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1 **A memetic algorithm approach to designing of precast-prestressed**
2 **concrete road bridges with steel fiber-reinforcement**

3 José V. Martí¹, Víctor Yepes² and Fernando González-Vidoso, M.ASCE³
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¹ Associate Professor, Institute of Concrete Science and Technology (ICITECH), *Universitat Politècnica de València*, 46022 Valencia, Spain. E-mail: jvmartia@upv.es

² Associate Professor, Institute of Concrete Science and Technology (ICITECH), *Universitat Politècnica de València*, 46022 Valencia, Spain. **Corresponding author.** Phone +34963879563; Fax: +34963877569; E-mail: vyepesp@upv.es

³ Professor, Institute of Concrete Science and Technology (ICITECH), *Universitat Politècnica de València*, 46022 Valencia, Spain. E-mail: fgonzale@upv.es

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Abstract

This paper describes the influence of steel fiber-reinforcement on the design of cost-optimized, prestressed concrete, precast road bridges, with a double U-shaped cross-section and isostatic spans. A memetic algorithm with variable-depth neighborhood search (MA-VDNS) is applied to the economic cost of these structures at different stages of manufacturing, transportation and construction. The problem involved 41 discrete design variables for the geometry of the beam and the slab, materials in the two elements, active and passive reinforcement, as well as residual flexural tensile strength corresponding to the fibers. The use of fibers decreases the mean weight of the beam by 1.72%, reduces the number of strands an average of 3.59%, but it increases the passive reinforcement by 8.71% on average, respectively. Finally, despite the higher cost of the fibers, their use is economically feasible since the average relative difference in cost is less than 0.19%.

Keywords: Heuristic optimization; precast beam; prestressed concrete bridge; steel fiber; structural design.

34 **Introduction**

35 For more than half a century, precast-prestressed concrete (PPC), pretensioned concrete beams
36 with cast-in-situ slabs has been one of the most common forms of structural systems when
37 building road bridges, given their cost effectiveness, especially when high production volumes
38 are possible (Yee 2001). Production control in precast plants not only provides better quality of
39 concrete products (geometry, facing, finishes, etc.), but it also reduces construction time. In this
40 context, standard PPC bridge beams are considered one of the key solutions to bridging
41 problems in the short-to-medium-span range, typically ranging from 10 m to over 40 m.

42 On the other hand, the stationary precasting industry offers optimal possibilities for steel
43 fiber-reinforced concrete (SFRC) as a cement-based composite material, whose use has
44 increased since steel fibers were introduced as effective concrete reinforcement in the 1960s.
45 Extensive research has shown that dispersed fiber-reinforced in concrete improves such
46 mechanical and fracture properties as tensile strength, energy absorption capacity, toughness,
47 seismic loads resistance, fatigue resistance, cracking resistance and ductility (ACI 1996). These
48 properties are influenced by parameters such as the type of fiber, aspect ratio (length/diameter),
49 fiber content, and distribution as well as their matrix properties. Nowadays, SFRC is
50 increasingly used in structural engineering applications, including pavements and overlays,
51 industrial floors, precast elements, hydraulic and marine structures, large industrial slabs,
52 tunnel linings and in bridge decks. Even though the use of SFRC allows for savings on
53 assembling operations related to conventional reinforcement and for reductions in labor force,
54 equipment use, and associated risks (de la Fuente et al. 2011), steel fibers are often considered
55 expensive. Additionally, reducing material weight through prestressing is essential due to
56 elevation and transportation requirements. This is where structural optimization of this type of
57 large and repetitive structures becomes particularly relevant.

58 Economic optimization of concrete structures is central in the practice of engineering, not only
59 for savings in materials but also for automating the engineering design process. Most realistic
60 structural optimization problems cannot be addressed by exact methods because computing
61 time becomes prohibitive when large numbers of variables are required. Fortunately, it is now
62 possible to use high-level frameworks which employ heuristics to find acceptable solutions at a
63 reasonable computational cost. Much research has been conducted with the so-called
64 metaheuristics methods in structural engineering (Hare et al. 2013). Design optimization of
65 prestressed concrete (PC) beams is a classical problem considered many years ago (Kirch
66 1973); however, as Hernandez et al. (2010) have recently suggested, most approaches for beam
67 and slab deck bridges found in the literature are not suitable for implementation in real life
68 engineering. While there is little research on optimization of PC structures (Ohkubo et al. 1998;
69 Sirca and Adeli 2005; Ahsan et al. 2012; Martí et al. 2013), the literature includes numerous
70 studies on optimizing real-life reinforced concrete (Yepes et al. 2012; Paya et al. 2008;
71 Martinez et al. 2010; Carbonell et al. 2011; Camp and Akin 2012; El Semelawy et al. 2012).
72 Sarma and Adeli (1998) reviewed research on cost optimization of concrete structures while
73 Hassanain and Loov (2003) did the same for concrete bridge structures. Regarding SFRC
74 structures, optimization techniques have been employed in recent years in the design of
75 fiber-reinforced concrete mixes (Baykasoglu et al. 2009; Ayan et al. 2011). However, the
76 literature includes very few works on the cost optimization of SRFC structures (Ezeldin and
77 Hsu 1992; Suji et al. 2008). This shows that there is ample research in SRFC cost optimization,
78 especially regarding prestressed fiber-reinforced concrete (PFRC) structures.

79 In this research, the interest of the authors in the cost optimization of PPC road bridges focuses
80 on the influence of steel-fiber reinforcement (SFR) on the optimal design of this type of
81 structures. The PPC bridge system studied consists of two simply-supported U-beams with a
82 cast-in-situ reinforced concrete slab for road traffic (Fig. 1). A large number of design variables

83 and constraints are considered, and a memetic algorithm with variable-depth neighborhood
 84 search is used. In the following sections, the numerical research and parametric study center on
 85 the influence of SFR on the optimum cost design of PPC U-beam bridges. After a description of
 86 the proposed optimization model, the optimization methodology is presented and verified
 87 comparing different lengths of the bridge analyzed as well as the PC and PFRC beams.

88 **Proposed Optimization Model**

89 The optimization of composite materials such as concrete involves the problem of selecting
 90 values for several variables to determine the minimum value for a function subject to design
 91 constraints:

$$\min C(x) \quad \text{subject to } g_j(x) \leq 0, \quad x_i \in (d_{i1}, d_{i2}, \dots, d_{iq_i}) \quad (1)$$

92 where $C(x)$ denotes the objective function, which represents the cost of building the structure as
 93 the sum of unit prices multiplied by the measurements of construction units, and $g_j(x)$ denotes
 94 the serviceability limit states (SLSs), the ultimate limit states (ULSs) as well as the geometric
 95 constraints of the problem. Each variable x_i can take on the discrete values listed in Eq. (1)
 96 because the final solution must be constructable.

97 The objective function considered, f_{cost} , is the cost function defined in the following equation:

$$f_{cost} = \sum_{i=1,r} c_i \times u_i(x_1, x_2, \dots, x_n) \quad (2)$$

99 where c_i = unit costs; u_i = amount of material and construction units, and r = total number of
 100 construction units. For this study, the basic costs, obtained from a survey of Spanish contractors
 101 and subcontractors of precast structures, are given in Tables 1, 2, 3 and 4 (Martí 2010). The
 102 cost-related input for placement of fiber reinforcement is included in the cost of the beam steel
 103 fiber (Table 1).

