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Structural Analysis of Eduardo Torroja's Fronton de Recoletos' Roof.

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Abstract

Eduardo Torroja's thin concrete shells stand among the best examples of structural engineering works of the 20th century. In a time when computers did not exist, Torroja's imagination and creativity were not constrained by the limits of the analytical methods available for structural design, and he was able to design and build economically innovative structures of the highest aesthetic quality. One of his major creations was the roof of the Fronton Recoletos, a unique two lobe thin shell that was destroyed during the Spanish Civil War. This paper reviews briefly the history of the Fronton, shows the results of a structural analysis of its roof by several Finite Element (FE) models of different complexity and precision, and compares FE results to those obtained by Torroja. FE results confirm the validity of Torroja's conceptual design, although he seems to have underestimated the internal forces and stresses in the roof. In addition, the paper analyzes in detail the influence on the behaviour of the roof from its support conditions and from the stiffening ribs that Torroja designed but that never were built. As a result, the paper enables a better understanding of one of the masterpieces of Structural Art, and of simplified and complex shell analysis models, which is useful for engineers' education as well as for future designs.

Keywords: Eduardo Torroja, Finite Element, Fronton Recoletos, Structural Art, Thin shell structures.

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1. Introduction.

Shell structures figure among the most exciting man built structures. Their attractiveness comes from the expressiveness, efficiency and good structural behaviour they have when properly designed as shown by the works of Candela [1], Isler [2], Nervi [3] or Torroja [4] in concrete and Dieste [5] in brickwork. In these cases, shells are also very sustainable structures, as they employ small amount of construction materials, require low maintenance, and do not have durability problems (see e.g. [6], [7]). However, thin shells are nowadays rarely considered as competitive alternatives in new designs as pointed out by Meyer and Sheer [8]. According to these authors, the difficulty to properly analyze and design this kind of structures is one of the causes of their popularity loss. Some books provide a detailed explanation of shell design (see e.g. Billington [9]) and very interesting research is being done in shell optimization and form finding (see [10], [11], [12] among others) but analyzing the work of the shell master builders is one of the most attractive and inspiring ways to learn about shell design and construction. This idea guided previous works that provided new insights into the roofs built by Candela [13], Tedesko [14] and Dieste [15]. This paper aims to increase the understanding of thin concrete shell construction through the study of one of its masterpieces: the roof designed by Torroja for the Fronton Recoletos in Madrid (Spain).

Eduardo Torroja (1899-1961) is one of the most important structural engineers of the 20th century [16, 17]. For almost forty years, he developed an intense activity as university professor, researcher, and consultant engineer [18-20]. He was especially outstanding in the design and construction of thin shell concrete structures, a technical field where his designs provoked enthusiasm by their audacity, efficiency, and aesthetics [21]. The Algeciras Market Hall (1934), the Zarzuela Hippodrome Roof (1935), and the Fronton Recoletos (1935) are his three major concrete shell projects. To build such remarkable structures, Torroja developed new analysis methods, built scale models, and monitored scale models and real structures to check their safety, learn about their structural behaviour, and improve later designs.

Torroja explained his main works and structural philosophy in his two major books [4, 22]. He also explained the details of the analysis and construction of Recoletos' roof in a report [23] written on the occasion of his appointment as a member of the *Real Academia de Ciencias Exactas, Físicas y Naturales* (Royal Academy of the Exact, Physical and Natural Sciences). Later works [19, 24] have briefly explained the architecture and, qualitatively, the structural behaviour of Recoletos' roof, but none of them has analyzed it exhaustively. This paper bridges this gap and explains the main lessons that can be learned from its design. To reach this goal, the roof is analyzed with different FE models of increasing complexity and precision and the results of these analyses are compared to those published by Torroja in [23]. Additionally, the influence of some design decisions not discussed by Torroja is analyzed in detail. In doing so, this work enables a better understanding of (a) one of the key works in the history of reinforced concrete construction, (b) the accuracy of different models that can be used in shell design, and (c) global shell behaviour, what is useful for future designs and engineers' education. The paper starts with a description of the Fronton Recoletos' roof and its engineering historical context. Then, the methods used by Torroja to design the roof and the main features of the FE analyses carried out by the authors are explained in detail and their results are compared. Next, the paper analyzes the influence in the structural behaviour of the roof of its support conditions and of different patterns of external stiffening ribs that Torroja designed but that were not built. Finally, the main conclusions of the work are drawn.

2. The Fronton Recoletos.

The Fronton Recoletos (see Fig. 1) was a sport facility designed by the architect Secundido Zuazo and the engineer Eduardo Torroja for the practice in Madrid of a game called Basque Pelota. This game is played by two teams on a large rectangular playing pitch enclosed by a frontal, a lateral, and a rear wall in a building or place called *frontón* in Spanish. Two teams participate in a match whose aim is to prevent the opponents from launching a ball correctly against the frontal wall [25]. The roof of a *frontón* is one of its most difficult structural elements to design because it must cover a large area without any interior support, allowing at the same time the entry of natural light, and leaving a certain clearance between the playing pitch and the roof. To fulfill all these functional requirements, the roof of the Fronton Recoletos was designed as a thin concrete shell structure. The roof covered a surface of 55 x 32.5 m and, in those areas where skylights were needed, the shell was replaced by a triangulated structure designed for the insertion of glass panes. This design was the result of architectural, economic, aesthetics and construction schedule constraints and was considered by Torroja much better than two other proposals based on transverse or longitudinal truss girders [4]. Fig. 2 shows a cross section and a plan view of the building.

The shell had an innovative and attractive shape defined by two joined cylindrical sectors or lobes of horizontal and parallel axes (Fig. 3). The shell directrix was defined by two circular arches of radii 12.2 m and 6.4 m which sprang from the outer supports (points A and B in Fig. 3) with a vertical tangent and joined orthogonally along a common line parallel to the their axes (point C in Fig. 3) defining the outline of a seagull. The thickness of the shell was only 8 cm except at the connection between the cylindrical sectors where it increased to 0.3 m to resist the transverse bending moments and to adequately cover the reinforcement bars found there. Two 55 m long skylights covering almost the whole length of the Fronton were built. The first one was located in the biggest cylindrical sector near the intersection between both lobes. The second one was placed in the smallest cylindrical sector near its connection with the outer wall. Reinforced concrete elements of the triangulated structure of the skylights had a depth of 0.3 m, a width of 0.17 m and a length of 1.4 m. The roof structure was supported at its two extreme longitudinal edges (lines represented by points A and B in Fig. 3) and at its two extreme directrixes (sections with Z coordinate equal to 0 and 55 in Fig. 2) by means of different kinds of structures described in Antuña [24]. These structures allowed for the free longitudinal dilatation of the shell (along its "Z" axis in Fig. 2) whereas they restrained the transverse displacements (along axis "X" and "Y") at the shell springings. Additionally, structures supporting the two extreme directrixes acted as a rigid diaphragm, and avoided any change of the shape and any vertical displacement of these directrixes.

The structure of the whole building was completed after only 90 working days [4]. During the spring of 1937 and due to the Spanish Civil War (1936-1939), the roof was subjected to both direct hits and severe vibrations coming from aerial bombing. These actions were not considered in the structural design of the roof and caused it severe deformations as well as the removal of several square meters of shell. Soon after the end of the war, Torroja made a proposal to repair the structure. This proposal was based on adding external reinforcement ribs to the roof and aimed to increase the overall rigidity of the structure as well as to recover the initial geometry through a prestressing of the ribs by means of turnbuckles. However, these repairs could not be finished because the roof collapsed in the night of the 15th August of 1939. A new roof project with conventional transverse trussed girders every 5.5 m was drawn up in September of 1939 and later built. The whole building of the Fronton Recoletos was demolished in 1973 and subsequently replaced by a residential building [24].

Several features made the original Recoletos' roof an outstanding structure, especially among barrel vault concrete shells. First of all, Recoletos' roof was bigger and more slender than previous barrel vaults designed by the German engineers (Dischinger and Finsterwalder) who pioneered the design and construction of this kind of constructions (see Table 1). Secondly, never before had an asymmetrical two lobes thin concrete vault of such dimensions been built. Thirdly, the continuous shell was replaced by a triangulated structure in highly stressed portions of the barrel vault to create the skylights, constituting a major innovation. Finally, Torroja's search for structural honesty, aesthetics, and ease of construction led him to design the roof without any edge beam at the intersection between the lobes of the shell and without any visible rib [26]. These edge beams were common in previous designs such as the market halls of Frankfurt (1926-27) and Budapest (1930), but their use at Recoletos would have affected the perception of the structural behaviour and lightness of the structure. All of the above-mentioned characteristics of the roof made it an icon but also made its structural design very difficult. That's why the building developer asked two eminent Spanish engineers, Eugenio Ribera and J.M. Aguirre, to supervise the design and write a report. The concluding remark of this report was:

"(…) we think that the construction of the *Fronton Recoletos* is not only feasible, but also that it will be a new success of our architectural technique. It is specially praiseworthy the decision of the designers of covering this space with such a vaulted roof that, being the biggest of its kind in the world, will put Spain in a pre-eminent place in the list of technical advances. It is also praiseworthy the attitude of the designers who worked hard to explore new solutions and directions that reflect a progressive advance instead of following very well known paths, much easier and implying less responsibility" [23]

The next section describes how Torroja faced and overcame the design challenge as well as the results he obtained.

3. Analysis of the roof carried out by Torroja

Shell structures were defined by F. Dischinger, as "structures formed by singly or doubled curved surfaces, the thickness of which is slight in comparison with the superficial area" [27]. This definition was later completed by other authors [28] who considered that, furthermore, the structure had to be made of a material resistant to compression and tension, the goal of this other condition being to distinguish shells from other structures such as medieval vaults which only can resist compressive stresses.

