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RESEARCH ARTICLE

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Selection of the level of vibration signal decomposition and mother wavelets to determine the level of failure severity in spur gearboxes

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Abstract

Spur gearboxes are an integral component in the operation of rotary machines. Hence, the early determination of the severity level of a failure is crucial. This manuscript delineates a methodology for selecting essential mother wavelets and filters from the wavelet transform (WT) to process the vibration signal within the time-frequency domain, aiming to ascertain the severity level of failures in spur gearboxes. Initially, information is garnered from the gearbox through vibration signals in the time domain, utilising six accelerometers. Subsequently, the signal is partitioned into various levels, and information from each level is extracted using diverse mother wavelets and their respective filters. The signal is segmented into sub-bands, from which the condition state is ascertained using an energy operator. After that, the appropriate level of wave decomposition is determined through ANOVA tests and post-hoc Tukey analyses, evaluating performance in failure classification via the Random Forest (RF) model. Upon establishing the decomposition level, the analysis proceeds to identify which mother wavelets and filters are most suitable for determining the severity level of different types of failure in spur gearboxes. Moreover, this study investigates the impact of sensor positioning and inclination on acquiring the vibration signal. This aspect is explored through factorial ANOVA tests and multiple comparisons of the data derived from the sensors. The RF classification model achieved exceedingly favourable results (accuracy >96% and AUC >98%), with minimal practical influence from the positioning and inclination of a sensor, thereby affirming the proposed methodology's suitability for this type of analysis.

KEYWORDS
classification models, feature extraction, fault severity, time-frequency domain, wavelet packets transform

1 INTRODUCTION

Spur gearboxes are deemed a pivotal element in the operation of rotary machinery, owing to their proficiency in power transmission within constrained spaces. Since gearboxes function under heavy load and high-velocity conditions over extended periods, their components are prone to failures. Thus, the early detection of potential failures is imperative to avert machine downtime or unforeseen maintenance activities.[1,2]

To extract salient information from engineering systems and their quintessential components, it becomes essential to examine many characteristics and deploy an array of analytical methods for scrutinising the mechanical signals emanating from these systems.[3] A mechanical signal frequently used for fault diagnosis is vibration, owing to the ease with which it can be acquired. However, this signal is inherently complex, encompassing stationary, non-stationary, and resonant components. Therefore, it requires selecting appropriate signal processing techniques through suitable data algorithms, which enable the extraction of its characteristics to diagnose the condition state of a machine.[4,5]

One approach to feature extraction from a signal involves its decomposition in the time-frequency domain and the subsequent extraction of statistical parameters pertaining to the machine's condition state. The wavelet transform (WT) represents a significant method for signal decomposition within the time-frequency domain, whereas the energy operator (EO) serves as a widely used condition indicator (CI) for the analysis of vibration signals in this domain.[6,7]

Various approaches exist regarding the detection and identification of a failure's severity level, with machine learning being the most employed due to its capability to detect or predict a failure based on the training process of the selected model.[8] Among several models or statistical classification algorithms, Random Forest (RF) is widely used to determine the severity level of a failure in gearboxes, as detailed in refs. [9, 10].

Scientific evidence indicates that addresses the analysis of vibration signals in the time-frequency domain via WT, feature extraction through CIs, and the employment of statistical classification models to ascertain the failure severity level in gearboxes, as elaborated in refs. [6, 11]. What remains unreported and constitutes one of the objectives of this work is a methodology for determining the appropriate decomposition level of the vibration signal in the time-frequency domain through the WT.

Previous research show that compares signal processing via the WT using different mother wavelets and filters to analyse vibration signals in the time-frequency domain, as presented in ref. [12], where the detection of cracked railway axles through vibration signal processing is addressed via a comparative study between the Daubechies 6 and Symlet 9 functions. What remains unreported is which mother wavelets and filters are most efficient for determining the severity level of a failure in spur gearboxes, starting from a combination of a vast array of mother wavelets and their filters for analysis.

Studies have been conducted, as reported by refs. [13, 14], determining the optimal position for placing a sensor on a gearbox. What needs to be documented is the determination of whether the position and inclination of a sensor significantly affect the acquisition of vibration signal information and, consequently, the accuracy of a classification model for the severity level of a failure.

Based on the preceding, the objectives outlined in this work were: (a) To propose a methodological approach for establishing the appropriate level of decomposition of the vibration signal in the time-frequency domain via the WT to determine the severity level of a failure in a spur gearbox. (b) To identify the most efficient mother wavelets and filters that allow for the measurement of the severity level of different types of failures using the RF classification model for a spur gearbox. (c) To ascertain whether the position and inclination of the sensor influence the feature extraction from the vibration signal.

This paper is structured as follows. Section 2 introduces the conceptual foundation underpinning this article. Section 3 details the experimental setup employed to obtain the vibration signals and outlines the proposed methodology for developing this work. Section 4 presents and analyses the results derived from the implemented methodology. Finally, Section 5 offers the conclusions.