104 The precast bridge is defined using 41 design variables. There are eight geometrical design
 105 variables representing the dimensions of the bridge: the depth of the beam (h_1), the width of the

106 beam soffit (b_1) and the thickness of the bottom flange (e_1), the width and thickness of the top
107 flanges of the beam (b_3 and e_3), the thickness of the webs (e_2), the thickness of the slab (e_4) and
108 the spacing between beams (S_v). Another two variables define the slab and the beam
109 compressive strength of the concrete. The design residual flexural strength of the concrete, $f_{R,3d}$,
110 is a variable necessary to calculate the sections subject to normal stresses in the ULSs.
111 Prestressing is defined by four variables: the number of strands in the top flanges, the number of
112 strands in the the bottom flange, and number of sections with strand sheaths (non-bonded steel)
113 in the second and third layers of the bottom flange. Lastly, 26 variables define the diameters,
114 spacing and lengths of the reinforcing bars following a standard set-up for the beam and the top
115 slab. Table 5 lists parameters established for the structure analyzed, and Fig. 2 shows the main
116 variables and parameters for the beam and slab. The slenderness of the beam is limited to a
117 minimum of $L/17$ due to aesthetic, ground and specific road transportation considerations,
118 where L is the span length. Otherwise, the optimization algorithm tends to increase the depth of
119 the beam continuously, and particularly for short span bridges. The model is flexible since
120 variables and parameters can easily be adapted to the given precast plant process specific needs.
121 The variable traffic load is taken as a uniformly distributed load of 4.0 kN/m^2 and a point load
122 of 600 kN , according to IAP-98 code regulation (Ministerio de Fomento 1998). A dead load is
123 assumed as a wearing surface of 0.09 m as well as a uniformly distributed load of $2 \times 0.5 \text{ kN/m}$
124 for concrete bridge barrier rails installed along the edge of the deck. Precast RC slabs of 0.06 m
125 width were considered for the formwork of the top concrete slab. The general exposure class
126 was Iib, according to the Spanish code on structural concrete (EHE-08) (Ministerio de Fomento
127 2008).

128 **The Structural Evaluation Module**

129 Structural constraints considered by the evaluation module followed standard provisions for the
130 Spanish design of this type of structure (Ministerio de Fomento 1998; 2008). Defining a given

131 structure, the structural evaluation module calculates the stress envelopes and checks all the
132 structural constraints. The ULSs for flexure and shear, as well as the geometric minimum
133 requirements, were verified. The calculations for the decompression limit state comprise
134 verifying that under the combination of actions corresponding to the phase being studied,
135 decompression does not occur in the concrete in any fiber in the section. Deflections were
136 limited to 1/1000 of the free span length for the quasi-permanent combination. The ULSs for
137 concrete and steel fatigue were considered in this research. Beam end diaphragms and D-region
138 reinforcement setups can be designed independently in order to resist local stresses and avoid
139 cracking; thus, this was not considered in the optimization process. However, the beam end
140 diaphragms were included for each beam in the structural model. The durability limit state is
141 checked according to the working life design, which was checked at each iteration. The
142 construction sequences and the long-term interaction between the precast beam and the
143 cast-in-place concrete (Marí and Montaler 2000) were considered to design the elements and
144 analyze the structural response of the bridge in each phase. Firstly, a structural model was used
145 for a linear elastic analysis of the beam before being connected to the slab. In this phase, the
146 elastic shortening of concrete was considered when calculating the short-term prestress loss.
147 Then, stress resultants and reactions were calculated taking into account long-term prestress
148 loss due to creeping and shrinkage of concrete and prestressing steel relaxation. A grillage
149 model was used to represent the mechanical characteristics of the bars in which the longitudinal
150 stresses due to the distortion of the cross-section were considered. The details of the structural
151 model can be found in the work by Martí et al. (2013). To evaluate the sections subject to
152 normal stresses in the ULSs from shear and bending forces, the recommendations indicated in
153 Annex 14 of the EHE-08 (Ministerio de Fomento 2008) were used. Regarding the specified
154 residual characteristic flexural strengths, the following series were used, expressed in N/mm^2 :
155 3.0 – 3.5 – 4.0 – 4.5 – 5.0 – 5.5 – 6.0 – 6.5 – 7.0. Common fiber dosages ranging from 40 kg/m^3

156 to 60 kg/m^3 can lead to those specified residual characteristic flexural strengths. In spite of the
 157 fact that the presence of steel fibers affects the compressive strength and the elasticity modulus
 158 (Bentur et al 1990, Nataraja et al 1999, Hatzigeorgiou et al 2005), the stress-strain curve for
 159 plain concrete was adopted in this study according to the EHE-08 recommendation (Ministerio
 160 de Fomento 2008) as it may be considered that the addition of fibers does not significantly alter
 161 the behavior of the concrete under compression. Thus, a rectangular calculation diagram in Fig.
 162 3, characterized by the design residual tensile strength, $f_{ctR,d}$, was used, where $f_{ctR,d} = 0.33f_{R,3,d}$
 163 and the elongation under maximum load $\varepsilon_{lim} = 20\%$ for bending. Skin reinforcement was not
 164 required according to the EHE-08 code because of the use of fibers with a structural function
 165 (Ministerio de Fomento 2008).

166 In order to prevent fragile fracture of the concrete, the contribution of the fibers to simple
 167 bending was limited following this expression (Ministerio de Fomento 2008):

$$168 \quad A_p \cdot f_{pd} \frac{d_p}{d_s} + A_s \cdot f_{yd} + \frac{z_f}{z} A_{ct} \cdot f_{ctR,d} \geq \frac{W_1}{z} f_{ctm} + \frac{P}{z} \left(\frac{W_1}{A} + e \right) \quad (3)$$

169 where $z_f A_{ct} f_{ctR,d}$ is the contribution of the fibers; z_f is the lever arm for the tension in the
 170 concrete; A_{ct} is the tensioned area of the concrete, and $f_{ctR,d}$ is the design residual tensile strength
 171 in the rectangular diagram. The minimum geometric ratio may be reduced by an equivalent
 172 mechanical quantity: $A_c F_{ctR,d}$.

173 According to the EHE-08 code (Ministerio de Fomento 2008), where there are bent
 174 longitudinal bars which are taken into account in the calculation as shear reinforcement, at least
 175 one-third of the shear strength must be provided by the contribution of the steel fibers or, where
 176 applicable, by the joint contribution of the steel fibers and vertical stirrups. The contribution of
 177 the fibers accounted for the load bearing capacity of the tie rods. The failure shear stress due to
 178 tension in the web, V_{u2} , is equivalent to:

$$179 \quad V_{u2} = V_{cu} + V_{su} + V_{fu} \quad (4)$$

180 where V_{cu} is the contribution of the concrete to the shear strength; V_{su} is the contribution of the
 181 transverse reinforcement of the web to the shear strength, and V_{fu} is the contribution of the steel
 182 fibers to the shear strength. V_{fu} can be evaluated as (Ministerio de Fomento 2008):

$$183 \quad V_{fu} = 0.7\xi\tau_{fd}b_0d \quad (5)$$

184 where $\xi = 1 + \sqrt{200/d}$ with d in (mm) and $\xi \leq 2$, and τ_{fd} is the design value for the increment in
 185 shear fiber strength, taking the value $\tau_{fd} = 0.5 \cdot f_{ctR,d}$ (N/mm²).