In a general way, two families of internal forces can appear in a shell: membrane forces and bending forces (see Fig. 4). The importance of each one of these families depends on the thickness and shape of the shell, its support conditions, and its loading. The smaller the bending behaviour is, the bigger the structural efficiency of the shell is, and the smaller its thickness can be. Until the extension of computer use for structural design, the theoretical calculation of the shell internal forces and displacements was a complex task which was only possible for some shapes (see e.g. [9] for some solutions). Therefore, shell design, especially at its beginnings, combined theoretical and experimental knowledge and required abilities to innovate and deal with code gaps and possible distrusts from project supervisors (see e.g. [1], [16], [29]).

Torroja based the analysis of Recoletos' roof on the methodologies developed by Dischinger and Finsterwalder for the design of the roofs of the market halls of Frankfurt and Budapest [30-32]. However direct application of the methods used in the design of those structures was not possible because (a) Recoletos' directrix was asymmetrical and without any edge beam at the intersection between the two arches of the directrix, and (b) Recoletos' larger size and higher rise made it necessary to consider wind loads. In addition, there was no theoretical design methodology able to take into account the real characteristics of the skylights and the variation of the cross section thickness in the intersection of the lobes.

Within this historical and technical context, Torroja analyzed Recoletos' shell considering it as a homogenous structure with uniform depth and elasticity modulus. He justifiably neglected longitudinal bending moments (M_z in Fig. 4), longitudinal shear forces (Q_z in Fig. 4) and torsional moments ($M_{z\theta}$ and $M_{\theta z}$ in Fig. 4), and proceeded to solve the structural problem in two steps. First of all, the loading was considered to be resisted entirely by membrane forces. The resulting forces and displacements at the shell boundaries were not compatible with the known boundary conditions, and, in a second step, forces and displacements were applied to the shell boundaries in the amount required to eliminate the incompatibilities resulting from the membrane forces. This second step introduced bending moments and shear forces in the shell and final forces and stresses were the sum of those obtained at each one of the two steps. The calculations carried out by Torroja were based on a fifty-four differential equation system obtained from the equilibrium and compatibility conditions of the shell's structure. Its solution provided the membrane stresses and bending forces in the shell as well as its deflections. Afterwards, Mohr's circles corresponding to the membrane stresses were drawn and used for obtaining the principal stresses, the stress trajectories and the isobars (Fig. 5a and 5b) along the middle surface of the shell. The most unfavourable section was the central directrix (section B-B in Fig. 2) where the maximum obtained deflections were around 15 cm at the connection between both lobes [26] and the structure internal stresses and forces varied between 0 and 5.7 MPa for compressive stresses, 0.6 and 7.8 MPa for tensile stresses, -10 KNm/m and 10 KNm/m for transverse bending moments and -3.4 KN/m and 2.5 KN/m for transverse shear as indicated in [23]. With all these results, the shell reinforcement was calculated and detailed.

The solution of the mathematical problem took several months [33] and was faced by two different teams. The results obtained by these teams did not perfectly match [34] and the theoretical work was supplemented with an experimental investigation on a reduced scale model (Fig. 6) with a scale factor of 1/10. Strains and deflections of the built shell were measured at the removal of the formwork and during the early stages of the roof life and were in a general good agreement with the expected values [4]. The analysis of the roof, its scale model, its construction, and even a discussion on the causes of its collapse were explained by Torroja in [23].

4. Analysis of the roof by the Finite Element Method.

4.1 Introduction.

This paper aims to delve into the behaviour of concrete shells through the analysis of one of its masterpieces. To reach this goal, this section starts by analyzing Recoletos' roof with four FE models of increasing complexity and precision and compares their results with those provided by Torroja. Then the influence of some parameters of the design (support conditions of the shell at its extreme directrixes, use of external ribs for stiffening the shell) is discussed.

4.2 Analysis of the roof.

The finite element (FE) linear elastic structural analysis is based on the development of four successive models of the structure named FEM-1 to FEM-4 with the commercial software Lusas [35] whose main characteristics are listed in Table 2. Differences between the FE models relate to the following factors: shell thickness, modelling of the skylights, shell support conditions, and loads applied. FEM -1 corresponds to the analysis done by Torroja as described in Section 3, whereas FEM-4 is the closer approach to the built structure. Lusas' QSI4 thin shell element and BMS3 beam element have been used for modelling the shell and the bars of the skylights, respectively. The QSI4 is a 4-node element with four degrees of freedom per node. This element is commonly used in the analysis of three-dimensional thin shell structures, and considers both membrane and bending behaviour. The BMS3 element is a 3-node straight beam with six degrees of freedom at the beam end nodes. Its geometric properties are constant and it includes the effect of shear deformations.

Material properties and loads have been taken from Torroja's report [23]. Therefore, the analysis used a Elasticity Modulus (*E*) of 29400 MPa and a zero value of the Poisson coefficient (ν), except in FEM-4 where a more realistic value of ν equal to 0.2 was used. Table 3 lists both the loads used by Torroja and those applied in each one of the FE models. These loads correspond to three elementary cases: dead load, snow, and wind.

Four types of results were used to compare Torroja's and FE models: deflections of the central directrix, isobars of membrane compressive and tensile stresses, and transverse bending moments along the central directrix. Torroja published the isobars and internal forces diagrams in [23]. He also included a drawing in [4] with a graphic scale comparing the deflections of one shell directrix for the theoretical and reduced scale models with the deflections measured in the built structure. However, when trying to analyze Torroja's deflections results a problem arose: he did not indicate the position of the directrix where the results had been obtained, neither the value of the deflections, nor the load combination used to obtain them. To solve this problem, the authors conducted a parametric study which concluded that deflections published by Torroja corresponded to the central directrix and to the load case "dead load + snow + wind-1". It must be mentioned that Torroja's results of deflections were graphically measured by the authors of this paper on the drawing published in [4] and that an error in the measured values of +/- 1.5 cm was unavoidable due to misreading.

Fig. 12a shows the deformed shape of the central directrix according to each one of the FE models as well as the deformations of the scale model and the built structure published by Torroja. Graphical comparison of the results shows a good agreement between FE and Torroja's results and validates the numerical models. In all cases, the area of the shell in the neighborhood of the connection between the two lobes experienced the biggest deflections. FEM-1 and not FEM-4, provided the closest approach to Torroja's results, although FEM-4 is the model most similar to the real built structure. This difference might be explained by the influence on the results of the real values of factors such as the Elasticity Modulus and the loads. To gain additional knowledge on the causes of this discrepancy, the authors studied two additional models: FEM-4* and FEM-4**. These models are equal to FEM-4 except for the value of E used in the structural analysis. FEM-4* used a value of E*=0.9E and FEM-4** used a value of E**=0.8E. Results of these new analyses are shown in Fig. 12b, whereas Table 4 lists the vertical displacement of the connection between the two lobes (point C in Fig. 3) for each one of the analyzed cases. FEM-1 and FEM-4** provide results which perfectly fit the values of the built structure, if a tolerance of 1.5 cm for the values of the displacements published by Torroja is considered as previously explained.

The isobars of compressive and tensile stresses in the middle surface of the shell resulting from the models FEM-2 and FEM-4** are shown in Fig. 13a to Fig. 13d. Fig. 13 does not include results from FEM-1 because this model does not include the real thickness of the shell in the junction of the two lobes and, therefore, overestimates the stresses. To simplify comparison and understanding of these diagrams, the areas where the FE models' stresses exceed the maximum values obtained analytically by Torroja, are colored in the darker blue for compressive stresses (Fig. 13a and 13c) and in red for tensile stresses (Fig. 13b and 6d). The overall shape of the isobars is very similar to that published by Torroja (Fig. 5a and 5b) and confirms Torroja's general layout of the reinforcement.

On the other hand, Table 5 and Fig. 14, 15 and 16 detail the principal stresses and transverse bending moments along the central directrix for all the models. In the big lobe, maximum compressive stresses range from the value of 5 MPa obtained by Torroja to values of 7.2 MPa (FEM-3), 7.6 MPa (FEM-4), 7.8 (FEM-4**) and 7.9 MPa (FEM- 2). In the small lobe, Torroja obtained a maximum compressive stress of 5.8 MPa whereas the maximum compressive stresses obtained by FEM-2, FEM-3 and FEM-4** are of 4.8 MPa, 4.4 MPa, and 3.7 MPa respectively. It is worth noticing that maximum compressive stresses in the big lobe are obtained in the same location in all the cases, but this situation is not repeated in the small lobe, where the maximum compressive stresses are closer to the shell springings than supposed by Torroja in a magnitude between 1 m and 2.1 m depending on the FE model considered. On the other hand, both Torroja and the FE models conclude that principal stresses in tension only appear at the lobes intersection and are perpendicular to the shell directrix. Values of these stresses range from 8 MPa (Torroja) to 28.7 MPa (FEM-1), 18.7 MPa (FEM-2), 16.7 MPa (FEM-3, 4**) and 16.6 MPa (FEM-4). To resist these high tensile stresses Torroja designed a tension chord perpendicular to the shell directrix made of square steel bars embedded in the concrete of the shell. It is necessary to say that results of FEM-1 tensile stresses at the connection between the two lobes are not valid because of the difference existing in this area between the real thickness of the shell and the thickness of the shell considered in FEM-1.