2 THEORETICAL BACKGROUND

This section elucidates the conceptual underpinnings essential for the advancement of this work. Section 2.1 explicates the procedure for feature extraction from a signal. Section 2.2 details the classification model utilized to determine the severity level of failures. Subsequently, Section 2.3 elaborates on the resampling methods employed to ascertain the hyperparameters and assess accuracy in the classification models.

2.1 | Feature extraction

The extraction of features from dynamic signals is pivotal in identifying and diagnosing failures in mechanical equipment such as gearboxes. If conducted improperly, it will lead to erroneous conclusions.[4,15]

Feature extraction from a signal can be performed either in the time domain as developed by ref. [16], in the frequency domain by transforming the signal from the time domain using the Fast Fourier Transform (FFT) as presented in ref. [17], or in the time-frequency domain by transforming the signal from the time domain using the WT as supported in refs. [18, 19].

Feature extraction using the WT is utilized as it is a suitable method for analyzing non-stationary signals such as vibration. It is particularly apt as it diagnoses abnormal changes in signal data through a variable window that uses a time interval to analyze high and low-frequency components. Several studies apply the WT to decompose vibration signals, enhancing the performance of fault detection and diagnosis in rotary machinery as detailed in refs. [20, 21].

2.1.1 | Wavelet transform

The WT is a mathematical tool that has garnered significant interest across various engineering fields, especially in processing non-stationary vibration signals and fault diagnosis. The WT decomposes the signal into wavelets of various scales in the time domain with variable window sizes, unveiling the local structure in the time-frequency domain.[11]

The advantages of the WT include its effectiveness in extracting transient signal features across the spectrum without necessitating a dominant frequency band. The WT employs essential functions known as mother wavelets, which undergo modifications to perform the analysis. These modifications are managed by scaling and shifting parameters, providing more excellent time resolution for high-frequency components and greater frequency resolution for low-frequency components.[22,23]

The WT can be principally classified into Continuous Wavelet Transform (CWT), Discrete Wavelet Transform (DWT), and Wavelet Packet Transform (WPT).[24]

2.1.2 | Continuous wavelet transform (CWT)

The CWT of a signal in the time domain $x(t)$ can be performed by a convolution operation between the signal $x(t)$ and the complex conjugate of a family of wavelets using the Equation (1).

$$cwt(s,\tau) = \frac{1}{\sqrt{s}} \int x(t)\psi^*\left(\frac{t-\tau}{s}\right)dt \tag{1}$$

where, s represents the scaling parameter, and τ is the shift parameter. $\psi^*(\bullet)$ is the complex conjugate of the scaled and shifted wavelet function $\psi(\bullet)$.

2.1.3 | Discrete wavelet transform(DWT)

In order to avoid redundant information generated by the parameters s and τ due to their continuous change in the CWT and in order to increase computational efficiency, it can be discretised using the dyadic scales $s = 2$ and $\tau = \kappa 2^j$ using the Equation (2).

$$dwt(j,\kappa) = \frac{1}{\sqrt{2^j}} \int x(t)\psi^*\left(\frac{t-\kappa 2^j}{2^j}\right)dt \tag{2}$$

The DWT can be implemented using a low-pass filter $h(\kappa)$ and a high-pass filter $g(\kappa) = (-1)^\kappa h(1-k)$ which are known as quadrature mirror filters (QMF) and depend on the wavelet function $\psi(t)$ and its scaling function $\phi(t)$ detailed in Equation (3) and Equation (4) respectively.

$$\phi(t) = \sqrt{2}\sum_{\kappa} h(\kappa)\phi(2t-\kappa) \tag{3}$$

$$\psi(t) = \sqrt{2}\sum_{k} g(\kappa)\phi(2t-\kappa) \tag{4}$$

Using the wavelet filters the signal is decomposed into a set of low and high-frequency components by the Equation (5) and Equation (6) respectively.

$$a_j,\kappa = \sum_{m} h(2\kappa - m)a_{j-1,m} \tag{5}$$

$$d_j,\kappa = \sum_{m} g(2\kappa - m)a_{j-a,m} \tag{6}$$

where a_j,κ is the approximation coefficient representing the low frequency components of the signal and d_j,κ is the detail coefficient corresponding to the high frequency components of the signal. These coefficients on the scale 2^j where j is the signal decomposition level are obtained by convolving the coefficients at the $(j-1)$ level with the low-pass and high-pass coefficients respectively.