186 The minimum quantity of shear reinforcement was provided where the following ratio was met
 187 (Ministerio de Fomento 2008):

$$188 \quad V_{su} + V_{fu} \geq \frac{f_{ct,m}}{7.5} b_0 d \quad (6)$$

189 Regarding longitudinal reinforcements, $(V_{su} + V_{fu})$ was used in the expressions instead of V_{su} .

190 **Proposed Optimization Methodology**

191 A Memetic Algorithm (MA) is a population-based approach to stochastic optimization that
 192 combines the parallel search of evolutionary algorithms with the local search of the solutions
 193 forming a population (Moscato 1989). The idea of using hybrid population-based and
 194 trajectory-based metaheuristics can improve effectiveness by combining diversification and
 195 intensification searches (Krasnogor and Smith 2005; Blum et al. 2011).

196 Regarding the local search strategy used within the memetic algorithm, in this paper we
 197 propose a variant of the Very Large-Scale Neighborhood Search (VLSN) algorithm. In
 198 particular, following the classification proposed by Ahuja et al. (2002), the variant selected
 199 belongs to a class of heuristics known as Variable-Depth Neighborhood Search (VDNS).
 200 Although one of the first applications of this strategy can be found for the resolution of vehicle
 201 routing problems (Lin and Kernighan 1973), this is the first time that this type of local search is
 202 used to optimize structures. VDNS is based on a local search which moves from solution to
 203 solution in the space of candidate solutions to reach a local optimum. Then, in order to escape

204 the local optimum, the movement is changed to a larger one, and the search continues until a
205 predefined number of movements, each one larger than the previous. Thus, in this paper, we
206 propose a novel hybrid MA-VDNS to solve structural optimization.

207 In the MA-VDNS algorithm proposed in this study, the first movement is defined by the
208 random change of a single variable, always choosing the new solution if it improves the
209 previous one. The second movement consists in a simultaneous random change of two
210 variables, and so on. In this case, a number of movements without improvement must be
211 defined to change from one movement to the next. Therefore, the MA-VDNS algorithm begins
212 with the random generation of a population, $N = 500$ solutions in this case. Each of these
213 solutions is improved by a VDNS local search until a local optimum is reached. To this end, the
214 algorithm begins changing only one variable, and when it takes ten consecutive movements
215 without improvement, the number of variables changing simultaneously is increased to a
216 maximum of eight. Then, a genetic algorithm is applied to this new improved population of 500
217 solutions. The next step is to create a new generation population of solutions from those
218 selected according to their fitness through crossover and mutation. Appropriate calibration of
219 MA-VDNS algorithm parameters is essential for good MA-VDNS performance. The
220 parameters used in this study are: a population of 500 solutions, probability of 0.50 and elitist
221 selection. A penalty cost is used to evaluate each solution within the evolution procedure;
222 however, the VDNS local search only accepts feasible solutions in order to avoid the early
223 divergence of the algorithm (no penalties are allowed). A VDNS local search is applied to each
224 and every one of the solutions of the new generation. The MA-VDNS will stop if the relative
225 difference between the mean and the minimum cost values at each generation is less than 5%,
226 up to 150 generations. Fig. 4 illustrates typical convergence of the mean and minimum cost
227 curves with the number of generations. Note that the code of the MA-VDNS algorithm can be
228 found in the web page of our research group (www.upv.es/gprc).

229 **Numerical Results and Parametric Study**

230 The MA-VDNS is used to perform a parametric study with different span lengths to analyze the
231 influence of SFR on cost-optimized precast road bridges. The algorithm was coded in Intel®
232 Visual Fortran Compiler Integration for Microsoft Visual Studio 2008. A typical MA-VDNS
233 run lasted 1300 min for an INTEL® Core TM i7 CPU X980 3.33 GHz. Five span lengths of 20,
234 25, 30, 35 and 40 m were considered for each of the two bridge beams, considering the
235 parameters defined in Table 5. The results of the parametric study indicated the design rules for
236 the PPC road bridges, with a double U-shaped cross-section and isostatic spans, including the
237 use of steel fibers. The algorithm was run nine times for each span length according to the
238 methodology proposed by Payá-Zaforteza et al. (2010) based on the extreme value theory. The
239 difference checked between the minimum cost obtained with the nine MA-VDNS runs and the
240 extreme value estimated using the three-parameter Weibull distribution that fits 300
241 MA-VDNS results is less than 3.4%. The average deviations of the mean with respect to the
242 minimum for different span lengths are 5.8% and 6.1% for PC and PFRC structures,
243 respectively (Table 6).

244 The primary economic, geometric and steel reinforcement characteristics were analysed.
245 Tables 7 and 8 summarize the features of the best solutions: Table 7 shows the solutions for the
246 geometry, concrete grade and amount of prestressing steel of the solutions, while Table 8 lists
247 the concrete and reinforcing steel measurements. The influence of steel fibers is discussed
248 together with those of a regression analysis. The functional relations between the variables are
249 valid approximations within the range of the observational data and therefore require careful
250 consideration when extrapolation is carried out. Fig. 5 shows that there is hardly any difference
251 between the average costs of the PC and the PFRC precast road bridges for span lengths ranging
252 from 20-40 m in steps of 5 m. Thus, the relative difference in terms of average cost between the
253 PFRC and the PC bridges with regard to the PC ones is no more than 1.54% (this is the case for

254 a 35 m span, with an average total cost of US\$132,135 and US\$134,199 for PFRC and PC
255 bridges, respectively). The average minimum cost per unit area is US\$298.62/m² and
256 US\$297.43/m² for the optimized PC and PFRC bridges, respectively, for span lengths ranging
257 from 20-40 m. In addition, the relative difference in terms of the overall cost per unit area
258 between the 20 and 40 meter spans for the optimized PFRC bridges is no more than 3.80%; in
259 case of optimized PC ones, this relative difference is no more than 4.67%. In this study,
260 decompression does not occur in the concrete in any fiber in the section; thus, the examined
261 beams are under compression due to prestress, while the most benefits of usage of steel fibers
262 have mainly to do with the improvement of concrete behavior in tension or flexure. The cost
263 variation as a function of the horizontal span leads to a high linear correlation. The average
264 PFRC bridge cost adjusts to $C = 4123.7 L - 9753.2$ with a regression coefficient of $R^2=0.9928$,
265 whereas the PC bridge adjusts to $C = 3915 L - 3609.9$ with $R^2=0.9967$. The cost increases as the
266 span lengthens given the higher material costs, necessary to resist increased slab forces and to
267 satisfy deflection requirements. The use of fibers has little effect on the average costs of the
268 precast road bridges despite the fact that PFRC is significantly more expensive than plain
269 concrete (e.g., according to Table 1, the beam concrete HP-45 costs US\$197.73/m³; however,
270 the fiber addition of 60 kg/m³ increases the initial cost by nearly 43.4%.) This is a significant
271 finding because the cost of using fibers is clearly advantageous without any loss of
272 competitiveness. Fig. 6 shows the relationship between the mean depth of the beam (h_1) and the
273 span lengths for PC and PFRC precast bridges. Again, the use of fibers has no significant
274 influence on the depth of the beam. This is explained by the fact that the ratio L/h_1 , although
275 limited to $L/17$ (see Table 5), was always lower than $L/18$. The mean depth of the PFRC beam is
276 2.41% less than the PC one. In the case of a 20 m span, the mean depth of the PFRC beam is less
277 than 0.05 m. The average depth of the beam adjusts to $h_1 = 0.0488 L + 0.1429$ with $R^2 = 0.9994$
278 in the case of PC bridges and to $h_1 = 0.0507 L + 0.0524$ with $R^2 = 0.9999$ when fibers are used.