Fig.16 shows the transverse bending moments according to Torroja and the FE models. Once again, the shape of all the diagrams is very similar but the maximum bending moments (in absolute value) obtained by Torroja (10.3 and 6.2 KN*m/m in the small and big lobes respectively) are smaller than those predicted by the FE models. These models give maximum absolute values of the bending moments between 15.8 KNm/m and 19.5 KNm/m in the small lobe, and between 13.0 and 22.1 KNm/m in the big lobe. These bending moments predicted by the FE models might be responsible for a longitudinal crack that appeared in the middle of the big lobe when the formwork of the shell was removed and that Torroja himself attributed to "a concentration of stresses due to bending moments" [23]. On the other hand, longitudinal bending moments from the FE models are negligible as supposed by Torroja. Finally, it is important to notice that simple models such as FEM-1 and FEM-2 reveal insight into the global behaviour of the shell and are much easier to build than the models that include a perfect definition of the skylights (FEM-3 and FEM-4). Therefore, this work shows that simplified models can be very useful for the preliminary design of concrete shells with skylights or of roofs with complex three-dimensional behaviour as the grid shells designed e.g. by Schlaich, Bergermann und Partner [36].

4.3 Influence of the support conditions of the roof at its extreme directrixes on its structural behaviour.

The extreme slenderness of Recoletos' roof was possible thanks to its three-dimensional behaviour derived from both the curvature of the roof and the support conditions at the extreme directrixes. These supports acted as a rigid diaphragm that prevented any deformation of the structure in the XY plane. To check the importance of these supports, the authors carried out a new FEM analysis (FEM-5). FEM-5 is very similar to FEM-2, the only difference between them being that in FEM-5 the supports responsible for the rigid diaphragm action were removed, and therefore, the extreme directrixes were free to deform.

Fig. 17 shows the transverse bending moments in FEM-5. Examination of this Figure reveals that: a) three-dimensional behaviour of the structure is no longer present as all the directrixes of the shell have exactly the same bending moments independently of their Z coordinate, b) transverse bending moments have been multiplied by a factor between 5.6 and 18 when compared to FEM-2 results and now range from a minimum value of -224.0 KN*m/m to a maximum value of 123.2 KN*m/m. The high values of the bending moments indicate that bending behaviour is now much more important than membrane behaviour and a considerable increase of the depth of the concrete section and of the steel reinforcement of the roof would be required to sustain the loads. Therefore the decision of building the rigid diaphragms was crucial for the correct structural behaviour of the roof. Without these diaphragms, the roof works as a pair of barrel vaults with a two-dimensional behaviour (in fact, as a pair of arches) from which the central support has been removed. This was the common interpretation of the structural behaviour of the roof among the layman (Torroja,[4]), an interpretation that, fortunately, was far from reality.

4.4 Influence of placing stiffening ribs in the exterior part of the shell.

As explained in Section 2, deformations due to the bombing suffered by the roof during the Spanish Civil War caused its collapse when the construction of some external stiffening ribs was about to begin. Those ribs were very important because they were intended to restore the structural capacity of the roof, but no structural analysis related to their design was found by the authors neither in the documents kept by the Torroja Archive nor in [23]. Furthermore, some disagreement surrounds the ribs as different designs have been explained by Antuña [24] and Torroja [23] on one side, and Torroja [4] on the other side. According to Antuña, the strengthening of the roof consisted in external ribs covering only the big lobe and placed every 5 m. This opinion is supported by a drawing existing in [23] where a section of the shell including the rib is shown but the spacing between two consecutive ribs is not indicated. On the other hand, a drawing of the repair proposal published by Torroja in [4] shows the whole roof repaired with stiffening ribs placed every 13.75 m and covering both the big and the small lobes.

Within this general context, this section aims to study the efficiency of the two alternative published designs. To reach this goal, two new FE models, namely FEM-6 and FEM-7, based on FEM-4 were built. These models included stiffening ribs as proposed by Antuña [24] and Torroja [4] respectively. In both cases the ribs were modelled as an overhanging beam with a depth of 0.45 m and a width of 0.3 m according to the construction drawings published in [23]. Fig.18 shows a 3D view of the two new models.

Fig. 19 shows the deformed shape of the central directrix of the shell with and without stiffening ribs. Maximum displacements occur at the intersection between the two lobes and have a maximum value of 9.1 cm (FEM-6) and 9.3 cm (FEM-7). These displacements are 12 % and 14 % lower than those of FEM-4 at the same point. Deflections in the small lobe are higher in FEM-6 than in FEM-7, as FEM-6 does not have any rib in the small lobe. The ribs also reduce considerably the transverse bending moments along the central directrix in the places where they exist as shown by Fig. 20 and Table 5. For example, the minimum bending moments in the big lobe in FEM-6 and FEM-7 are -2.2 and -4.2 KN*m/m. These values represent only 16 % and 31 % respectively of the most unfavourable bending moment given by FEM-4 at the same location. Figs. 13e to 13h show the isobars of compressive and tensile stresses on the middle surface of the shell, whereas Table 5 contains the maximum values of these stresses in the central directrix. Comparison of these results with Fig. 6c shows that the construction of the ribs does not produce significant changes in the stress distribution, even though compressive stresses are smaller in FEM-6 and FEM-7 as the area in darker blue is smaller in Fig. 6e and 6g than in Fig. 6c. It can also be seen that a small concentration of compressive stresses appears in areas where the shell joins the ribs.

In conclusion, both ribs patterns have a substantial positive effect in the global structural behaviour of the shell as they reduce both displacements and transverse bending moments. The solution defined by FEM-7 seems especially interesting because it reduces bending moments in both lobes and these bending moments were the cause of some cracks in the built structure as explained at the end of Section 4.2. Furthermore, this solution employs less material and formwork making it more economical and easier to build.

5. Conclusions and future work

This paper analyzes the structural behaviour of the roof of the Fronton Recoletos, an elegant and innovative structure designed by the Spanish engineer Eduardo Torroja in 1935. The FE modeling of the roof validates the conceptual design done by Torroja although the internal forces and stresses in the shell given by the FE models are bigger than those predicted by him. In addition, the paper enables a better general understanding of the structural design of thin concrete shells, and of the important role that some elements such as diaphragms and stiffening rings play in their behavior. At the same time, this study highlights how FE models with shell elements can be useful tools for the design of complex three-dimensional trussed structures and points out areas where additional research about Recoletos' roof is needed. More specifically, this future research should focus on the nonlinear and time-dependent behaviour of the roof (consideration of buckling, creep and shrinkage) and on its shape optimization.

Recoletos' roof stands as a great example of structural design and collaboration between the architect and the engineer compared to the present when some landmark buildings and bridges are designed following mainly aesthetic reasons and without too much consideration to structural requirements (see e.g. [37], [38], [39]). The roof was the result of a careful study of alternatives that considered both aesthetics and costs; it was built on schedule and was aesthetically superb. In addition, sustainability was considered in the design by (a) using small amounts of construction materials and a reusable formwork and (b) designing skylights that enabled a reduced consumption of electricity for lighting. All these characteristics turn Recoletos into a masterpiece of Structural Art, a practical application of Torroja's ideals of structural honesty and simplicity, and an inspiring work for future designs.

But the story of Recoletos is also the story of an extraordinary human being, Eduardo Torroja. A person of incredible courage and talent, who was able to design, calculate by hand, and construct a unique roof that surpassed contemporary designs, using his structural knowledge and intuition. Recoletos also speaks about Torroja's innovative and curious character that led him to create the first structural monitoring company in Spain and to place measuring devices in the roof to learn from its real behaviour. Furthermore, Recoletos is also an example of Torroja's altruism and generosity as he considered the roof a way to develop science, and he wrote many publications to share the knowledge he gained from this work with the scientific-technical community. This kind of writing is of special value today because it enables the engineers of the present to learn from the master pieces of the past and because it also encourages present-day designers to write about their technical work and aesthetic motivations for the present and future generations benefit. But above all, Recoletos shows us the humility of a genius who was able to reflect on his design and, recognizing that the existence of the reinforcement ribs could have avoided the collapse of the roof, wrote "had I to build it again (Recoletos' roof), I should provide reinforcement ribs" [4]. All these features make Torroja an outstanding example to the engineers and architects of all ages.

