \pi_T \quad (60)$$

Therefore, by increasing the value of φ, which is determined based on the bargaining power of the parties, the amount of the retailer's share of the total profit of the supply chain increases and the amount of the supplier's share of this profit decreases.

4. Numerical Analysis

In this section, we will examine the behavior of the developed models as well as the efficiency of the HCRSC using numerical examples, analyzing the sensitivity of functions and variables to some parameters. Also, the way of sharing the income and the numerical terms of acceptance of the contract by the parties are analyzed quantitatively.

With the conducted numerical studies, it is determined how the values of the decision variables change in the transition from the decentralized decision-making structure to the coordinated decision-making structure, and also how the profit functions of the supply chain members change. Changing the decision-making structure from the traditional mode to the centralized mode, the profit of the supply chain increases. However, the retailer's profit in the case of centralized decision-making will be less than his profit before decentralized decision-making, which will seriously challenge his commitment to centralized decision-making. We will further analyze this issue.

4.1. Numerical example

In order to further investigate the efficiency of the developed models, in this section we present a numerical example with three data sets. In this analysis, the values of the decision variables in different situations and conditions and as a result their effects on the profit functions of the supply chain and each member are examined independently. In this example, it is checked how each member behaves in each system (centralized, decentralized and coordinated).

On the right side of the presented table, coordinated, decentralized and centralized structures are presented, and on the left side of this table, the data of three examples are available.

One of the justifications for entering into a contract and developing coordination systems is that in some cases of centralized systems, the profit of one member is lower than the profit of that member in a decentralized state, and this makes that member of the supply chain not willing to cooperate in the form of a centralized system. The second example is an example for this condition. In the second example, the profit of the supplier in the centralized system is equal to 88.67, while the profit of this member of the supply chain in the decentralized system is equal to 269.70. Also, in order to check the efficiency of the HCRSC, it is possible to pay attention to the amount of improvements made in the profit of these members under the decision making in the coordinated system compared to the decision making in the decentralized system.

In the first example, the retailer's profit will increase by 4% and the supplier's profit by 59% compared to the centralized case. The lower improvement made in the retailer's profit compared to the improvement made in the supplier's profit can be justified according to the value calculated for φ_{min}=0.3849. Because by moving from φ_{min} to φ_{max}, the retailer's profit will increase and the supplier's profit will decrease. Also, in order to check the efficiency of the HCRSC, it is possible to pay attention to the amount of improvements made in the profit of these members under the decision making in the coordinated system compared to the decision making in the decentralized system. In the second and third examples, the profit improvement values in the coordinated system compared to the decentralized system can be seen for both members of the supply chain. Therefore, with the numerical analysis, it can be concluded that in the case of acceptance of the contract by the parties, there will always be an improvement in the profit functions of each member independently. Of course, the important point is that the amount of improvement created is a function of the contract parameter, φ, which can be determined based on the bargaining power of the members.

4.2. Sensitivity Analysis

4.2.1. Sensitivity analysis to the contract parameter

As previously explained, the profit-sharing mechanism in the coordinated system is developed based on the contract parameter, denoted as φ, which is determined based on the bargaining powers. By

Table 3. Three numerical example datasets.

Examples	a	φ	w	wco	b	E	n	t	u	c
EX1	100	0.4	15	4	5	10	12	5	5	10
EX2	200	0.5	13	6	7	12	16	6	7	12
EX3	300	0.6	18	8.4	10	18	20	7	6	14

Table 4. Optimal decision variables and profits.

Examples	p	e	o	Retailer's Profit	Supplier's Profit	Supply chain's profit	STRUCTURE
EX1	19.23	3.85	4.62	92.31	138.46	230.7692	COORDINATED
EX2	26.61	6.39	7.31	423.81	423.81	847.62	
EX3	24.34	3.10	4.02	496.20	330.80	826.9921	
EX1	19.74	1.97	2.50	88.82	87.00	175.82	DECENTRALIZED
EX2	25.78	3.84	2.50	421.30	269.70	643.62	
EX3	25.19	2.16	1.56	470.88	266.18	737.06	
EX1	19.23	3.85	4.62	106.51	124.26	230.7692	CENTRALIZED
EX2	26.61	6.39	7.31	758.96	88.66	847.62	
EX3	24.34	3.10	4.02	558.95	268.04	826.9921	

increasing the value of φ, the retailer gains more profit from the contract. Conversely, by increasing φ, the supplier receives less benefit and therefore seeks to reduce the value of this parameter. Figure 1 illustrates the changes in the profit functions with varying values of the contract parameter, as well as the approach to profit sharing within the supply chain.