279 In both cases, the value of R^2 is near 1.0 which means that the line fits the data almost perfectly.
 280 The use of SFR in the beam leads to an average 0.86% reduction in the slab thickness (e_4).
 281 Regarding the average number of strands in relation to the span, Fig. 7 illustrates a clear
 282 difference when the span is lengthened from 35 m to 40 m using the SFR. The number of
 283 strands is reduced by 3.59% on average, which means that steel fiber tensile strength can reduce
 284 some of the prestressing action. Regardless of the span length considered, an average reduction
 285 of 4.10 strands is achieved when fibers are used, which is equivalent to 775.06 kg. The average
 286 number of strands in the PC bridges adjusts to $\#strands = 1.2444 L + 11.178$ with $R^2 = 0.9564$,
 287 whereas for those with PFRC the adjustment is $\#strands = 1.0933 L + 13.8222$ with $R^2 = 0.959$.
 288 In both cases, the relationship is quite strong. There is a slight average reduction (0.86%) in the
 289 mean characteristic compressive strength of concrete in the beam ($f_{c,beam}$) as a function of the
 290 span when using fibers. There is no significant difference in $f_{c,beam}$ when using PFRC in the
 291 beam, with a range between 35 MPa and 40 MPa. The concrete grade used is relatively high
 292 although the highest concrete grade considered in the optimization problem was 50 MPa. In
 293 regards to the slab, the values of the concrete grade are quite similar to the beam, except for the
 294 35 m span length. There is no clear difference in the width of the beam soffit (b_1) when using
 295 fibers in the beam; thus, the relative difference between the optimized PFRC and the PC bridges
 296 is no greater than 0.52%. The mean width of the PC beam soffit adjusts weakly to $b_1 = 0.0081 L$
 297 $+ 1.1647$ with $R^2 = 0.42$, whereas $b_1 = 0.0031 L + 1.3173$ with $R^2 = 0.216$ for PFRC beams. In
 298 Fig. 9, the tendency is to increase the thickness of the bottom flange (e_1) in accordance with the
 299 span length; notwithstanding, using PFRC in the beam entails an average reduction of 3.25% in
 300 e_1 . Although there is an increasing trend for e_1 when the span length is greater than 25 m and 35
 301 m in the case of PC and PFRC beams, respectively. The mean thickness of the bottom flange
 302 adjusts to $e_1 = 0.0023 L + 0.1076$ with $R^2 = 0.6417$, whereas with PFRC, it is $e_1 = 0.0017 L$
 303 $+ 0.1198$ with $R^2 = 0.7492$.

304 Regarding the ratio of the volume of concrete (v_c) and the surface of the slab (s_s), Fig. 10
305 illustrates the amount of concrete tends to increase with the span length. In fact, the mean
306 volume-to-surface ratio for PC beams has a strong adjustment to $v_c/s_s = 0.002 L + 0.2251$ with
307 $R^2 = 0.9367$; this means that approximately ninety-three percent of the variation can be explained
308 by the span length. Using fibers, the volume of concrete related to the surface of the slab is
309 lower than that for PC beams when the span length is longer than 25 m. This ratio for PFRC
310 beams has a better fit to a line trend: $v_c/s_s = 0.0015 L + 0.2358$ with $R^2 = 0.8936$. There is a very
311 slight reduction in the amount of concrete with the span length using fibers in the beam, as well
312 as an average reduction of 1.27% in the volume of concrete per unit surface area of slab. In the
313 case of PC beams, the average amount of concrete required is $0.286 \text{ m}^3/\text{m}^2$, whereas this value
314 ratio is $0.282 \text{ m}^3/\text{m}^2$ for PFRC beams, which means a relative reduction of 1.5%.

315 By analyzing the ratio between the passive reinforcement (p_r) of the bridge and the surface of
316 the slab (s_s), using PFRC in the beam entails a significant increase (average 27.6%) in p_r/s_s
317 when the span length is 40 m. While it seems logical that the passive reinforcement increases as
318 the span lengthens to resist increased slab forces and to satisfy deflection requirements,
319 surprisingly, the amount of passive reinforcement required is higher when steel fibers are used.
320 This is hard to explain since the fibers contribute to increasing the bending and shear strengths
321 of the beam. However, MA-VDNS leads to a 1.72% reduction in the concrete volume (Table 8)
322 due to the high cost of PFRC which implies passive reinforcement increase. It is worth noting
323 that MA-VDNS can find near-optimal solutions that have similar costs, but are quite different
324 in other respects. Table 7 shows that the characteristic compressive strength of the slab concrete
325 ($f_{c,slab}$) of PC for 40 m case is larger than that of PC what is offset by the slab reinforcement
326 (Table 8). On the other hand, the concrete cross-section should not be reduced too much
327 because fibers reduce the number of strands (Fig. 7), this leading to increase the cross-sectional
328 moment of inertia by reducing the thickness of the beam bottom flange. To sum up, increasing

329 the passive reinforcement and reducing concrete volume and the number of strands minimize
330 PFRC cost. The results in Table 7 show no relationship between these variables and the span
331 length of the beam. The thickness of the webs (e_2) was 0.10 m in almost all cases and included
332 fibers. Using SFR in the beam leads to a 16.38% reduction in the average width of the beam top
333 flanges (b_3) as well as a 21.79% reduction in the average thickness of the beam top flanges (e_3).
334 A particularly relevant aspect related to the transport and placement of the precast concrete
335 structures is the weight of the beam (w_b), which varies as a function of the horizontal span and
336 leads to a high linear correlation, as shown in Fig. 11. Although using steel fibers in the beam
337 slightly reduces (1.72%) the mean weight of the beam, the mean weight savings is 2,567.22 kg
338 when the span length is 40 m, and thus a significant 3.27% reduction is found. The mean weight
339 of the PC beam adjusts to $w_b = 2616.4 L - 29617$ with $R^2 = 0.9801$, while when fibers are used,
340 this weight adjusts to $w_b = 2541.3 L - 28235$ with $R^2 = 0.9907$. However, there is a considerable
341 difference when comparing the weights of the optimized beams; in fact, cost-optimized PFRC
342 beams weigh 6.7%, 6.2% and 5.7% less than the PC ones when the span lengths are 20 m, 25 m
343 and 40 m, respectively.