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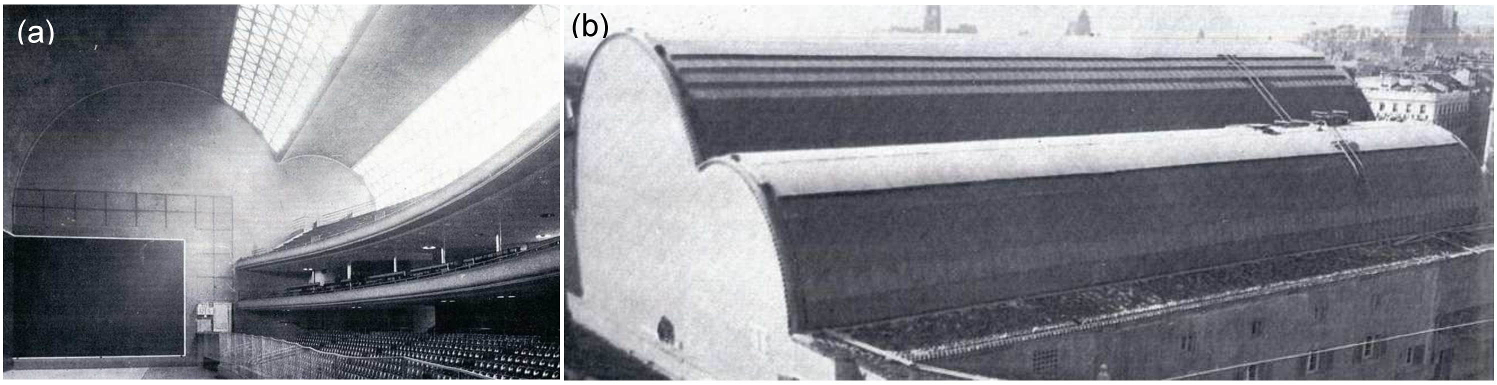
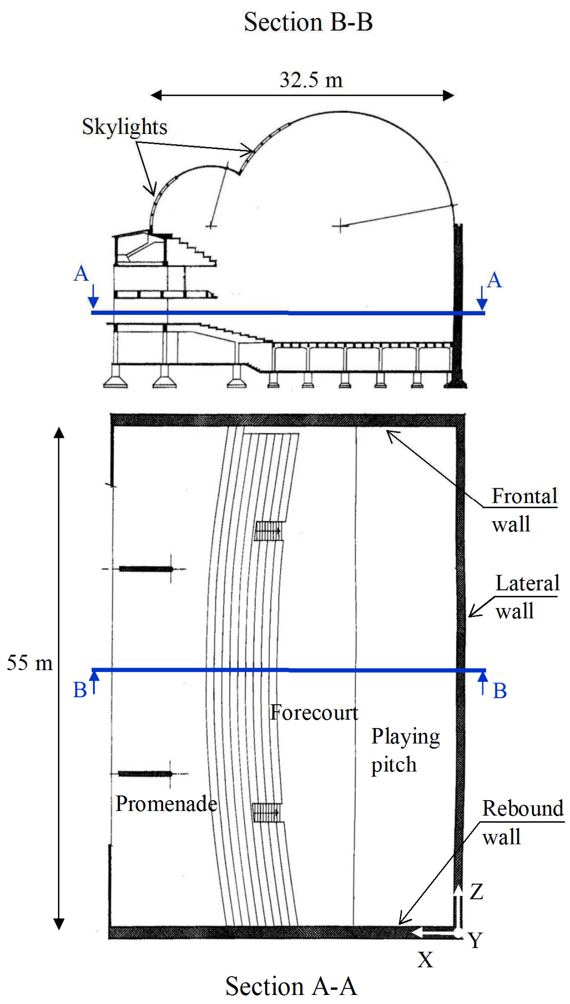


Figure 1. (a) Interior view and (b) exterior view of the Frontón Recoletos. Courtesy of Archivo Torroja – CEHOPU.



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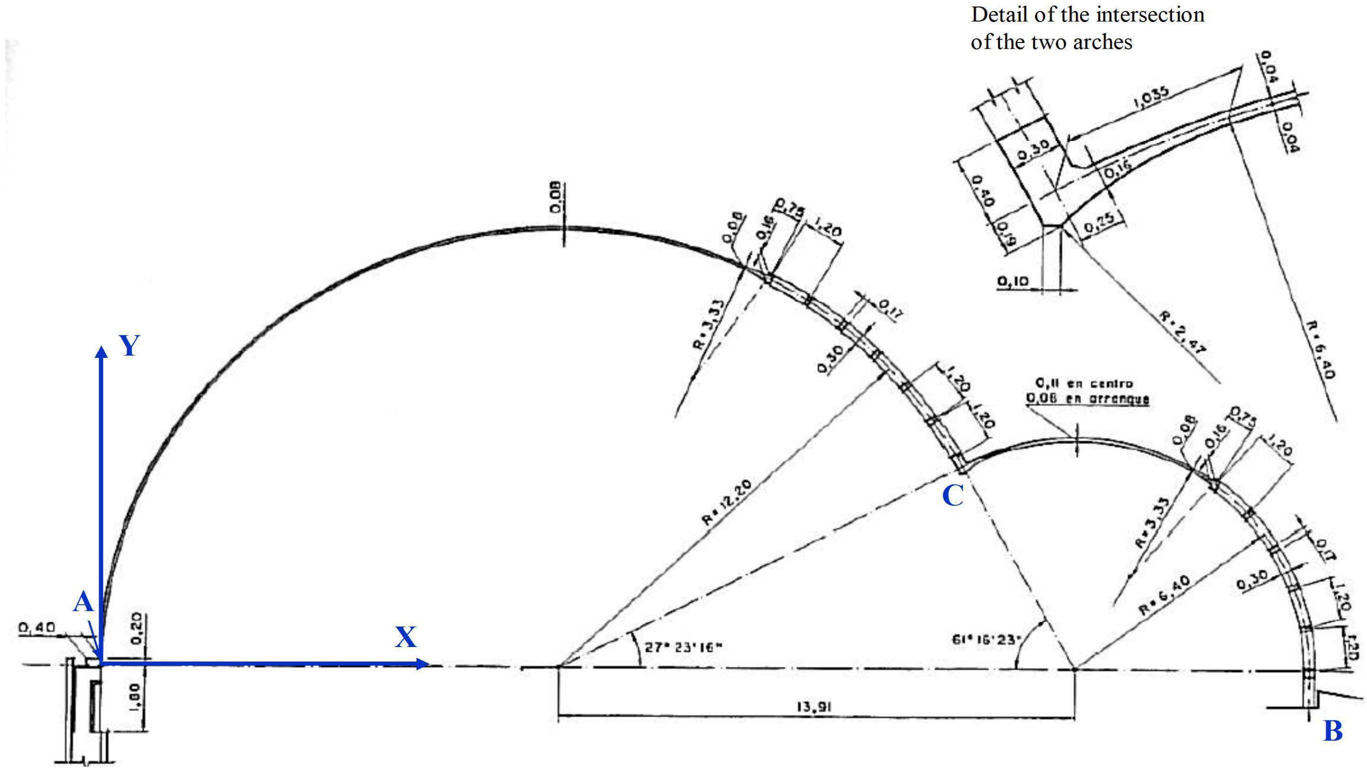

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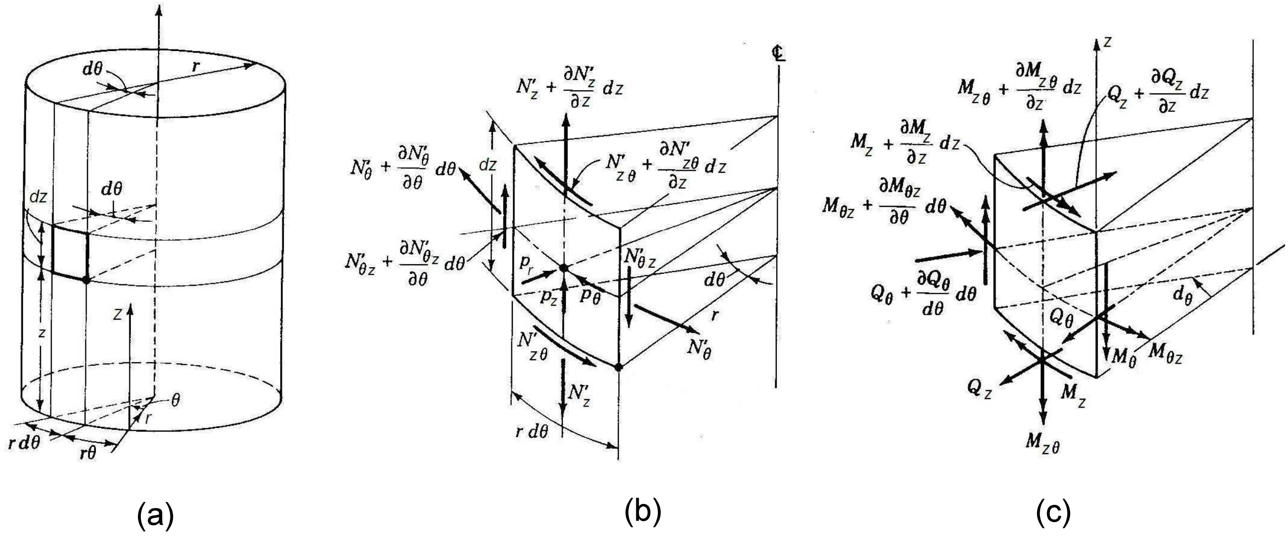