2.1.4 | Wavelet packet transform (WPT)

WPT is a generalisation of wavelet decomposition that offers a broader range of possibilities for signal analysis with better matching. It provides a level-by-level transformation of a signal from the time domain to the frequency domain. It is computed using a recursion of filter operations, leading to decreasing time domain resolution and increasing frequency domain resolution.[25]

Using the WPT, the information obtained can be decomposed in a more detailed way by replacing the wavelet function $\psi(t)$ with $u_{2n}(t)$ and the scaling function $\phi(t)$ with $u_{2n+1}(t)$ to obtain the Equation (7) and Equation (8) respectively.

$$u_{2n}(t) = \sqrt{2}\sum_{\kappa} h(\kappa)u_n(2t-\kappa) \tag{7}$$

$$u_{2n+1}(t) = \sqrt{2}\sum_{\kappa} g(\kappa)u_n(2t-\kappa) \tag{8}$$

Whereby the signal is decomposed into the low-frequency approximation (LFA) represented by $d_{j+1,2n}$ in the Equation (9) and into the high-frequency detail (HFD) represented by $d_{j+1,2n+1}$ in the Equation (10) for a decomposition level j with 2^j subbands where m represents the wavelet coefficient number.

$$d_{j+1,2n} = \sum_{m} h(m-2\kappa)d_{j,n} \tag{9}$$

$$d_{j+1,2n+1} = \sum_{m} g(m-2\kappa)d_{j,n} \tag{10}$$

In Figure 1, a second level decomposition level $(j = 2)$ is detailed in which 2^j subbands are generated.

2.2 | Classification model

2.2.1 | Random Forest (RF)

RF is a classification model depicted in Equation (11) consisting of different classifiers that follow a tree structure. For each (ith) tree an independent random vector (V_i) is generated. Each tree uses a training set and votes for the most popular category in the input vector (x). The error in the classifier detailed in Equation (12) is a function of the margin (mg), which measures the average number of votes in the random vectors (X, Y) that allow classification for the appropriate class and $(P_{X,Y})$ indicate the probability in the (X, Y) space.[26]

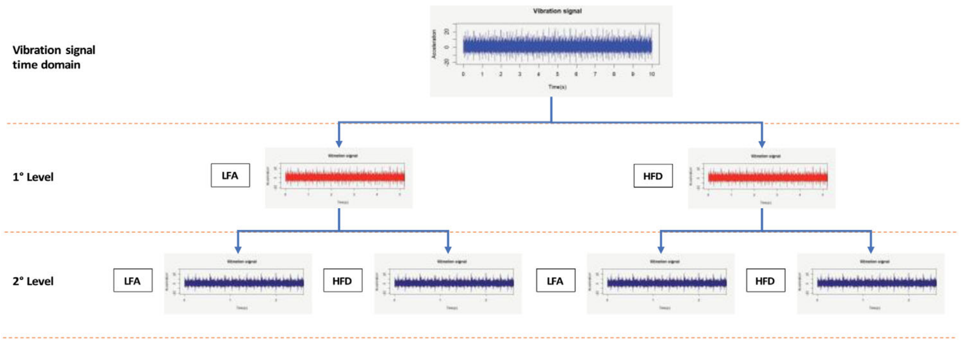


FIGURE 1 Vibration signal descomposition with WPT. WPT, wavelet packet transform.

$$RF = h(x, V_i)_{i=1}^{N}, \quad i = 1, 2, 3 \ldots \tag{11}$$

$$E = P_{X,Y}(mg(X, Y)) \tag{12}$$

2.2.2 | Hyperparameters

Hyperparameters (λ) are tuning parameters necessary to achieve optimal performance in the execution of a machine learning algorithm and can be determined by a tuning procedure through a resampling method.[27] For the RF classification model, the main λ-value is ntry = number of candidate variables extracted at each split.

2.3 | Resampling

Resampling methods are fundamental in statistics and machine learning algorithms. They involve drawing repeated samples from a dataset and resampling a model of interest on each sample to obtain information about the fitted model. The main resampling methods include repeated hold-out, bootstrapping, leave-one-out cross-validation (LOOCV), and k-fold cross-validation.[27]

To determine the hyperparameters in the RF classification model for the decomposition of the vibration signal, k-fold cross-validation was used. This resampling method was used because of the interactions the subsets of variables have with the machine learning model.

The repeated hold-out technique was used to evaluate the classification models' performance. This approach divides the dataset into a percentage (p) for training and a percentage ($1 - p$) for testing. This resampling method was chosen because it allows us to analyze the results' variability and obtain performance metrics of the classification models over multiple process repetitions.[28]

3 | MATERIALS AND METHODS

3.1 | Experimental bench

This work was developed with the vibration signal obtained from the testbench layout represented in Figure 2. It has a 1.5 kW, 1200 rpm, three-phase 220 V motor and a 1.5 kW frequency inverter. The motor is coupled to a single-stage spur gearbox; the gears have Z1 = 32 and Z2 = 48 teeth. The load is simulated by an 8.83 kW electromagnetic brake on the output shaft. The vibration signal was obtained through six accelerometers (A1–A6) in (m/s^2). Accelerometers A1, A4, A5 and A6 were installed in a vertical position defined by the z-axis. A1 and A4 were placed on the drive shaft of the gearbox, while A5 and A6 were placed on the output shaft. Accelerometer A2 was installed inclined 45° concerning the x and z axes, while A3 was installed inclined 45° concerning the x, y and z axes. The vibration signal is fed to a computer, which collects the data using Labview and Matlab software. We work with the vibration signal obtained with the accelerometers placed vertically for the work presented below.