344 **Concluding Remarks**

345 In this paper we study the influence of steel fibers on cost-optimized PPC road bridges,
346 typically formed by two isostatic beams, with a double U-shaped cross-section. A memetic
347 algorithm with variable-depth neighborhood search, abbreviated as MA-VDNS, is used in this
348 study. This algorithm combines the synergy effects of the MA and VDNS. The algorithm
349 eliminates the conventional design process of trial and error, in which engineers follow iterative
350 procedures to design PPC bridges. The analysis reveals that despite the higher cost of the fibers,
351 and considering that decompression does not occur in the concrete in any fiber in the section,
352 the relative difference between the optimized PFRC and the PC bridges is less than 5.36% in the
353 worst case studied, which means that using SFR is economically feasible. The parametric study

354 shows a good correlation between the cost, depth of the beam, weight of the beam and number
355 of strands for PRFC and PC bridges and the beam span length, which can be useful for
356 practicing engineers. The use of fibers in the beam leads to an average reduction of 0.86% and
357 2.41% in the thickness of the slab and in the depth of the beam, respectively. On average, the
358 number of strands is reduced by 3.59%, which means that steel fiber tensile strength can release
359 some of the prestressing action. Using PFRC in the beams leads to an average 0.86% reduction
360 in the compressive strength of the concrete used in the beam and a 2.53% increase in the
361 compressive strength of the concrete used in the slab. There is a very slight reduction in the
362 amount of concrete with the span length using fibers in the beam, as well as an average
363 reduction of 1.27% in the volume of concrete per unit surface area; however, this reduction is
364 above 6% for the cost-optimized solutions. Surprisingly, using PFRC in the beam results in an
365 average 8.71% increase in the passive reinforcement required per unit surface area of slab
366 despite the fibers increasing the beam strength. This can be explained by the lower concrete
367 volume due to the the high cost of PFRC. Finally, in the cost-optimized beams, using PFRC
368 reduces the mean weight of the beam slightly (1.72%); however, this reduction is above 6% for
369 the cost-optimized solutions. This value might be relevant for the transport and placement of
370 these precast beams. To conclude, the methodology described herein is quite flexible and may
371 be further modified for use with a continuous U-beam bridge systems or other types of bridge
372 systems considering both superstructure and substructure as well as for high strength concrete
373 with steel fiber beams.

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379 of the manuscript.

380 **Notations**

381 *The following symbols are used in this paper:*

a_{bar} = Concrete bridge barrier width

A_{ct} = Tensioned area of the concrete

As_6 = Top longitudinal passive reinforcement of the slab

As_7 = Bottom longitudinal passive reinforcement of the slab

b_1 = Width of the beam soffit

b_3 = Width of the top flange of the beam

C = Total cost of bridge

c_i = Unit costs

d = Beam effective depth

e_1 = Thickness of bottom flange of the beam

e_2 = Thickness of the webs

e_3 = Thickness of top flange of the beam

e_4 = Thickness of slab

E_{nt} = Bearing center to beam face distance

$f_{c,beam}$ = Characteristic compressive strength of concrete in the beam

$f_{c,slab}$ = Characteristic compressive strength of concrete in the slab

$f_{ctR,d}$ = Design residual tensile strength

f_{pk} = Active prestressing steel (Y1860-S7)

$f_{R,3d}$ = Design residual flexural strength of the concrete

f_{yk} = Passive reinforcing steel (B-500-S)

g_j = Structural constraints

h_1 = Depth of beam
 i_4 = Bottom flange division
 I_a = Web inclination
 L = Span length
 n = Number of design variables
 N = Number of solutions in a population
 N_{ai} = Top active reinforcement of the beam
 N_{as} = Bottom active reinforcement of the beam
 n_{i3} = Inclination, bottom flange tablet
 n_{s3} = Inclination, top flange tablet
 Q_m = Concrete bridge barrier loads
 r = Number of construction units
 s_3 = Top flange division
 S_v = Spacing between beams
 t_1 = Transverse reinforcement of the bottom flange of the beam
 t_2 = Transverse reinforcement of the web of the beam
 t_3 = Transverse reinforcement of the top flange of the beam
 t_4 = Top transverse reinforcement of the slab
 t_5 = Bottom transverse reinforcement of the slab
 T_d = Transport distance (one way)
 t_{ws} = Thickness of wearing surface
 u_i = Amount of material and construction units
 V_{cu} = Contribution of concrete to shear strength
 V_{fu} = Contribution of steel fibers to shear strength