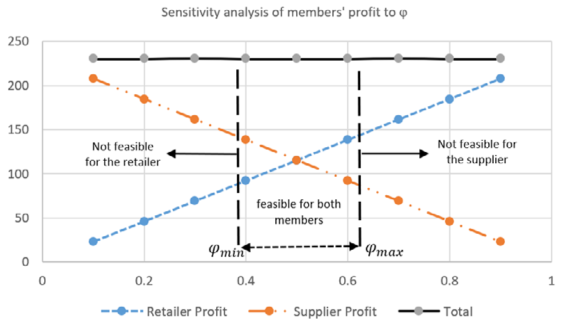

Figure 1. Feasible intervals for both members.

However, as explained in Section 3.3.1, each member of the supply chain considers a specified limit, denoted as φ, for accepting the contract. In Figure 1, if the value of the contract parameter is less than φ_{min}= 0.3849, the retailer will not agree to accept the contract and participate in it. This is because, at φ=0.3849, the retailer's profit will be precisely the same as their profit in the decentralized system. If the value of the contract parameter is less than this limit, the retailer's profit will be less than their profit in the decentralized structure. On the other hand, if the value of the contract parameter exceeds 0.623, the supplier will not accept the contract. Therefore, the acceptance interval of the contract is within the range of [φ_{min}, φ_{max}]=[0.3849,0.623]. Choosing any φ within this range leads to coordination in the supply chain and increased profit for all parties compared to the decentralized system. It is worth noting that, as demonstrated by the presented equations, the profit of the entire supply chain remains constant for different values of φ.

4.2.2. Sensitivity analysis of profit functions to CSC level

The green quality coefficient in the final demand function, as denoted in the model with t, reflects the significance that customers place on environmental issues and the product's green quality. This highlights the level of consumer awareness regarding the product's environmental impact and related concerns.

In Figure 2, as anticipated, an increase in CSC increases in the profit of both members, and consequently, the profit of the entire supply chain increases. This is because an increase in this parameter corresponds to an increase in demand for a specific level of green quality, resulting in greater profitability for the supply chain. Furthermore, by comparing the two figures, it is evident that in the coordinated system, due to the retailer sharing part of the green quality costs, the supplier's profit raises more quickly with an increase in the green quality impact factor. Additionally, the total profit of the supply chain in the coordinated system (Figure on the right) exceeds the total profit of the supply chain in the decentralized system (Figure on the left).

4.2.3. Sensitivity analysis of the acceptance interval of the contract to CSC level

As outlined in equations 44 and 48, both φ_{min} and φ_{max} are dependent on the variable t, meaning that the effectiveness of the contract is influenced by changes in customers' sustainability consciousness (CSC). To assess the impact of CSC on contract efficiency, we examine how φ_{min} and φ_{max} change as the level of CSC increases. This analysis allows us to better understand how changes in CSC affect the optimal acceptance interval of the contract and its overall effectiveness.

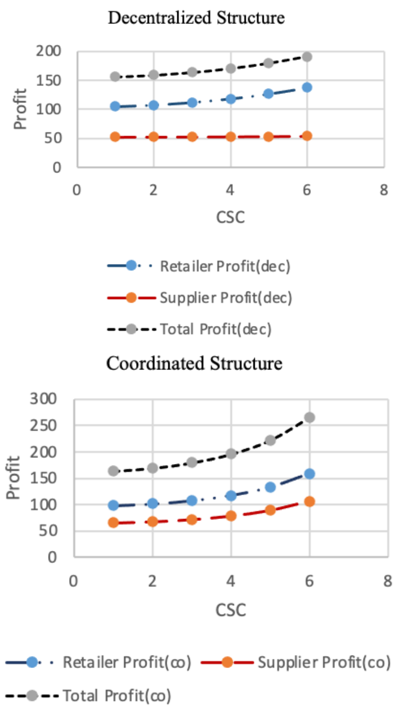

Figure 2. Impact of profit functions to CSC level.