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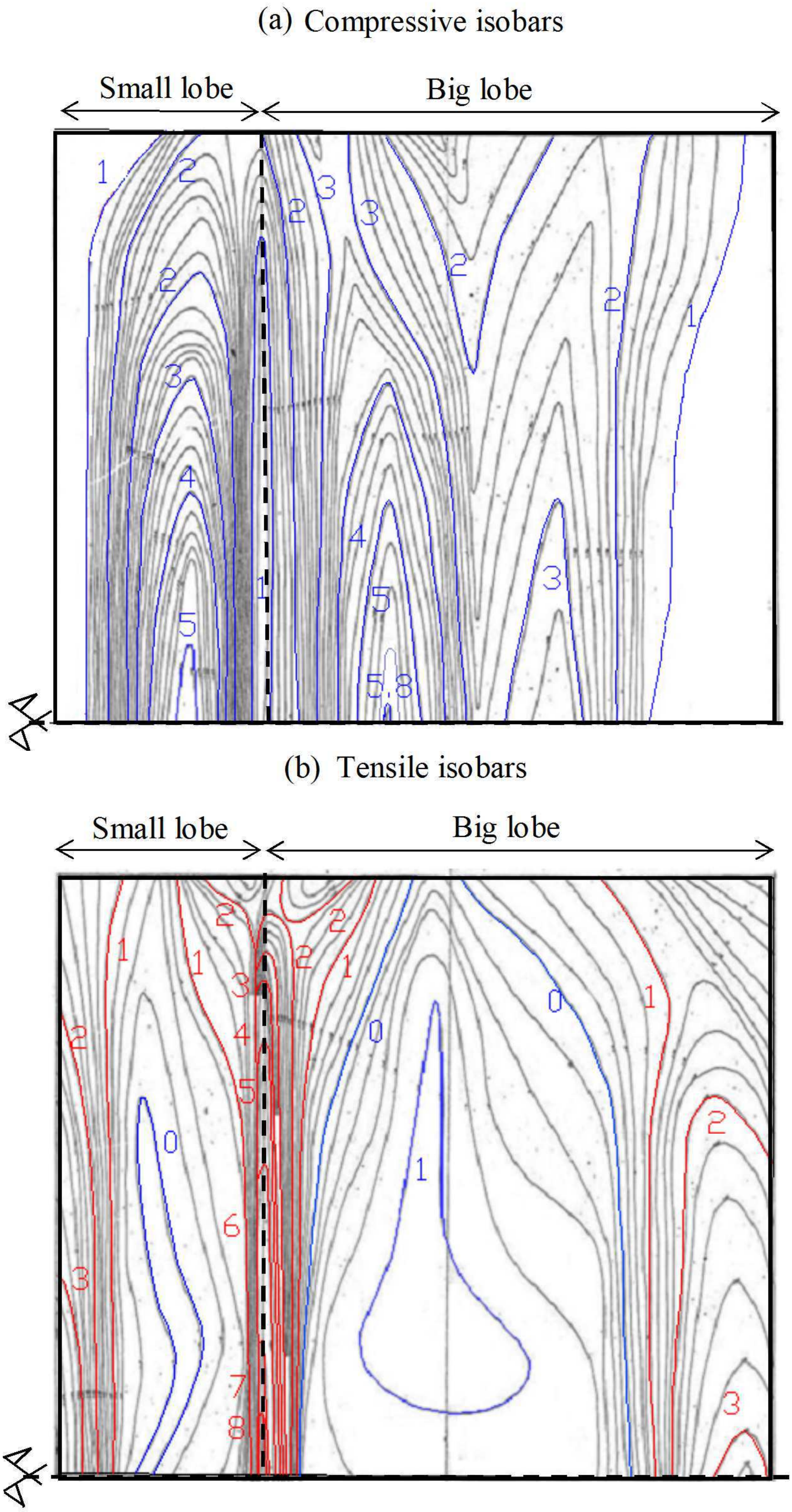

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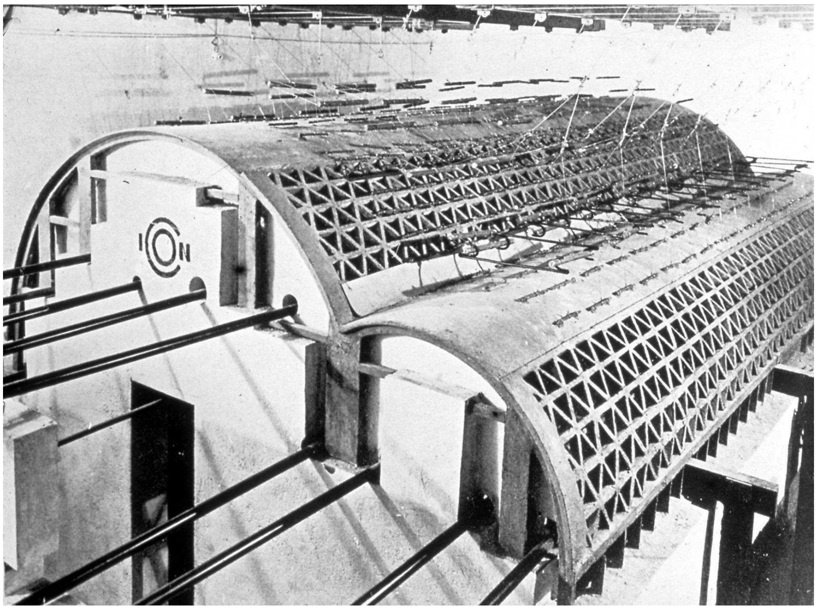

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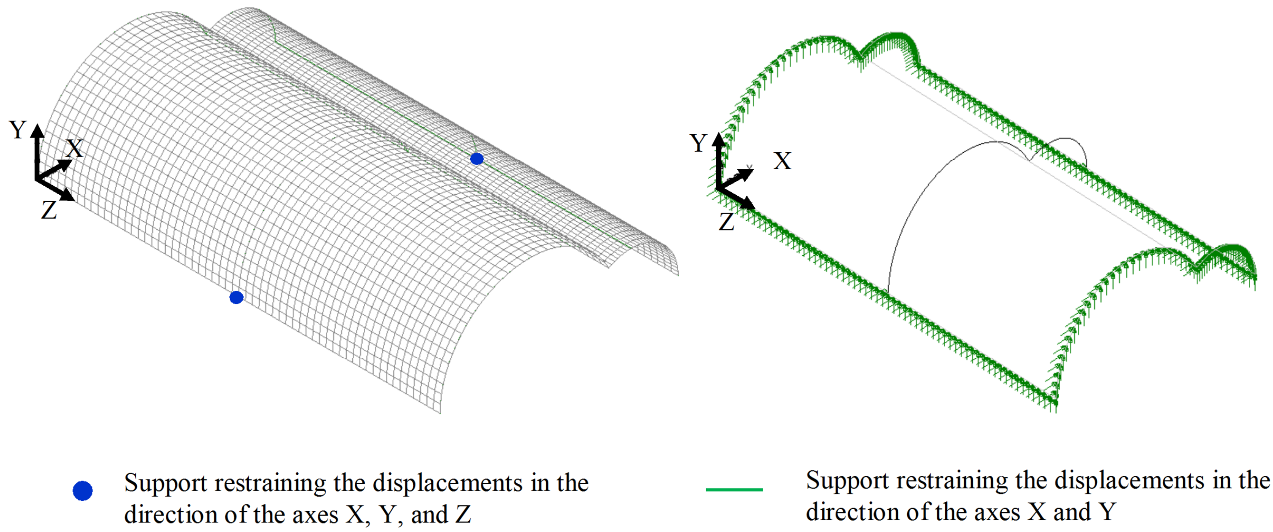

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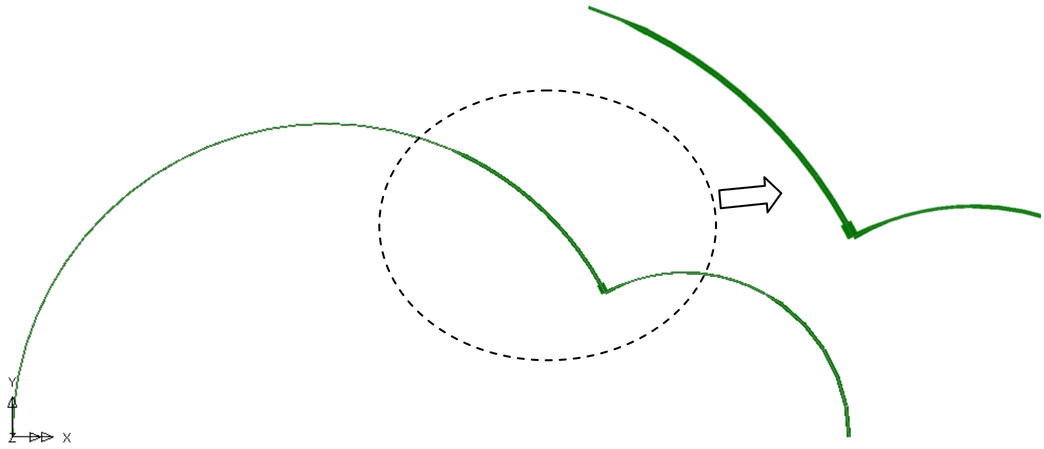

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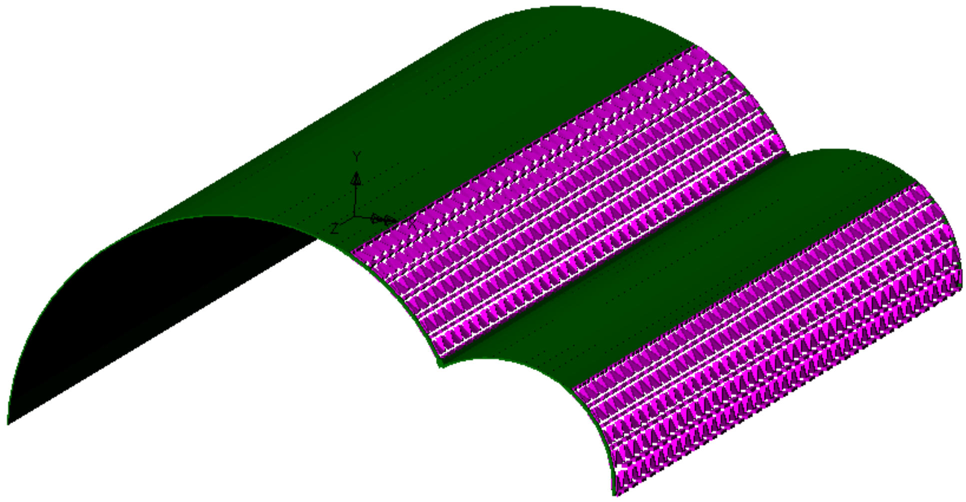

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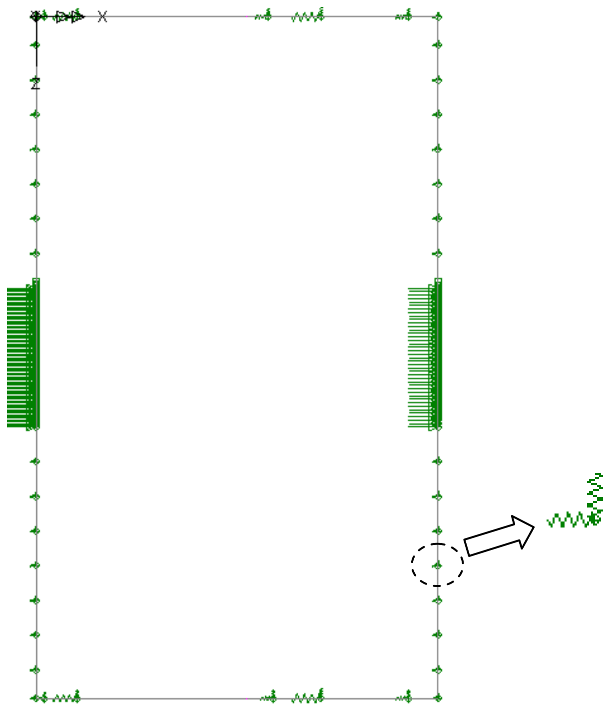