V_{su} = Contribution of transverse reinforcement of the web to shear strength

V_{u2} = Failure shear stress from tension in the web

W = PC precast bridge width

x_1, \dots, x_n = Design variables

z_f = Lever arm for tension in the concrete

ϵ_{lim} = Elongation under maximum load

τ_{fd} = Design value for the increment in shear strength from the fibers

Φ_r = Beam surface reinforcement

Φ_s = Strand diameter

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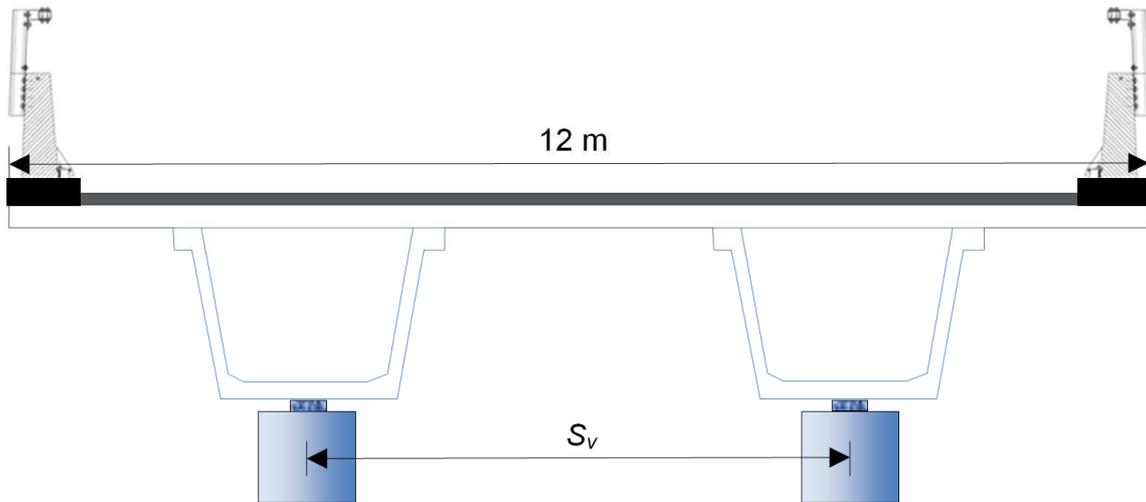
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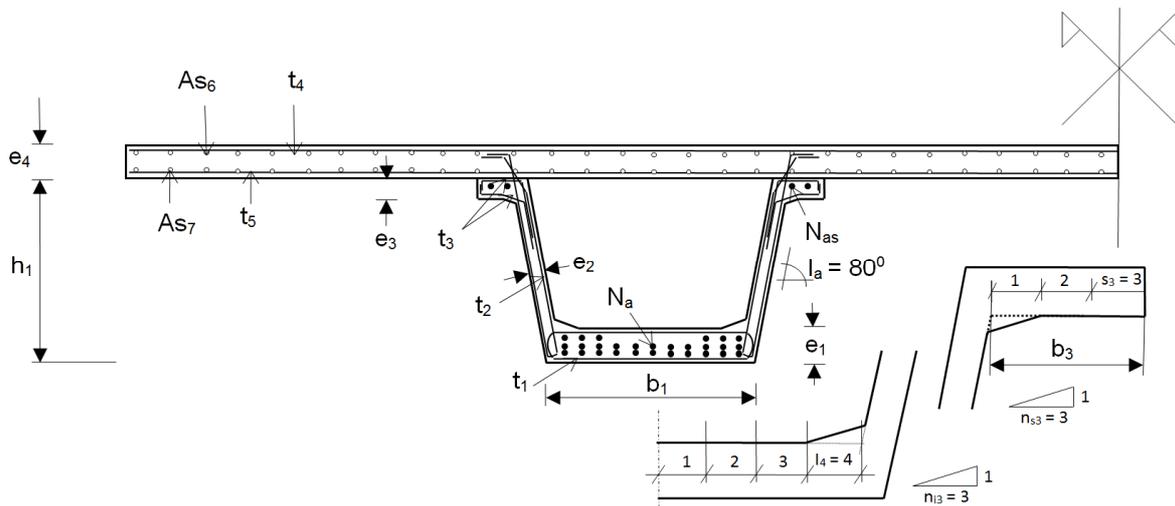
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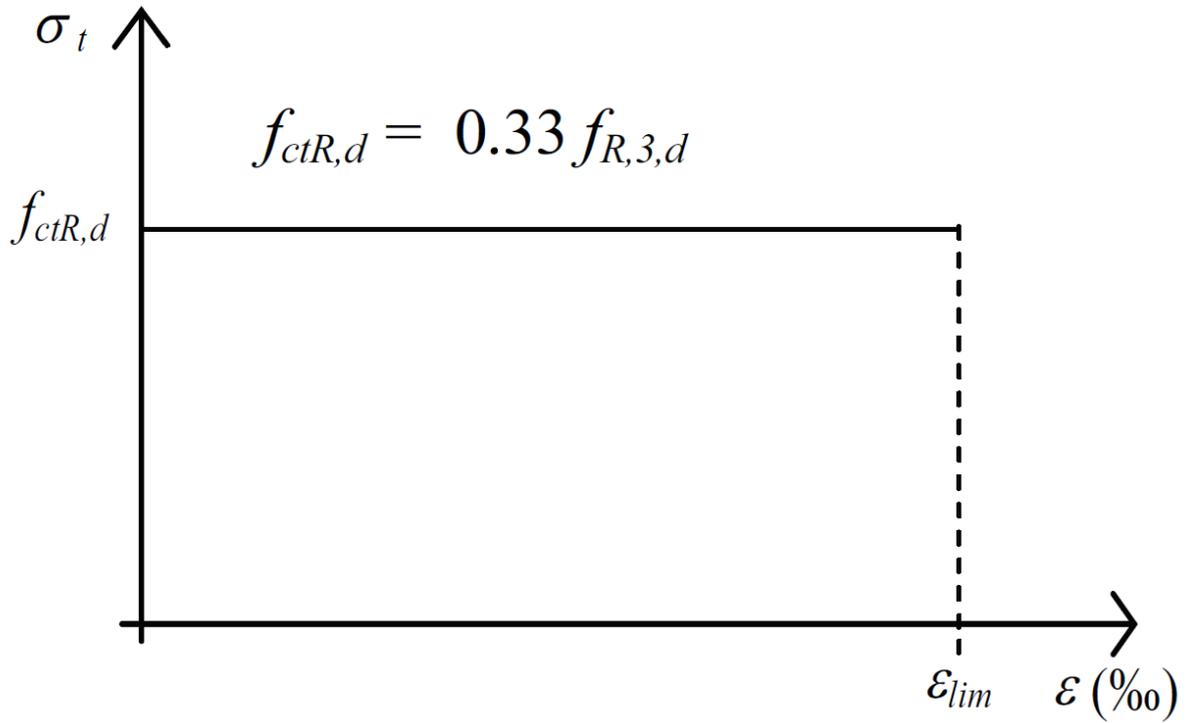


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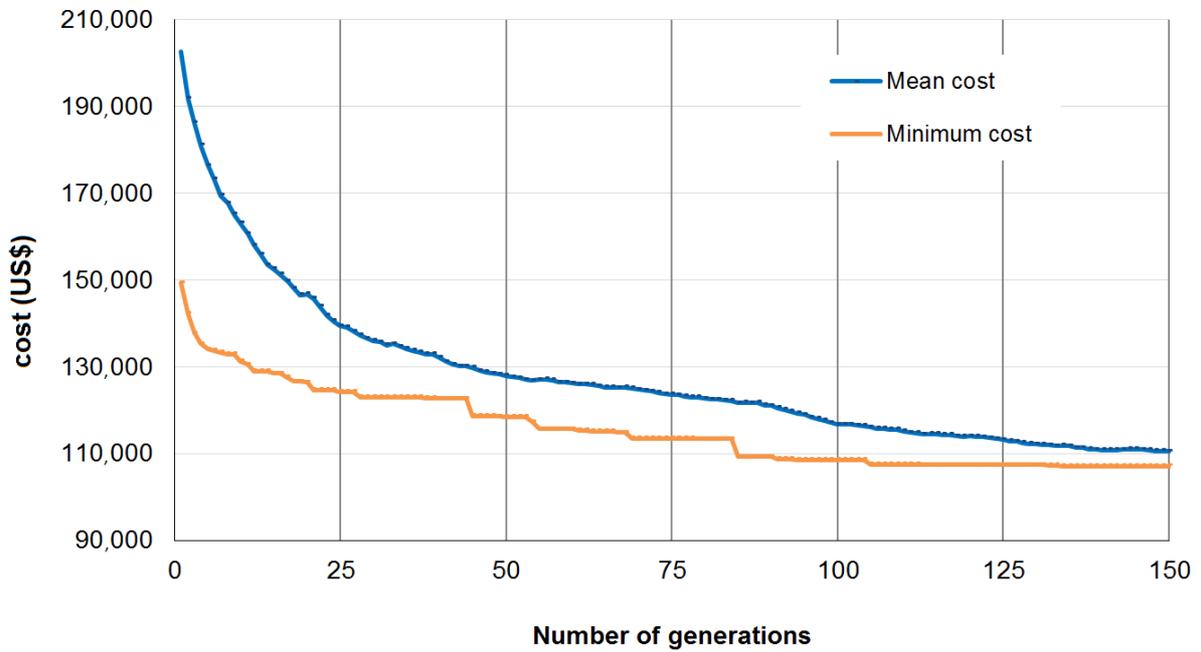


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Fig. 3. Rectangular calculation diagram of concrete with fibers