[Figure 3](#) demonstrates that as the level of CSC increases, the acceptance range of the contract parameter widens, resulting in improved contract efficiency. This finding highlights the importance of considering CSC when determining the optimal acceptance interval of the contract to ensure its effectiveness in promoting sustainable practices and enhancing supply chain coordination.

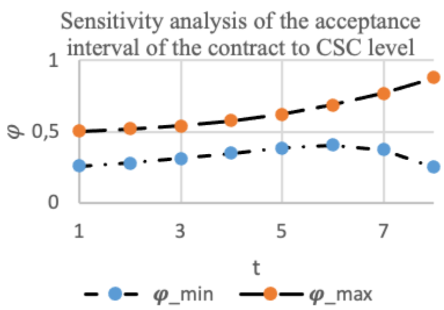

Figure 3. Impact of CSC on acceptance interval φ.

4.2.4. Sensitivity analysis of the acceptance interval of the contract to sales effort elasticity coefficient in demand

In addition to the impact of CSC, the sensitivity coefficient of sales effort in the demand function can also affect the efficiency of the contract. As illustrated in Figure 4, an increase in the value of u results in a wider acceptance interval for the contract, similar to the behavior observed in Figure 3. Consequently, if the advertising sensitivity coefficient in the demand function increases, the acceptance period of the contract widens, leading to increased efficiency of the contract. This highlights the importance of considering multiple factors when determining the optimal acceptance interval of the contract to achieve supply chain coordination and maximize profits.

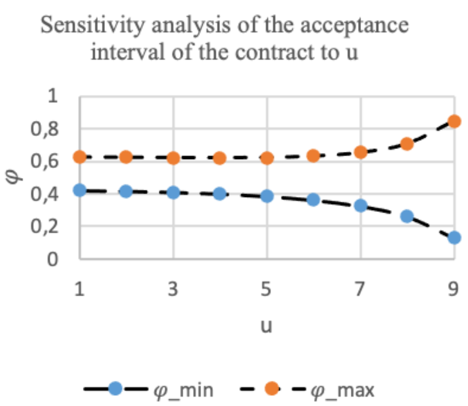

Figure 4. Impact of u on acceptance interval of φ.

4.2.5. Sensitivity analysis to sales effort and green quality cost coefficients

In this section, we will investigate the influence of sales effort and green quality cost coefficients on decision-making in the supply chain.

4.2.5.1. Sensitivity analysis to the green quality cost coefficient

As the cost coefficients rise, the values of these two variables are likely to decrease, which in turn will lead to a decrease in the profit of individual supply chain members and ultimately, the overall profit of the entire supply chain.

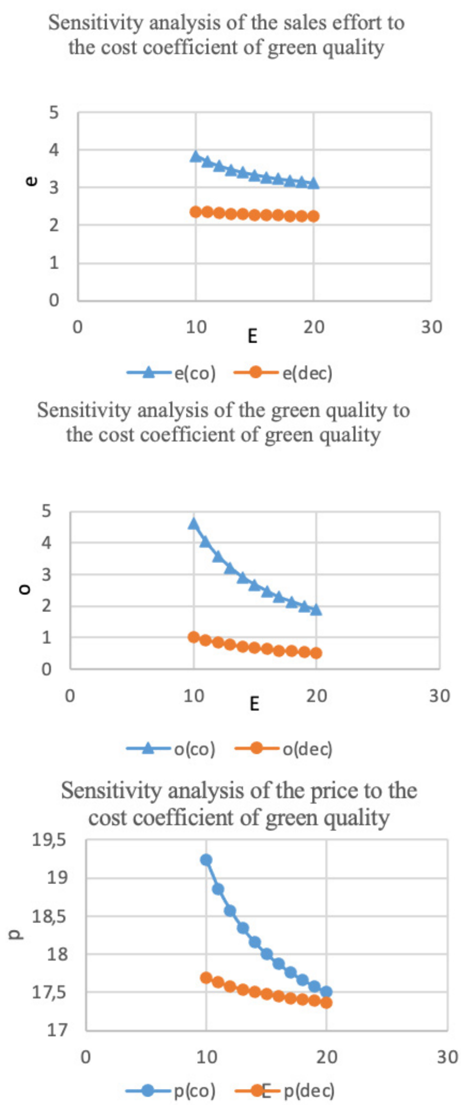

Figure 6. Sensitivity analysis of decision variables to green quality cost coefficient.