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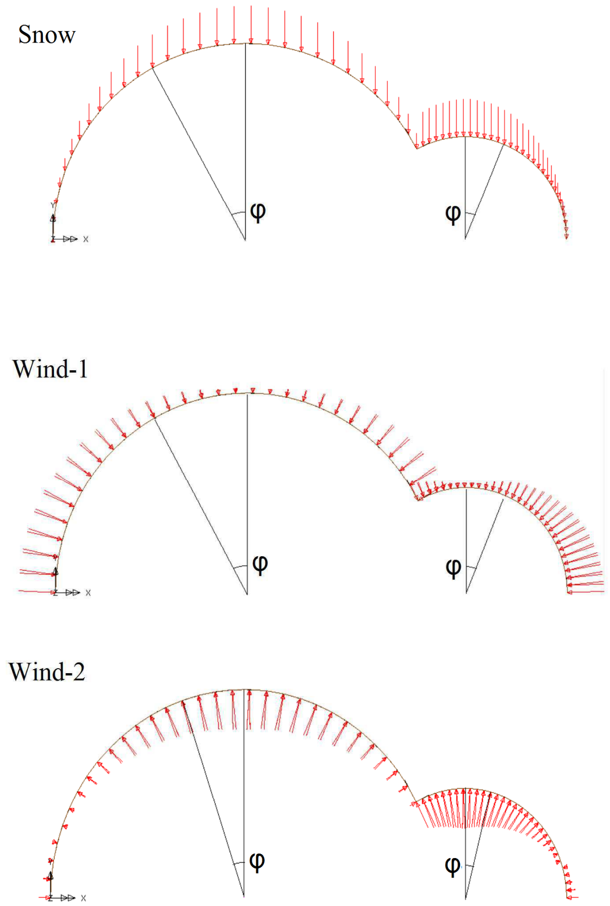

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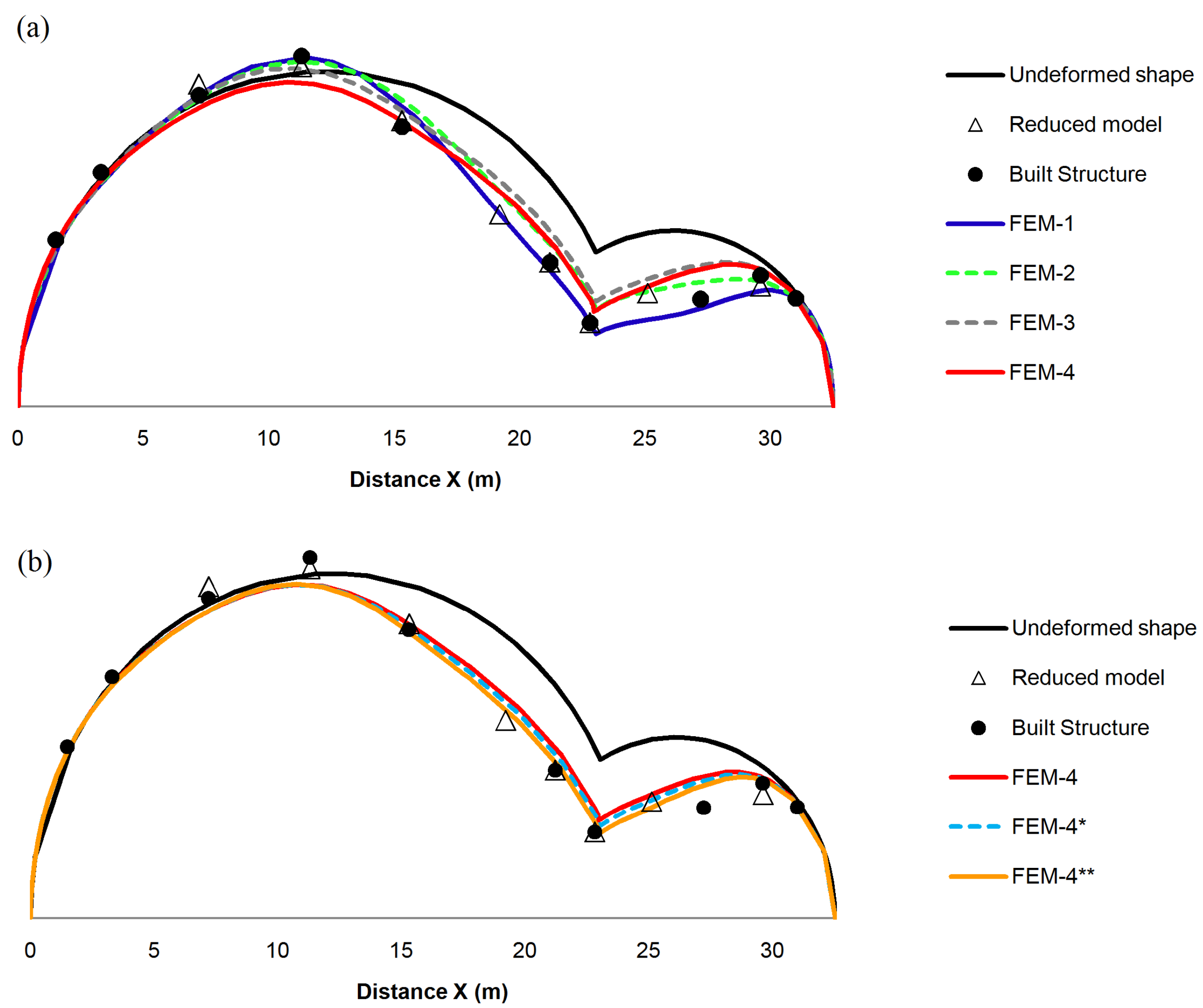

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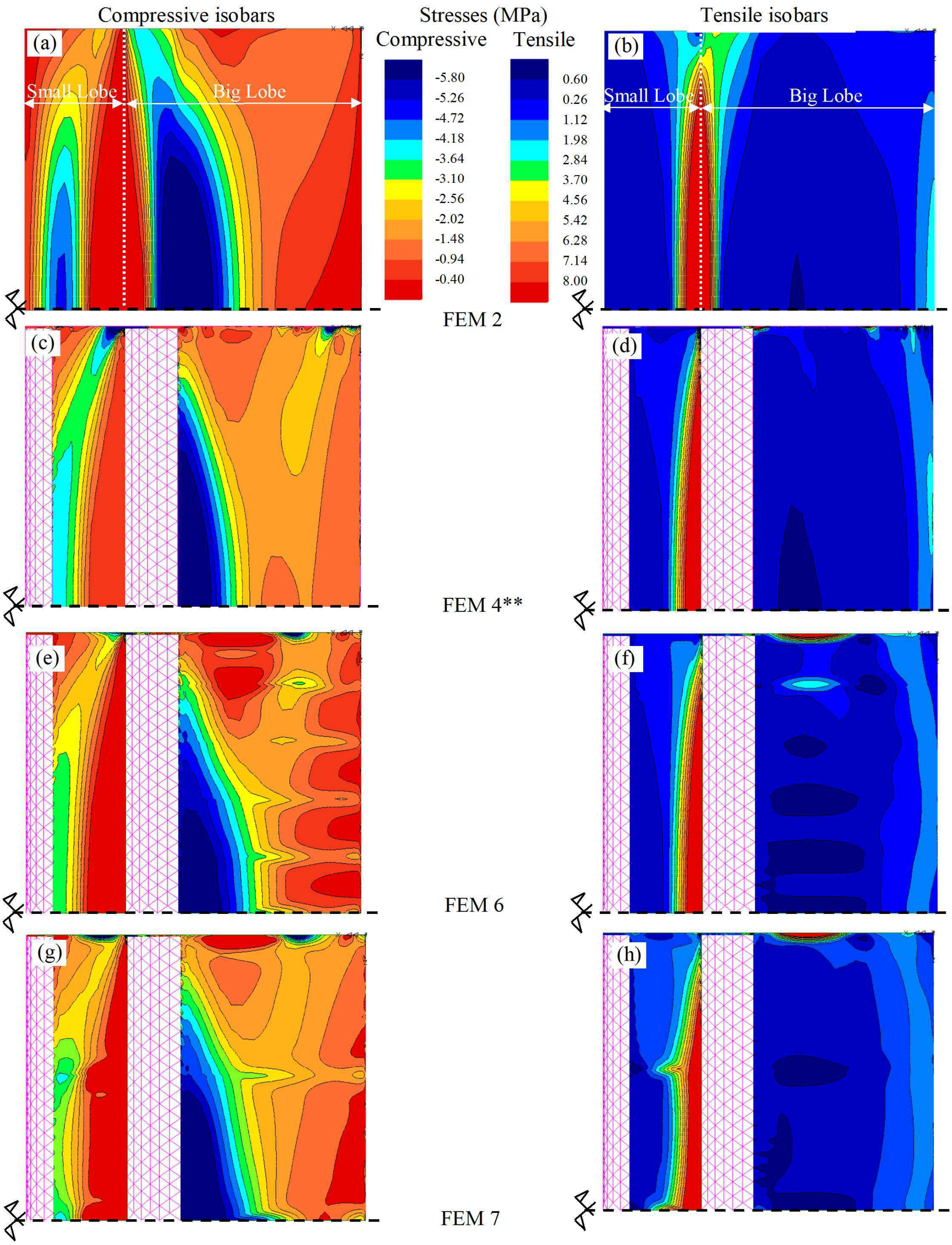