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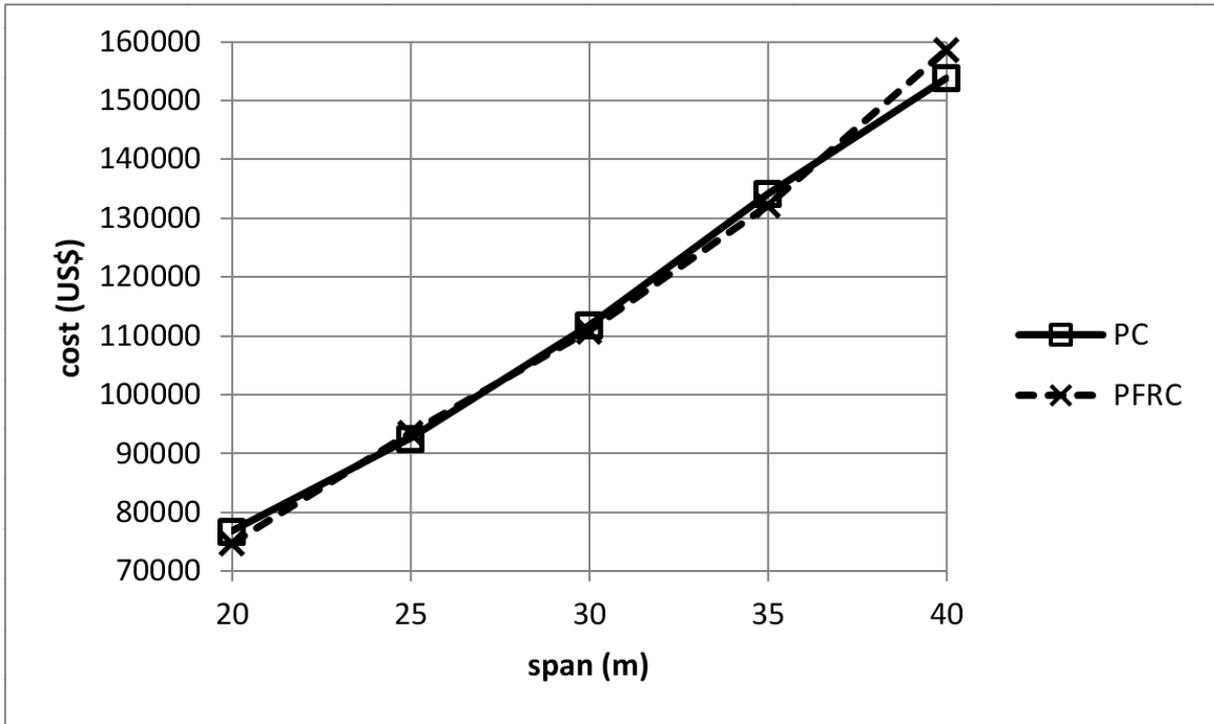
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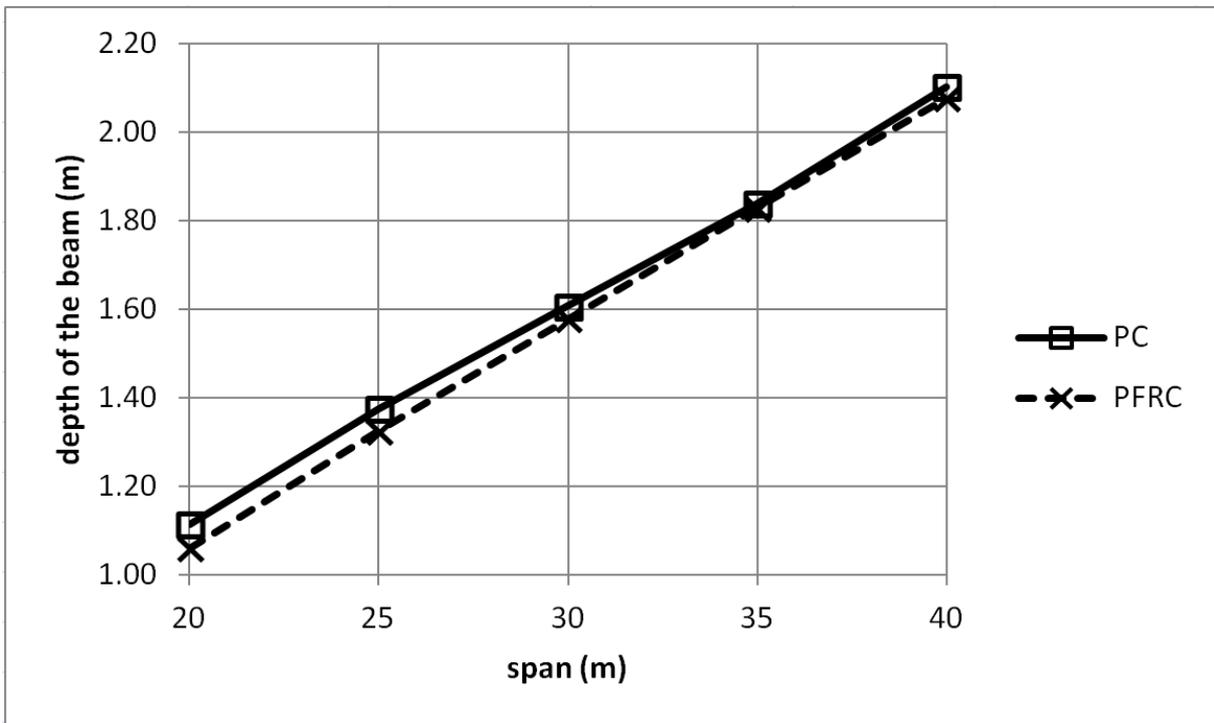


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Fig. 5. PC and PFRC precast road bridges mean costs for different span lengths

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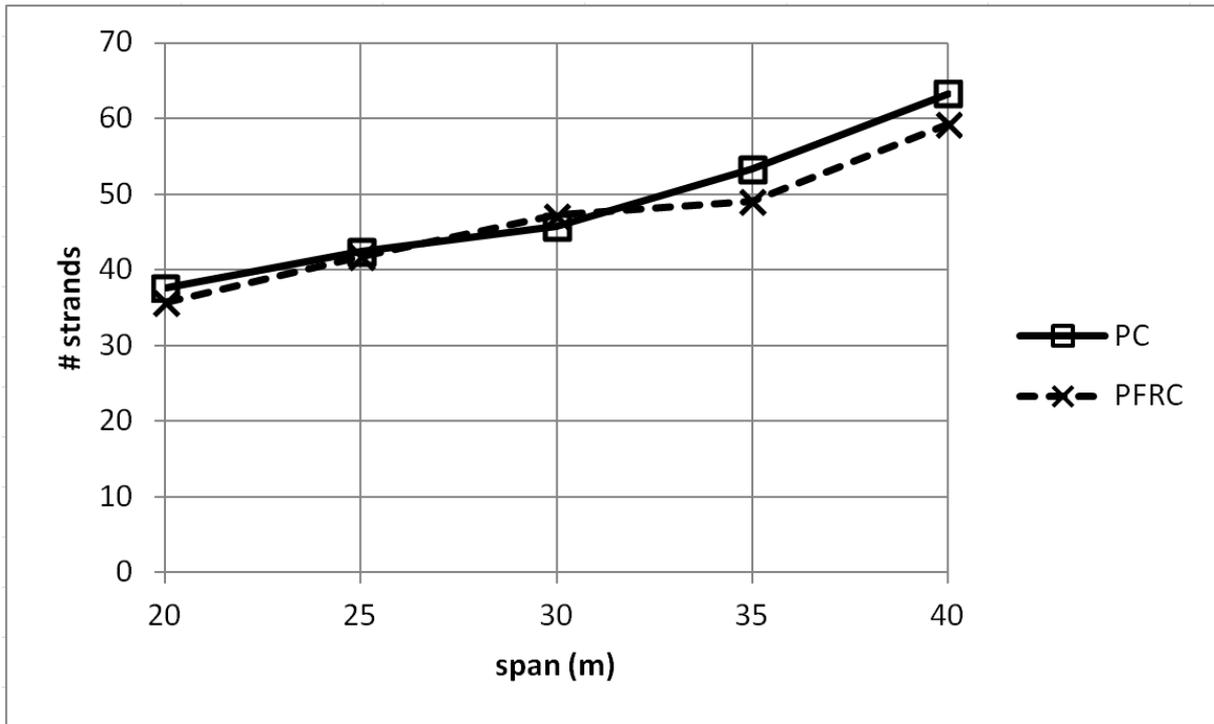