As the green quality cost coefficient increases, the values of sales effort, price and green quality decrease. This decrease occurs at a faster rate in coordinated system.

Also, as the decision variables decrease in value, the changes in profit for supply chain members are as follows:

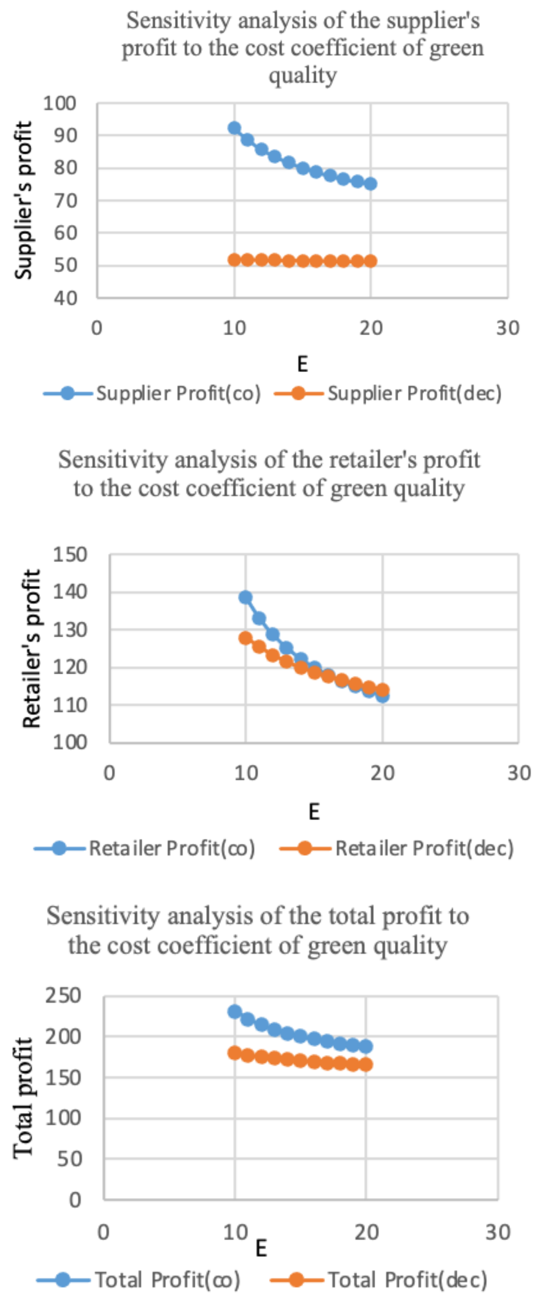

Figure 7. Sensitivity analysis of profit functions to green quality cost coefficient.

Furthermore, it is clear from the figures above that the profit values of members in the coordinated system are higher than those in the decentralized system. Also, it is imperative to acknowledge the pivotal role of green quality costs borne by retailers within the coordinated system. As the significance of these costs escalates, retailer is compelled to allocate substantial resources towards testing and certification procedures, thereby elevating its financial outlay. Consequently, this heightened expenditure directly impacts retailer's profit, rendering them comparatively lower in the context of the coordinated system vis-à-vis the decentralized approach.

4.2.5.2. Sensitivity analysis to the sales effort cost coefficient

As we saw earlier, the relationship between variables and functions changes when the green quality cost coefficient increases. Similarly, as the sales effort cost coefficient increases, the values of profit functions and variables in relation to sales effort cost coefficient will also decline (Figure 8).

In the decentralized system, the value of the green quality is completely independent of sales effort cost coefficient because $\partial o^*_{dec}/\partial n$ is equal to zero. In other cases, the changes of the variables are decreasing with the increase of sales effort cost coefficient. It is also obvious that all the decision variables in the coordinated system have higher values compared to the values of these variables in the decentralized system.

The changes in profit values in two coordinated and decentralized systems are shown in Figure 9.

Except for the profit function of the supplier in the decentralized system, the rest of the profit functions in the two decentralized and coordinated systems are decreasing.

4.2.6. Sensitivity Analysis of profit functions to decision variables

In Section 3, we thoroughly examined the optimality conditions of decision variables in profit functions. In this section, we will delve into the sensitivity of profit functions to decision variables and analyze their impact. It is evident that increasing or decreasing the values of decision variables should result in concave profit functions (Figure 5).