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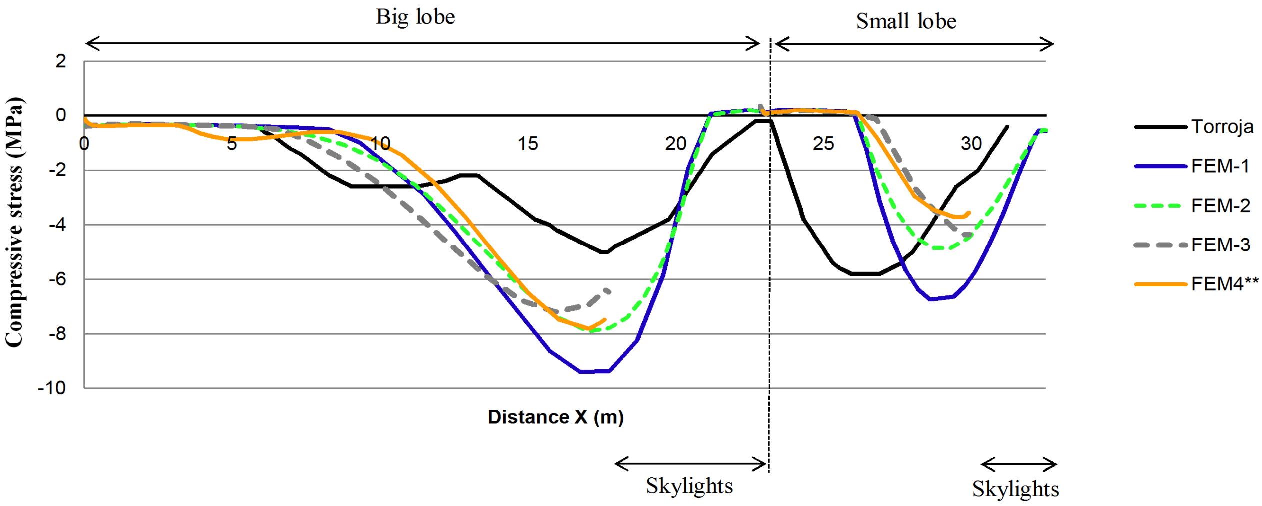

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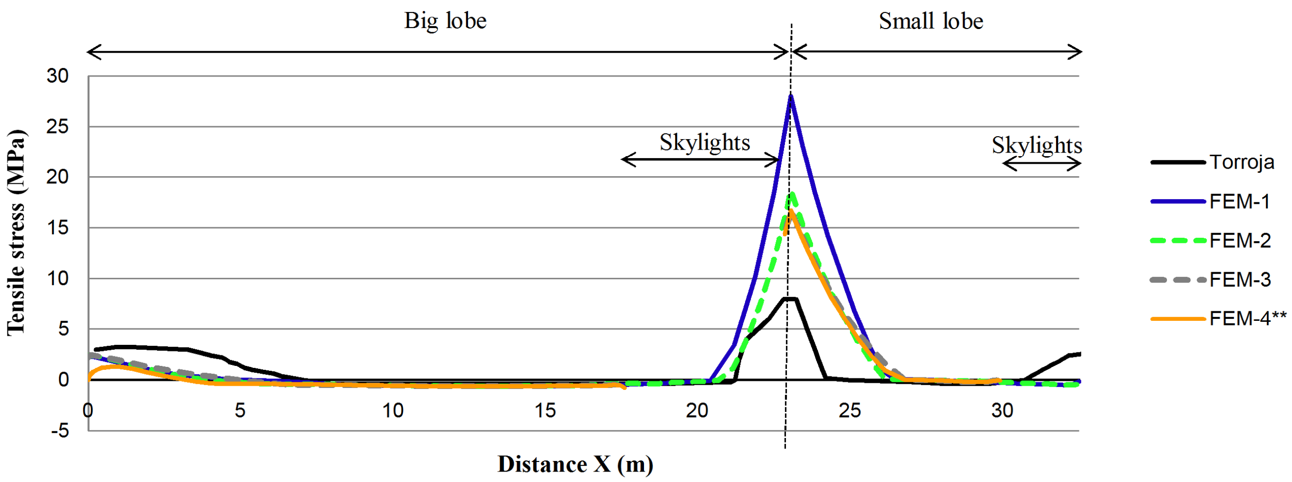

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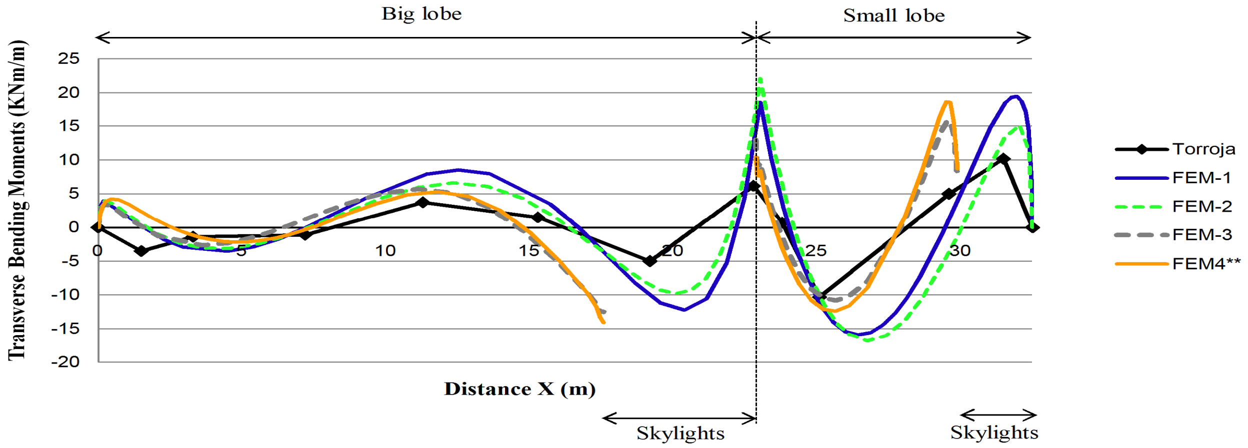

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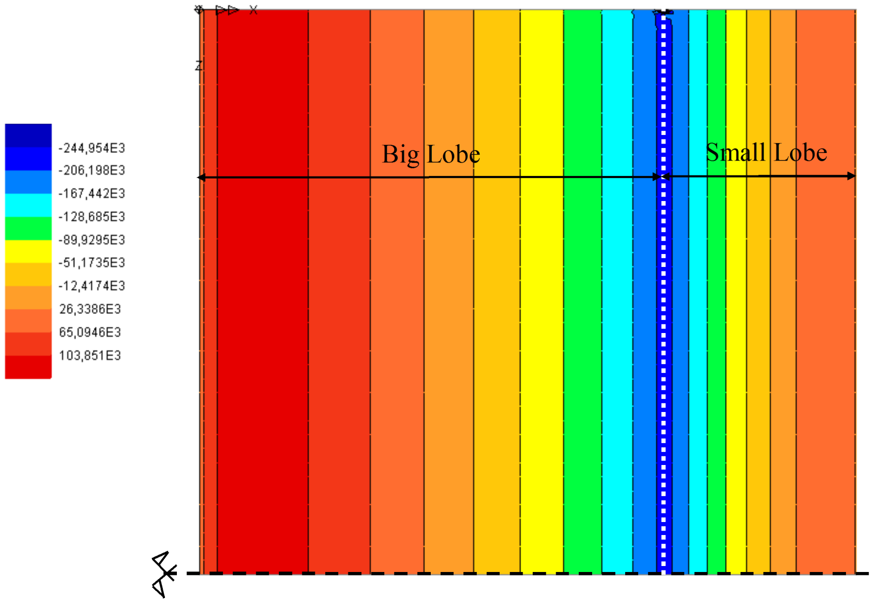

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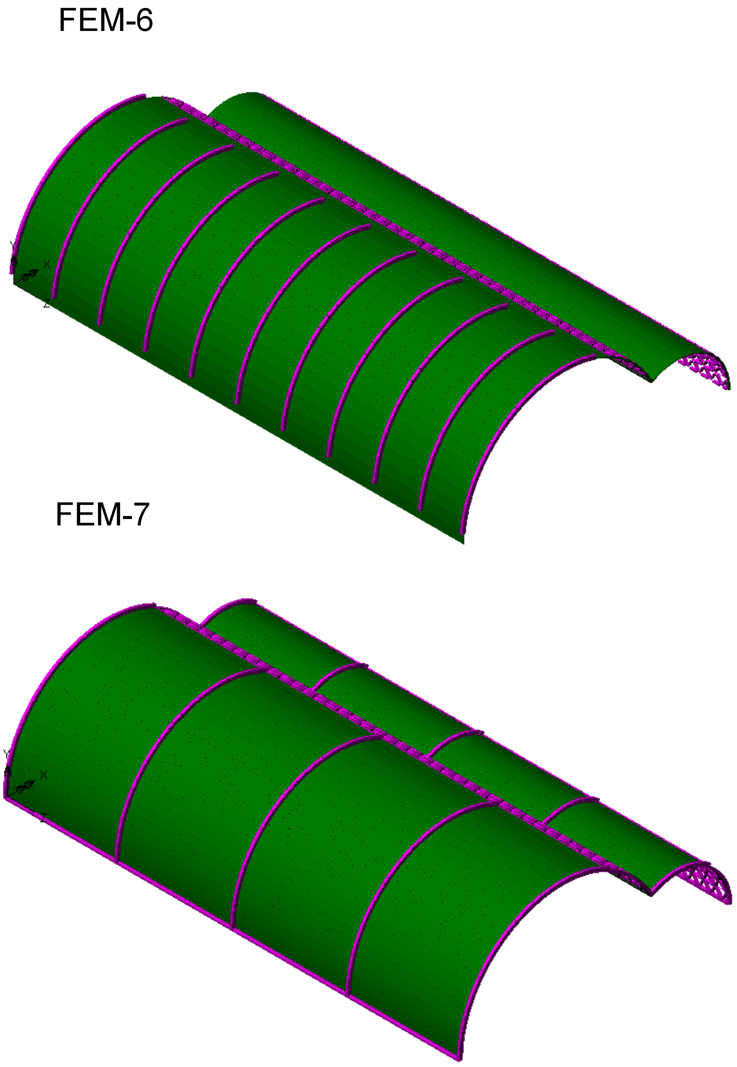