Four types of failure were artificially simulated on the Z1 gearbox: break failure was simulated by milling the tooth (Figure 3A), while cracking (Figure 3B), pitting (Figure 3C), and scuffing (Figure 3D) were simulated by electrical erosion.

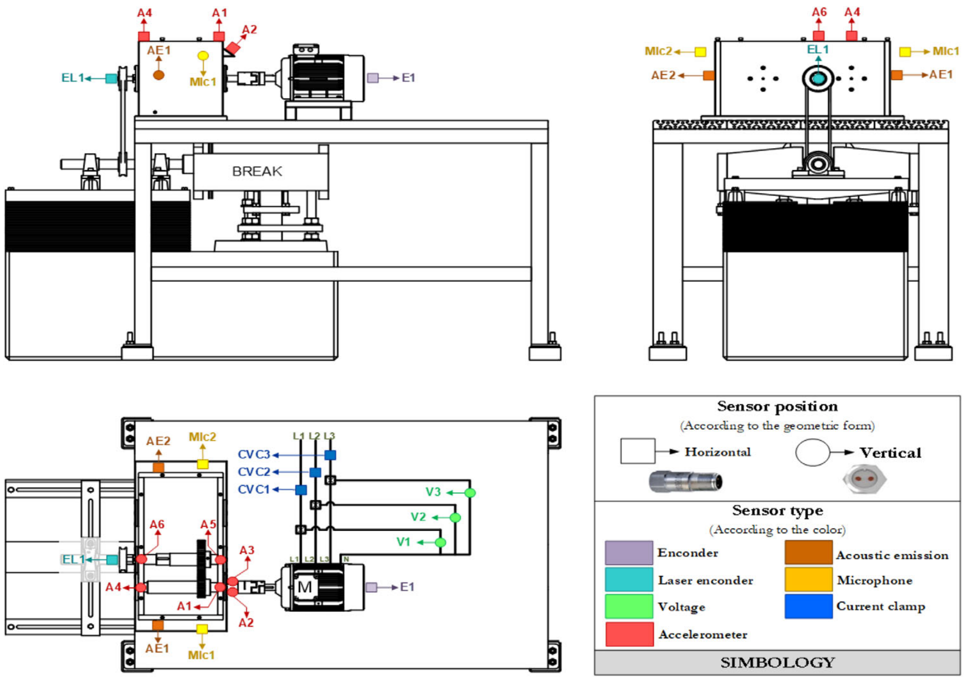


FIGURE 2 Testbench layout.

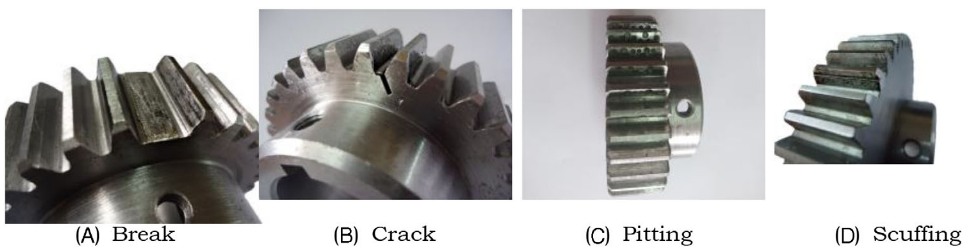


FIGURE 3 Failure type.

For each type of failure, tests were carried out under normal working conditions (P1) and nine severity levels (P2–P10). Using the frequency inverter, the motor speed F1 = 8 Hz, F2 = 14 Hz and F3 = 20 Hz was modified, and with the electromagnetic brake, the load conditions L1 = 0V, L2 = 10 V and L3 = 20 V were modified. By having 10 severity levels, changing the motor speed the load condition and repeating the experiment ten times, a database (DB) with 900 observations for each accelerometer is obtained. The testbench also has an encoder (E1), a laser encoder (EL1), two acoustic emission sensors (EA) and two microphones (Mic) for the gearbox and the power supply lines to the motor with three voltage meters (V) and three electric current clamp (CVC).

3.2 | Methodology

As part of the methodology, the data were processed and analysed using the R software[29] and its integrated development environment, RStudio. The approach developed in this work is depicted in Figure 4 and comprises the following stages:

3.2.1 | Data acquisition

The vibration signal in the time domain was obtained using six accelerometers (A1–A6) placed at different positions and inclinations on the straight gearbox. Each accelerometer has a sampling capacity of 50 k-samples/s, and the signal was measured over 10 s, resulting in a total of 500 k acceleration data points.