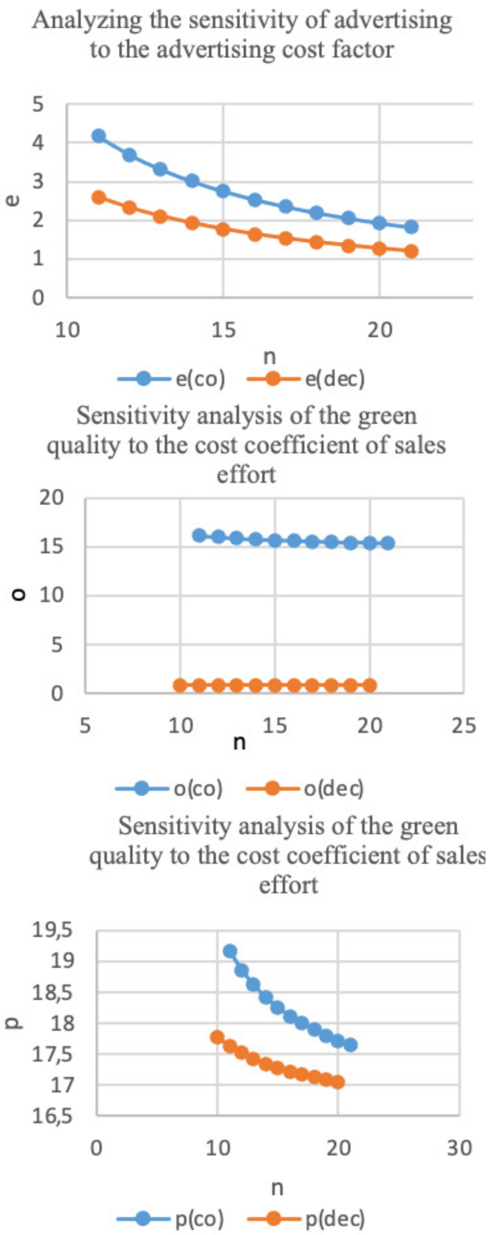

Figure 8. Sensitivity analysis of decision variables to sales effort cost coefficient.

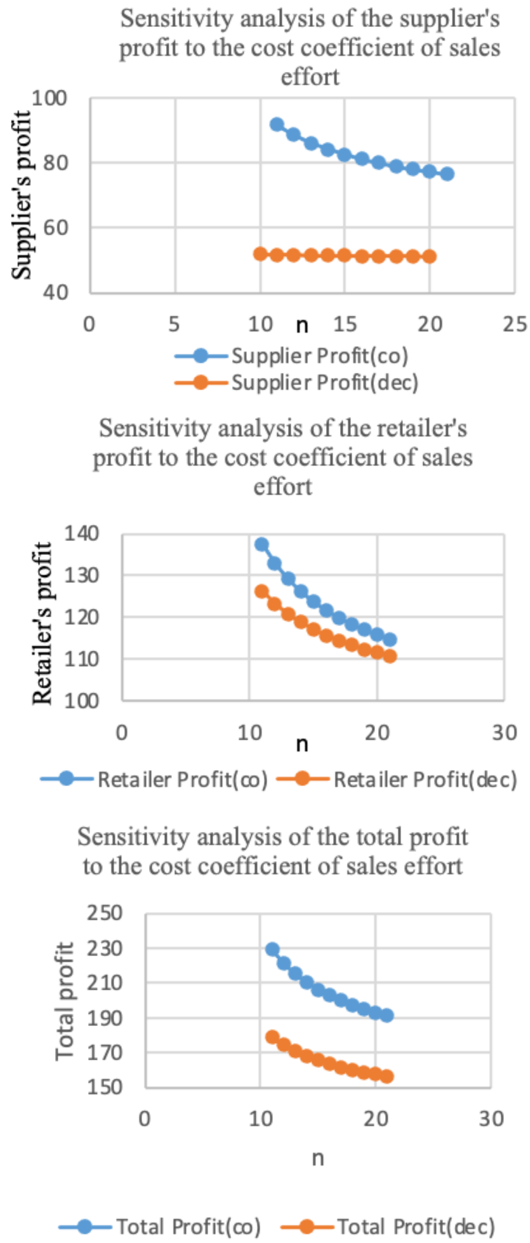

Figure 9. Sensitivity analysis of the profit functions to sales effort cost coefficient.