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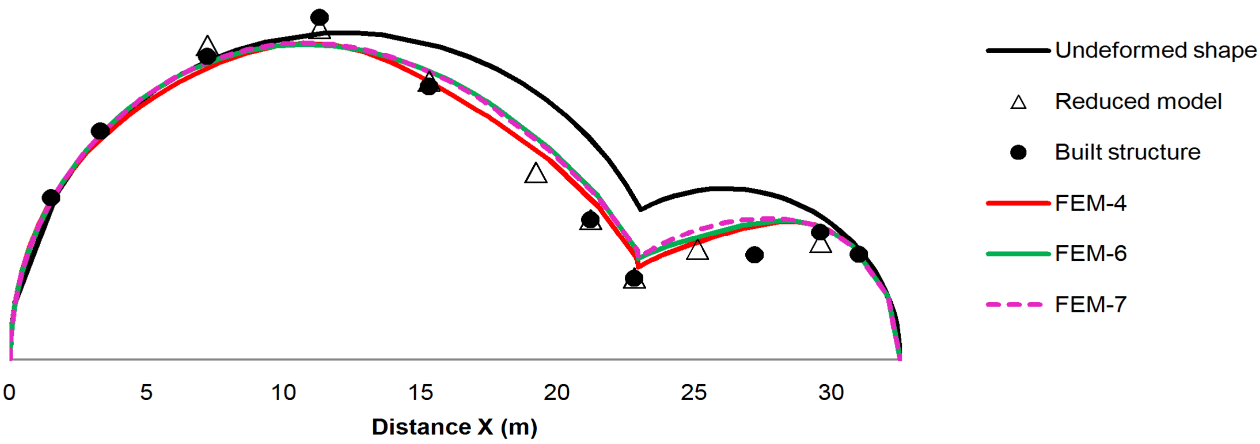

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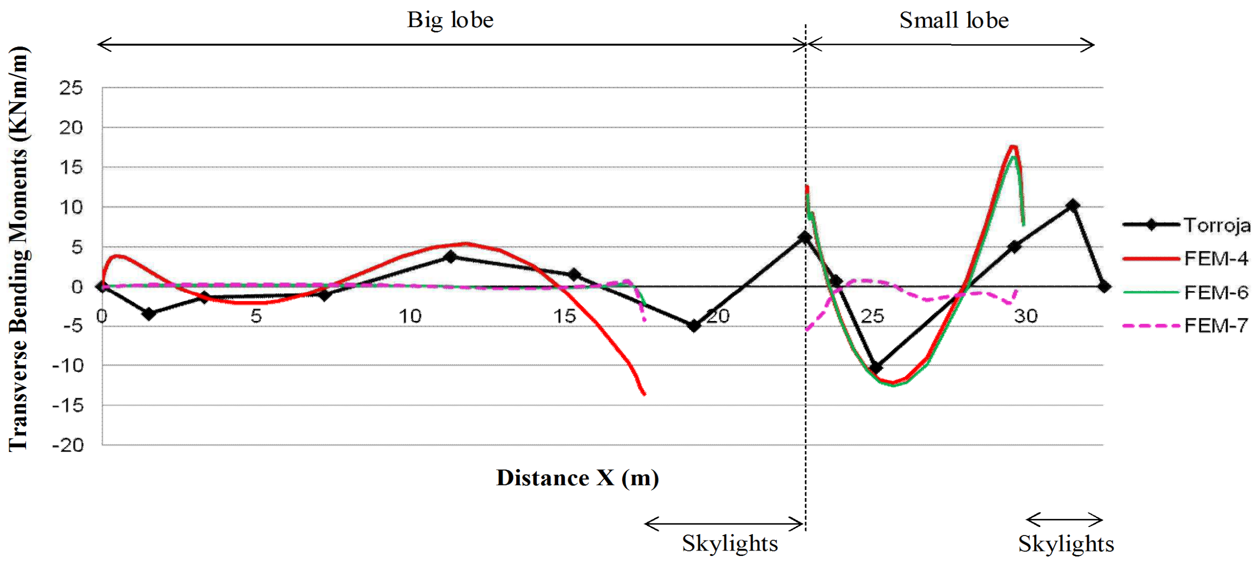

Figure 20. Transverse bending moments in different models. Positive bending moments produce tension in the top face of the shell.

List of Tables.

Name of the construction	Year	Type	w (m)	L (m)	d (m)	h (m)	w/h	L/h
Frankfurt's Market Hall	1926-27	Circular barrel	14	37	4	0.07	200/1	530/1
Budapest's Market Hall	1930	Shallow circular barrel	12	40	1.9	0.06	200/1	665/1
Frontón Recoletos	1935	Intersected circular barrels	32.5	55	12.2 – 6.4	0.08	400/1	680/1

Notation: h = shell thickness; w= width of a barrel shell, i.e., the span of a barrel shell's arch; L: length of a barrel shell

Table 1. Comparison of the barrel roofs of the Frankfurt's Market Hall, Budapest's Market Hall and Frontón Recoletos.

FE model	Shell thickness	Skylights modelling	Supports	Joints	Number of Shell elements	Beam Elements
FEM-1 (Fig. 7)	Constant and equal to 8 cm	Shell of 8 cm of thickness	Supports restraining only displacements in the directions of the axes X and Y. Additionally, supports of the central directrix also restrain movements in the Z direction (see Fig. 7)	3721	3600	0
FEM-2 (Fig. 8)	Variable (see Fig. 3)	Shell of 8 cm of thickness	idem FEM-1	4327	4200	0
FEM-3 (Fig. 9)	idem FEM-2	Beams of 0.17 m of width, 0.3 m of depth and of 1,4 m length	idem FEM-1	44211	30230	38255
FEM-4 (Fig. 9 and 10)	idem FEM-2	idem FEM-3	Springs with stiffness corresponding to that of the real supports	44211	30230	38255

Table 2. Main features of the FE models of the Frontón Recoletos' roof.

Analysis	Dead load	Snow[1]	Wind[1]
Torroja [23]	Constant[2] and equal to 2.45	Snow = 0.637cosφ	Two hypotheses: - Wind without suction effects: Wind-1=0.981sinφ - Wind with suction effects: Wind-2=0.392sinφ– 1.274cosφ
FEM-1	idem Torroja	idem Torroja	Wind-1, Wind-2
FEM-2	idem Torroja	idem Torroja	Wind-1
FEM-3, FEM-4	Shells: idem Torroja Beams[3]: 0.98	Shells: Idem Torroja Beams[3]: 0.255 cosφ	Wind-1

[1] See Fig. 11.
[2] The dead load used by Torroja is an average value which includes the self-weight of : 1) the concrete shell (with a thickness of 9 cm for taking into account possible construction errors), 2) the glass panes of the skylights, and 3) the insulation material used in the roof.
[3] Loads acting on the beams are obtained according to the tributary area of each beam.

Table 3. Loads used by Torroja and loads used in the FE models. Surface loads acting on shell elements are expressed in KN/m^2 and linear loads acting on beams are expressed in KN/m.

Model	Absolute displacement (cm)	Absolute displacement - displacement of the built structure (cm)
Torroja Theoretical	-13	-1.3
Reduced Scale Model	-15	0.7
Built structure	-14.3	0
FEM-1	-14.9	0.6
FEM-2	-10.4	-3.9
FEM-3	-8.9	-5.4
FEM-4	-10.6	-4.3
FEM-4*	-11.7	-2.6
FEM-4**	-13.0	-1.3

Table 4. Displacements of the connection between the two lobes of the shell.

Model	Big Lobe			Small Lobe		
	Compressive Stress	**Tensile Stress**	**Bending Moment**	**Compressive Stress**	**Tensile Stress**	**Bending Moment**
Torroja Theoretical	5.0	8.0	6.2	5.8	8.0	10.3
FEM-1	9.4	28.7	18.5	6.7	28.7	19.5
FEM-2	7.9	18.7	22.1	4.8	18.7	16.8
FEM-3	7.2	16.7	13.0	4.4	16.7	15.8
FEM-4	7.6	16.6	13.6	3.5	16.6	17.5
FEM-4*	7.8	16.8	14.1	3.7	16.8	18.6
FEM-4**	7.8	16.7	14.1	3.7	16.7	18.7
FEM-5	0	0	224.0	0	0	224.0
FEM-6	6.6	15.8	2.2	3.5	15.8	16.2
FEM-7	6.5	16.2	4.2	4.2	16.2	5.5

Table 5. Maximum principal stresses and transverse absolute bending moments in the central directrix. Stresses are given in MPa and bending moments in KN*m/m. All the magnitudes are given in their absolute value.