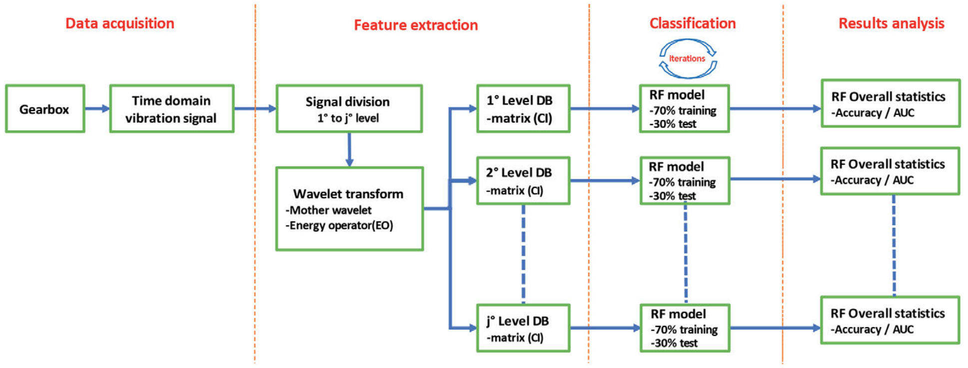


FIGURE 4 Methodology.

3.2.2 | Feature extraction

Feature extraction of the vibration signal in the time-frequency domain was performed using the WPT by decomposing the signal from level 1 to j and extracting the condition state from each subband using the CI EO. The formula to calculate the EO is detailed in the Equation (13).[30]

$$EO = \frac{N^2 \sum_{i=1}^{N} (\Delta y_i - \Delta \bar{y})^4}{\left[\sum_{i=1}^{N} (\Delta y_i - \Delta \bar{y})^2\right]^2} \tag{13}$$

Where,
$\Delta y_i = x_{i+1}^2 - x_i^2$,
$\Delta \bar{y} = mean\ \Delta y$, and
x_i = each acceleration data point.

For each level of decomposition, a combination of different mother wavelets and their filters, which are available in the R package,[31] was employed. From this package, the function (DaubExPhase) was used for the Daubechies' extremal phase wavelets with filters ranging from 1 to 10. The function (DaubLeAsymm) was utilised for the less asymmetric Daubechies' wavelets with filters from 4 to 10. The mother wavelet Coiflets with filters from 1 to 5. The mother wavelet Lawton with filter 3 for the complex-valued wavelets and the function (LinaMayrand) for the Lina-Mayrand family of Daubechies' complex-valued wavelets with filters taking the form x.y, where x represents the vanishing moment number and y the solution number, with available filters being 3.1, 4.1, 5.1, 5.2, 5.3, and 5.4. We obtained 29 possible combinations of mother wavelets and filters for each subband. In this way, a DB with $2^j * 29$ variables and 900 observations was formed for each j decomposition level of the signal.

3.2.3 | Classification

The DBs obtained in the 3.2.2 step for each level of vibration signal decomposition were used for the failure severity level RF classification models. For which the hyperparameter $mtry$ required for the RF classification model was first determined using the k-fold cross-validation resampling method with k-fold = 10.

Repeated hold-out resampling was performed to analyse the performance classification model by dividing the DBs into 70% for training and 30% for testing. Through this process, a vector of 1 k observations for each accelerometer and the corresponding type of failure was obtained, with the values of accuracy in the RF classification model considering all the variables of the databases for each level of decomposition j of the vibration signal.

TABLE 1 Average accuracy by level, accelerometer and failure type.

Level(j)	Failure	A1	A2	A3	A4	A5	A6	mtry
1	Break	0.6630	0.7444	0.7889	0.7148	0.7815	0.7704	18
	Crack	0.6852	0.8741	0.8481	0.8481	0.7926	0.8074	
	Pitting	0.8407	0.8778	0.8593	0.863	0.8593	0.8556	
	Scuffing	0.7000	0.8333	0.8370	0.8000	0.8222	0.8407	
2	Break	0.8741	0.9185	0.9296	0.9296	0.9370	0.9333	27
	Crack	0.9630	0.9111	0.9370	0.9333	0.9481	0.9074	
	Pitting	0.9259	0.9481	0.9407	0.9593	0.9667	0.9519	
	Scuffing	0.8630	0.8889	0.9370	0.9259	0.9481	0.9741	
3	Break	0.9037	0.9704	0.9852	0.9481	0.9778	0.9519	180
	Crack	0.9852	0.9815	0.9889	0.9593	0.9741	0.9593	
	Pitting	0.9741	0.9778	0.9926	0.9815	0.9815	0.9704	
	Scuffing	0.9630	0.9778	0.9704	0.9556	0.9630	0.9667	
4	Break	0.9778	0.9926	0.9889	0.9926	0.9815	0.9852	53
	Crack	0.9999	0.9704	0.9815	0.9963	0.9963	0.9926	
	Pitting	0.9926	0.9815	0.9741	0.9963	0.9999	0.9889	
	Scuffing	0.9889	0.9778	0.9926	0.9815	0.9926	0.9815	
5	Break	0.9963	0.9926	0.9926	0.9926	0.9926	0.9926	30
	Crack	0.9963	0.9963	0.9999	0.9999	0.9999	0.9999	
	Pitting	0.9963	0.9926	0.9963	0.9963	0.9963	0.9999	
	Scuffing	0.9963	0.9852	0.9926	0.9889	0.9963	0.9999	