5. Managerial insights

First, the profits of both members of the supply chain with the coordinated structure and the HCRS contract were higher than those in the decentralized structure. The numerical analysis also showed that the sales efforts and green quality values would be higher in coordinated conditions than in decentralized conditions. Therefore, by adopting an HCRS contract, members can increase not only the satisfaction of the customers but also their profits.

Second, under the HCRS contract, in addition to the fact that the level of green quality and sales effort is higher, the price level is lower. In other words, by choosing a contract, chain members can increase the

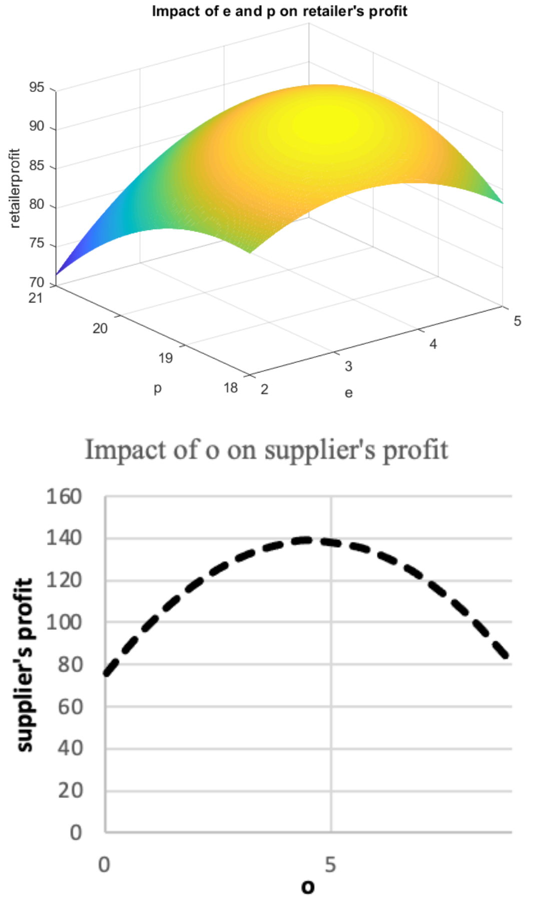

Figure 10. Impact of decision variables on the profit functions in coordinated structure.

responsiveness of the supply chain and reduce the price of the product. Both of these factors increase customer satisfaction.

Third, Although the profit of the entire supply chain is the same in the coordinated and centralized system, only by adopting the HCRS contract and under the coordinated approach, it is possible to distribute the appropriate profit. Otherwise, one of the members may not be willing to accept the cooperative approach under the centralized structure.

Finally, it was also shown in the analysis that the efficiency of the contract is much higher for higher values of CSC and sales effort coefficient. Therefore, the supply chain management can improve the efficiency of the contract by taking actions such as rolling out plans to persuade customers to care more the environment and also employing more effective advertising methods.