3.2.4 | Analysis of accuracy results in the classification model

The appropriate level of decomposition of the vibration signal in the time-frequency domain using WPT was determined by analysing the accuracy vectors obtained in each DB in the 3.2.3 step. An analysis of variance (ANOVA) and Tukey post-hoc test were performed to determine if there are significant differences in the average accuracy value obtained by the RF classification model between the j decomposition levels of the signal. The appropriate vibration signal decomposition level is determined by the highest average accuracy value obtained from this analysis.

Once the appropriate vibration signal decomposition level has been determined, the DBs are analysed again, considering the 29 combinations of source waves and filters one by one, resulting in 29 accuracy values for each accelerometer and failure type. The most efficient source wave and filter are considered for each type of failure and accelerometer, that is, the one with the highest average accuracy value in the RF classification model.

Subsequently, all DBs with these mother and filter wavelets are analysed to determine their efficiency using the average accuracy and AUC value in the RF classification model for all accelerometers and failure types.

The obtained accuracy vectors were scrutinised to ascertain the influence of the sensor's position and inclination on the vibration signal. By employing two factorial ANOVA along with post-hoc Tukey evaluations, the first analysis aimed to establish the impact of the accelerometer's position on the vibration signal, while the second analysis focused on the sensor's inclination.

4 | RESULTS AND DISCUSSION

The first objective of this work was to make a methodological proposal to establish the appropriate level of decomposition of the vibration signal in the time-frequency domain using the WT to determine the severity level of a failure in a spur gearbox.

By developing the process detailed in 3.2.3, the hyperparameters (λ) for each level of decomposition of the vibration signal were determined, and the average accuracy values in the severity level of failure were determined using the RF classification model for each accelerometer and type of failure, thus obtaining the Table 1. The signal was decomposed to level 5 because all the average accuracy values in the classification model exceeded 99%.

TABLE 2 Average accuracy by level.

	Level (j)				
	1	2	3	4	5
Accuracy	0.8045	0.9313	0.9693	0.9877	0.9954

TABLE 3 Accuracy of main mother wavelets and filter by failure and accelerometer.

Failure	MWF	A1	A2	A3	A4	A5	A6
Break	Lawton 3	0.9630	0.9704	0.9704	0.9704	0.9667	0.9556
Crack	Coiflets 4	0.9815	0.9481	0.9519	0.9926	0.9852	0.9704
Pitting	Lawton 3	0.9815	0.9704	0.9778	0.9778	0.9889	0.9852
Scuffing	Lawton 3	0.9741	0.9593	0.9519	0.9704	0.9741	0.9704

Abbreviation: MWF, Mother wavelet and filter.

TABLE 4 Accuracy with fusion Lawton 3 and Coiflets 4.

Failure	A1	A2	A3	A4	A5	A6
Break	0.9629	0.9759	0.9757	0.9804	0.9765	0.9791
Crack	0.9908	0.9779	0.9840	0.9878	0.9935	0.9822
Pitting	0.9862	0.9693	0.9804	0.9908	0.9959	0.9887
Scuffing	0.9862	0.9693	0.9805	0.9908	0.9959	0.9831

Subsequently, when the ANOVA test was performed, it was determined that there are significant differences (F-value = 145.5; p-value < 0.001) in the average values of classification accuracy for the five signal decomposition levels (Table 2). When the Tukey post-hoc test was performed, it was determined that there was no significant difference between the $j = 4$ and $j = 5$ (p-value = 0.98) decomposition levels, so the $j = 4$ level was taken as the appropriate decomposition level since the average classification accuracy values are higher than 98%.

In ref. [6], they used WPT to analyse the vibration signal in the time-frequency domain of a spur gearbox, for which they performed the signal decomposition process up to the $j = 6$ level, obtaining 2^6 subbands. In ref. [32], the vibration signal was taken to the $j = 8$ decomposition level, obtaining 2^8 subbands to analyse, thus increasing the computational time of information processing. In contrast, in this work, it was determined using an ANOVA and Tukey post-hoc test that the $j = 4$ decomposition level is optimal for analysing the vibration signal in the time-frequency domain for spur gearboxes.

In ref. [10], they used the $j = 4$ decomposition level but it is not justified because this additional decomposition level merges in a DB the CI obtained in the time, frequency and time-frequency domain for the analysis of the vibration signal in spur gearboxes. In this work we focus only on the analysis of the vibration signal in the time-frequency domain using the WPT.

The second objective of this work was to determine which are the most efficient mother wavelets and filters to measure the severity level of different types of failure using the RF classification model, for which, once the vibration signal decomposition level $j = 4$ was defined, the DBs of this level were analysed again. The 29 possible combinations of mother wave and filter were considered one by one with the R package,[31] therefore, each DB was reduced to $2^j = 16$ variables and 900 observations. The value (λ) $mtry = 6$ was determined, and the average accuracy values per accelerometer and failure type were obtained. The result of this process, considering only the highest average accuracy values, is detailed in the Table 3.

For the failures Break, Pitting, and Scuffing, the mother wavelet that achieves the highest accuracy in the classification model is Lawton with filter 3, whereas, for the Crack failure, it is the Coiflets mother wavelet with filter 4.

To assess the classification performance derived from the synergistic application of these two mother wavelets and filters (Lawton 3 and Coiflets 4), a re-examination of all failure types and accelerometers was undertaken using the databases consisting of 32 variables and 900 observations that were produced. Initially, the hyperparameter (λ) $mtry = 18$ was determined, succeeded by the evaluation of accuracy (Table 4) and AUC multiclass (Table 5).

The level of accuracy obtained with the fusion of the data obtained with the Lawton 3 and Coiflets 4 fusion exceeds 96% and the multiclass AUC exceeds 98%. Therefore, these mother wavelets combined with these filters are suitable for

TABLE 5 AUC multiclass with fusion Lawton 3 and Coiflets 4.

Failure	A1	A2	A3	A4	A5	A6
Break	0.9805	0.9871	0.9828	0.9889	0.9852	0.9887
Crack	0.9948	0.9861	0.9878	0.9932	0.995	0.9915
Pitting	0.9950	0.9826	0.9894	0.9961	0.9982	0.9942
Scuffing	0.9950	0.9826	0.9894	0.9961	0.9982	0.9916

TABLE 6 ANOVA factorial test for position.

Factor	*F*-value	*p*-Value
A	156.10	<0.001
Failure	939.60	<0.001
A:Failure	568.60	<0.001

Abbreviations: A, Accelerometer; ANOVA, analysis of variance.

TABLE 7 ANOVA factorial test for inclination.

Factor	*F*-value	*p*-Value
A	454.60	<0.001
Failure	2845.60	<0.001
A:Failure	343.30	<0.001

Abbreviations: A, Accelerometer; ANOVA, analysis of variance.

analysing the vibration signal in the time-frequency domain and determining the severity level of different types of failure in spur gearboxes.

In the work done by ref. [32], they used the Daubechies mother wavelet with filters 1 to 16 to analyse the vibration signal in gearboxes in the time-frequency domain, and the results in the classification process vary from 85%. In contrast, it was determined in this work that combining the two mother wavelets with their Lawton 3 and Coiflets 4 filters makes it possible to determine the severity level of a failure in spur gearboxes and obtain classification accuracy values of over 96%.

In refs. [6, 10], they used five mother wavelets with their filters for the analysis of the vibration signal in the time-frequency domain, using Daubechies 7, Symlet, Coiflets 4, Biorthogonal 6.8 and Reverse Biorthogonal; in addition, they fuse the CI obtained in the time-frequency domain with those obtained in the time and frequency domain. In the present work, we analyse 29 combinations of mother wavelets and filters to determine the combination that allows us to obtain the best signal information by analysing only the time-frequency domain.

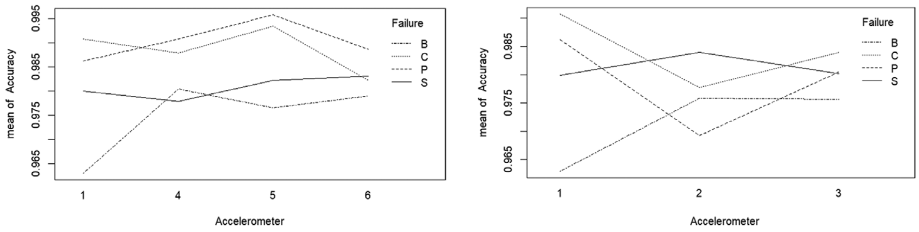
The third objective of this research was to ascertain whether the sensor's positioning and inclination influence the feature extraction process from the vibration signal within the context of accuracy in a classification model. To this end, ANOVA tests and post-hoc Tukey analyses were utilized, considering the accelerometer and the type of failure as factors.

The outcomes observed in accelerometers A1, A4, A5, and A6 were analysed to determine the impact of the sensor position. The ANOVA test ascertained the presence of significant differences (p-value < 0.001) in the average accuracy values for classification among the four accelerometers and for the four types of failure. It was also determined that an interaction exists between the accelerator-failure factors (p-value $< 0{,}001$). The results of the tests are detailed in Table 6.

Regarding the influence of sensor inclination, the results obtained in accelerometers A1, A2, and A3 were analysed. The ANOVA test determined significant differences (p-value < 0.001) in the average accuracy values for classification across the three accelerometers and the four types of failures. It was also determined that there is an interaction between the accelerometer-failure factors (p-value < 0.001). The results of the tests are detailed in Table 7.