6. Conclusion

This research paper delves into optimization models concerning sales effort, pricing, and green quality variables across three distinct structures: centralized, decentralized, and coordinated. We commence by laying out the model framework and equations governing the centralized system. Following the determination of optimal values for decision variables within this framework, we proceed to scrutinize the optimality conditions associated with these variables. A similar analytical process is then carried out for decision variables within the decentralized structure. Throughout Section 3, we underscore the significance of centralized decision-making as the most effective and optimal approach for steering the entire supply chain. Nonetheless, inherent challenges within this structure, such as insufficient consolidation of ownership across supply chain entities and potential profit diminishment for individual members compared to decentralized operations, necessitate a strategic response. In response to these challenges, we introduce an optimization model for decision variables elucidated through the Harmonized Cost-Revenue Sharing (HCRS) contract. This contractual framework encompasses key aspects including contract specifications, optimal decision variable determinations, member acceptance criteria, contract efficacy, and the profit-sharing mechanism (known as the Nash bargaining problem). The implementation of a sensitivity analysis in this study yields actionable managerial insights with practical implications for industry practitioners. Our analysis distinctly reveals that heightened consumer awareness regarding sustainability matters, denoted by the Customer Sustainability Consciousness (CSC) level, correlates with increased quality variables and sales efforts. Consequently, this uptick translates to enhanced profitability for every supply chain participant as well as the collective chain itself. To facilitate a comprehensive understanding of the variables within the profit functions, the paper integrates graphical representations that showcase the evolution of decision variables and their impacts on profit outcomes. These visual aids offer a lucid depiction of the profit functions, aiding in the visualization of how alterations in decision variables influence financial performance. However, while this study has provided valuable insights into the implementation of a hybrid cost and revenue sharing (HCRS) contract for promoting sustainability in supply chain operations, it is essential to acknowledge certain limitations that may impact the generalizability and applicability of the findings. One key limitation is the simplifying assumptions made in the modeling approach, such as assuming deterministic demand and a single sales period. In reality, supply chain dynamics are often subject to uncertainty, seasonality, and external market influences, which could affect the performance of the proposed coordination mechanisms. Future research should explore more realistic scenarios that consider stochastic demand patterns and multi-period decision-making to enhance the robustness of the model. Additionally, the numerical examples presented in this study are based on theoretical parameters and simplified scenarios. While these examples serve to illustrate the potential benefits of the HCRS contract, further validation through empirical data and real-world case studies would strengthen the credibility of the proposed model. Conducting field experiments or implementing the HCRS contract in a live supply chain setting could provide valuable insights into its practical feasibility and effectiveness. In conclusion, while this study contributes significantly to the understanding of supply chain coordination for sustainability, these limitations highlight the need for continued research efforts to refine the model, incorporate more realistic assumptions, and validate the findings through empirical studies to ensure the practical relevance and scalability of the proposed approach. Additionally, future research could explore the impact of dual-channel and multi-channel structures, particularly in cases where sales are made through online channels.

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Appendices:

Appendix 1: Examining the trend of changes in p^*_{dec}, e^*_{dec} and o^*_{dec} compared to CSC

$$\frac{\partial p_{dec}{}^*}{\partial t} = \frac{(2tn(w-c))}{E(-u^2+2bn)} \tag{61}$$

$$\frac{\partial e_{dec}{}^*}{\partial t} = \frac{(2tu(w-c))}{E(-u^2+2bn)} \tag{62}$$

$$\frac{\partial o_{dec}{}^*}{\partial t} = \frac{(w-c)}{E} \tag{63}$$

Since always $w>c$, if $2nb > u^2$, then $\partial p^*_{dec}/\partial t > 0$ and $\partial e^*_{dec}/\partial t > 0$, hence, p^*_{dec} and e^*_{dec} are incremental to CSC. Also, $\partial o^*_{dec}/\partial t > 0$ and o^*_{dec} is always incremental to CSC.

Appendix 2: Checking the effectiveness of the contract: checking whether the interval [φ_{min}, φ_{max}] is non-empty

The denominator of both fractions is equal. In order to remove denominators from both unequal sides, it should be checked whether the sign of the denominator of the fractions is positive or negative. Considering that $p^*_{co} > 0$ and $A \geq 0$:

- If $E\frac{(o^*_{co})^2}{2} + n\frac{(e^*_{co})^2}{2} > (p^*_{co} - c)A$, the denominator of the fraction is always positive.
- If $E\frac{(o^*_{co})^2}{2} + n\frac{(e^*_{co})^2}{2} < (p^*_{co} - c)A$, the denominator of the fraction is always negative.

Let's compare the forms of two fractions. If the denominator of two fractions is positive:

$$\varphi_{max} > \varphi_{min} \xrightarrow{Multiply\ by\ the\ denominator} -\left[(c - w_{dec})B - E\frac{(o^*_{co})^2}{2} + E\frac{(o^*_{dec})^2}{2} - n\frac{(e^*_{co})^2}{2} + (p^*_{co} - c)A\right] > n\frac{(e^*_{dec})^2}{2} - \left(p^*_{dec} - w_{dec}\right)B \rightarrow$$

$$\left[(w_{dec} - c)B + E\frac{(o^*_{co})^2}{2} - E\frac{(o^*_{dec})^2}{2} + n\frac{(e^*_{co})^2}{2} - (p^*_{co} - c)A\right] > n\frac{(e^*_{dec})^2}{2} - \left(p^*_{dec} - w_{dec}\right)B \tag{64}$$