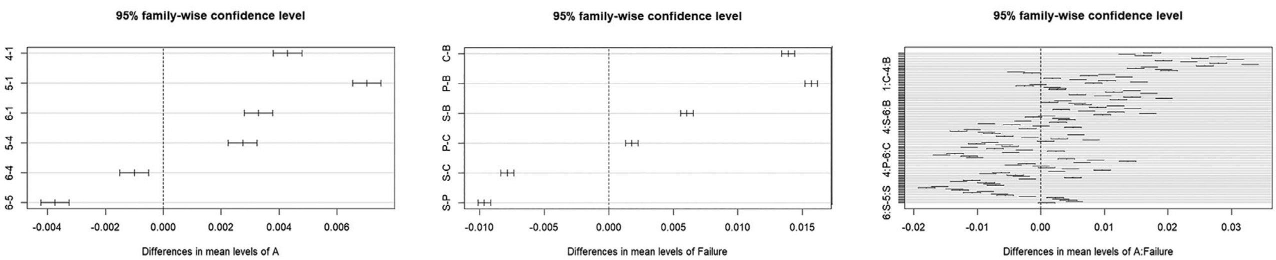
In Figure 5A and Figure 5B, graphical schematics of the interaction between the accelerometer factors and the type of failure are presented when analysing the accuracy in the classification model for position and inclination, respectively.

Regarding the post-hoc Tukey tests, diagrams for the detailed analysis of position and inclination are presented in Figure 6 and Figure 7, respectively, emphasising the significant differences and interactions between factors. It is noted that the differences between the accelerometer factors and types of failure are mostly significant, with few exceptions. However, these differences, being less than 2% as specified in Table 4, lack practical importance due to the interaction between the factors. This point is particularly pertinent as the possibilities for installing a sensor in a gearbox are often limited by the presence of other equipment and attached devices.



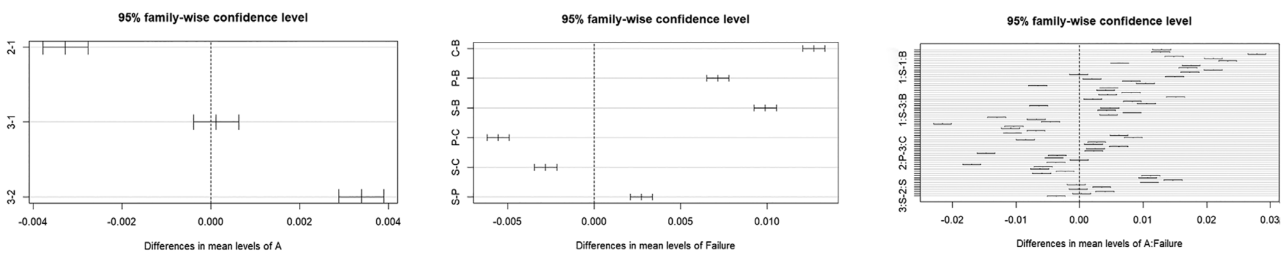
(A) Interaction position (B) Interaction inclination

FIGURE 5 Interaction accelerometer-failure.



(A) Accelerometer (B) Failure (C) Interaction A:Failure

FIGURE 6 Post-hoc and interaction factors for position.



(A) Accelerometer (B) Failure (C) Interaction A:Failure

FIGURE 7 Post-hoc and interaction factors for inclination.

In refs. [13, 14], the optimal position for placing a sensor within a gearbox is studied and determined by shifting it along the x, y and z axes to extract the best vibration signal information. While this position is optimal, access to it is often not feasible due to the physical layout or the mounting within the gearbox. Our study has found significant differences in the sensor's position and inclination, but the results indicate minimal practical significance.

5 | CONCLUSIONS

In this research, a methodological proposal was made to establish the appropriate level of decomposition of the vibration signal in the time-frequency domain using the WT to determine the severity level of a spur gearbox failure. With the proposed methodology, it was determined that the level of decomposition of the vibration signal for the analysis in the time-frequency domain using WPT is $j = 4$ when comparing the values of the accuracy in the classification models using factorial ANOVA and Tukey post-hoc tests between the different levels of decomposition.

In the conducted research, it was determined that the most efficient mother wavelets and filters for analysing the severity level of different types of failures in spur gearboxes are Lawton with filter 3 and Coiflets with filter 4 in combination, as the accuracy values in the RF classification model exceed 96%.

When comparing the accuracy values of classification models through factorial ANOVA tests and post-hoc Tukey tests, it was determined that there are significant differences in the position and inclination of the sensor. These differences, being less than 2%, need more practical significance. Therefore, when installing a sensor in a gearbox and moving it along either the drive axis or the driven axis with some inclination, the information obtained from the vibration signal will be minimally affected. All these factors make the developed methodology ideal for condition monitoring of spur gearboxes.

FUTURE WORKS

Further work will test the proposed methodology to analyse the signal from the spur gear gearbox's acoustic emission, voltage, noise and electric current sensors.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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