We move expressions with a positive sign to one side of the inequality and expressions with a negative sign to the other side of the inequality:

$$\rightarrow \left[(w_{dec} - c)B + E\frac{(o^*_{co})^2}{2} + n\frac{(e^*_{co})^2}{2} + \left(p^*_{dec} - w_{dec}\right)B\right] > n\frac{(e^*_{dec})^2}{2} + E\frac{(o^*_{dec})^2}{2} + \left(p^*_{co} - c\right)A$$

$$\rightarrow \left[\left(p^*_{dec} - c\right)B + E\frac{(o^*_{co})^2}{2} + n\frac{(e^*_{co})^2}{2}\right] > n\frac{(e^*_{dec})^2}{2} + E\frac{(o^*_{dec})^2}{2} + \left(p^*_{co} - c\right)A \tag{65}$$

And if the denominator of two fractions is negative:

$$\varphi_{max} > \varphi_{min} \xrightarrow{Multiply\ by\ the\ denominator} -\left[(c - w_{dec})B - E\frac{(o^*_{co})^2}{2} + E\frac{(o^*_{dec})^2}{2} - n\frac{(e^*_{co})^2}{2} + (p^*_{co} - c)A\right] < n\frac{(e^*_{dec})^2}{2} - \left(p^*_{dec} - w_{dec}\right)B \rightarrow$$

$$\left[(w_{dec} - c)B + E\frac{(o^*_{co})^2}{2} - E\frac{(o^*_{dec})^2}{2} + n\frac{(e^*_{co})^2}{2} - (p^*_{co} - c)A\right] < n\frac{(e^*_{dec})^2}{2} - \left(p^*_{dec} - w_{dec}\right)B \tag{66}$$

Note: By multiplying the sides of an inequality by a negative number, the direction of the inequality changes.

$$\rightarrow \left[(w_{dec} - c)B + E\frac{(o_{co}^*)^2}{2} + n\frac{(e_{co}^*)^2}{2} + \left(p_{dec}^* - w_{dec}\right)B \right] < n\frac{(e_{dec}^*)^2}{2} + E\frac{(o_{dec}^*)^2}{2} + \left(p_{co}^* - c\right)A$$

$$\rightarrow \left[\left(p_{dec}^* - c\right)B + E\frac{(o_{co}^*)^2}{2} + n\frac{(e_{co}^*)^2}{2} \right] < n\frac{(e_{dec}^*)^2}{2} + E\frac{(o_{dec}^*)^2}{2} + \left(p_{co}^* - c\right)A \quad (67)$$

Therefore, if one of the following conditions is met, the interval $[\varphi_{min},\varphi_{max}]$ will not be zero:

- If $E\frac{(o_{co}^*)^2}{2} + n\frac{(e_{co}^*)^2}{2} > (p_{co}^* - c)A$,

  $$\left[\left(p_{dec}^* - c\right)B + E\frac{(o_{co}^*)^2}{2} + n\frac{(e_{co}^*)^2}{2} \right] > n\frac{(e_{dec}^*)^2}{2} + E\frac{(o_{dec}^*)^2}{2} + (p_{co}^* - c)A$$

  then $\varphi_{max} > \varphi_{min}$.

- If $E\frac{(o_{co}^*)^2}{2} + n\frac{(e_{co}^*)^2}{2} < (p_{co}^* - c)A$,

  $$\left[\left(p_{dec}^* - c\right)B + E\frac{(o_{co}^*)^2}{2} + n\frac{(e_{co}^*)^2}{2} \right] < n\frac{(e_{dec}^*)^2}{2} + E\frac{(o_{dec}^*)^2}{2} + (p_{co}^* - c)A$$

  then $\varphi_{max} > \varphi_{min}$.

Data Availability

Data sharing is not applicable to this article as no new data were created or analyzed in this study.

Authors contribution

Mahdi Nakhaeinejad is responsible for the conceptualization of the study, data collection process, performed analyses, and interpreted the results, ensuring the accuracy and reliability of the findings.

Abolfazl Dehghan was responsible for the conceptualization of the study, designing the research methodology, and developing the experimental framework.

Nayerh Sadat Vaziri conducted the data collection process, performed statistical analyses, and interpreted the results, ensuring the accuracy and reliability of the findings.