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Fig. 6. Mean depth of the PC and PFRC beams for different span lengths

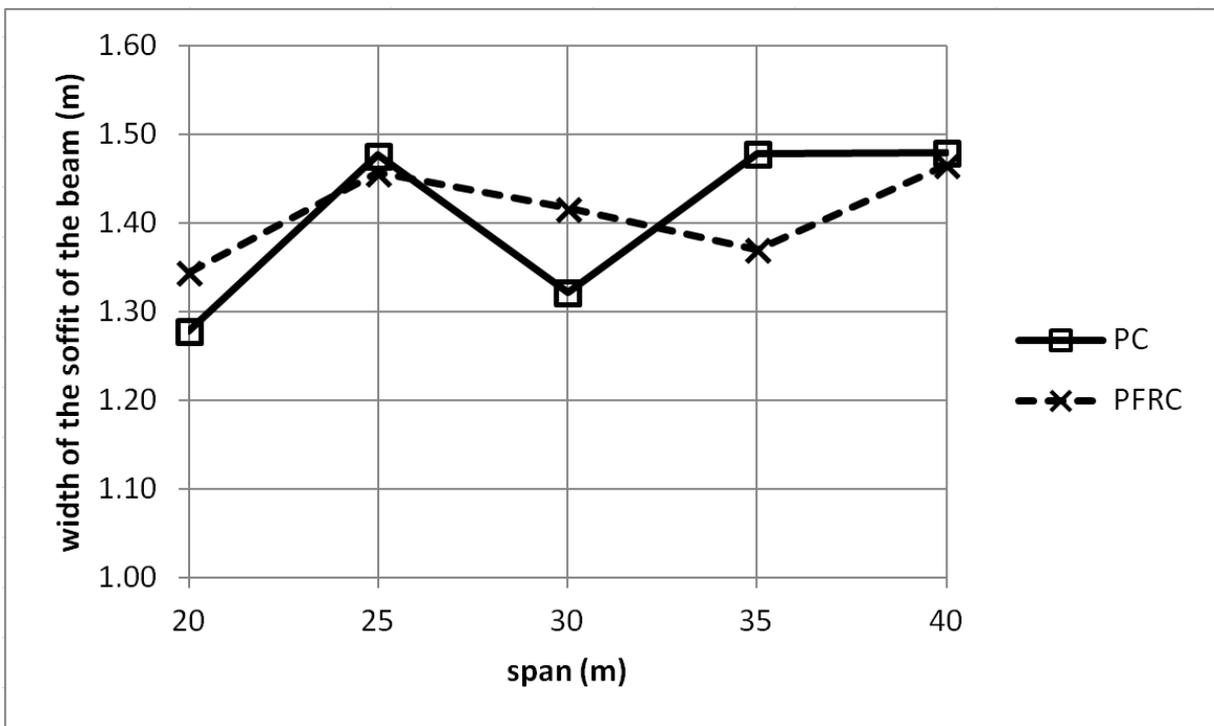
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514 **Fig. 7.** Average number of strands in relation to the span lengths and use of fibers

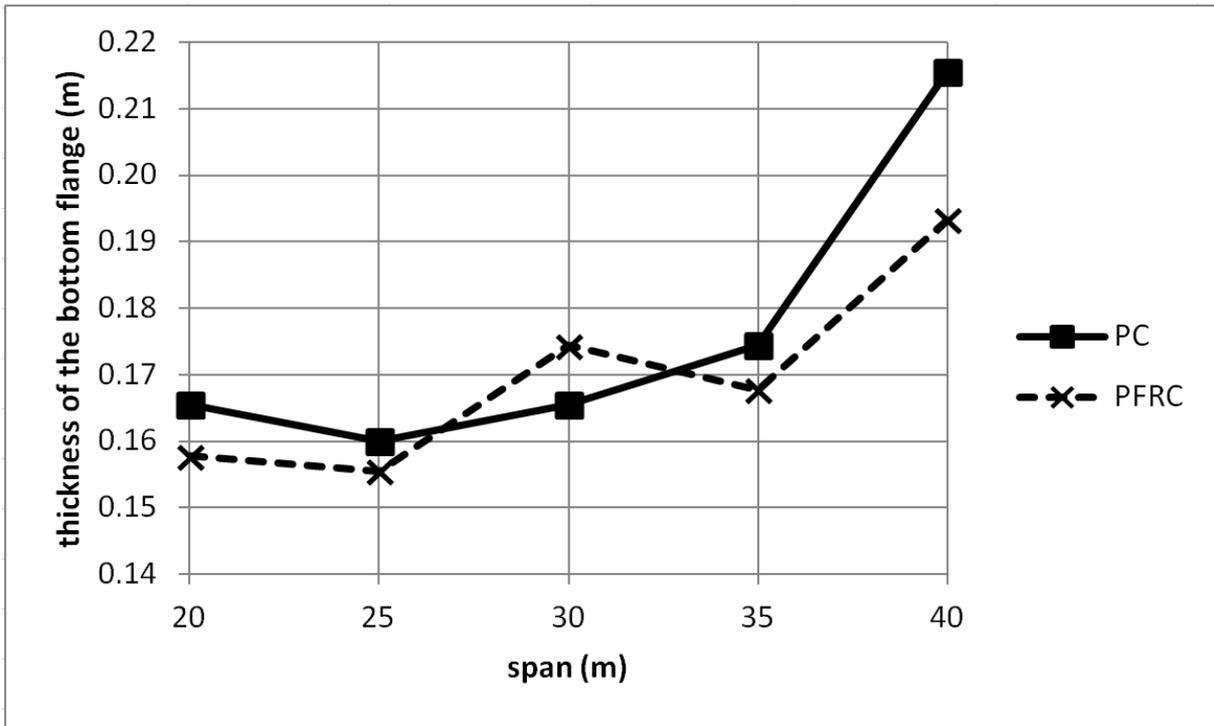
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517 **Fig. 8.** Width of the soffit of the beam in relation to the span lengths and use of fibers

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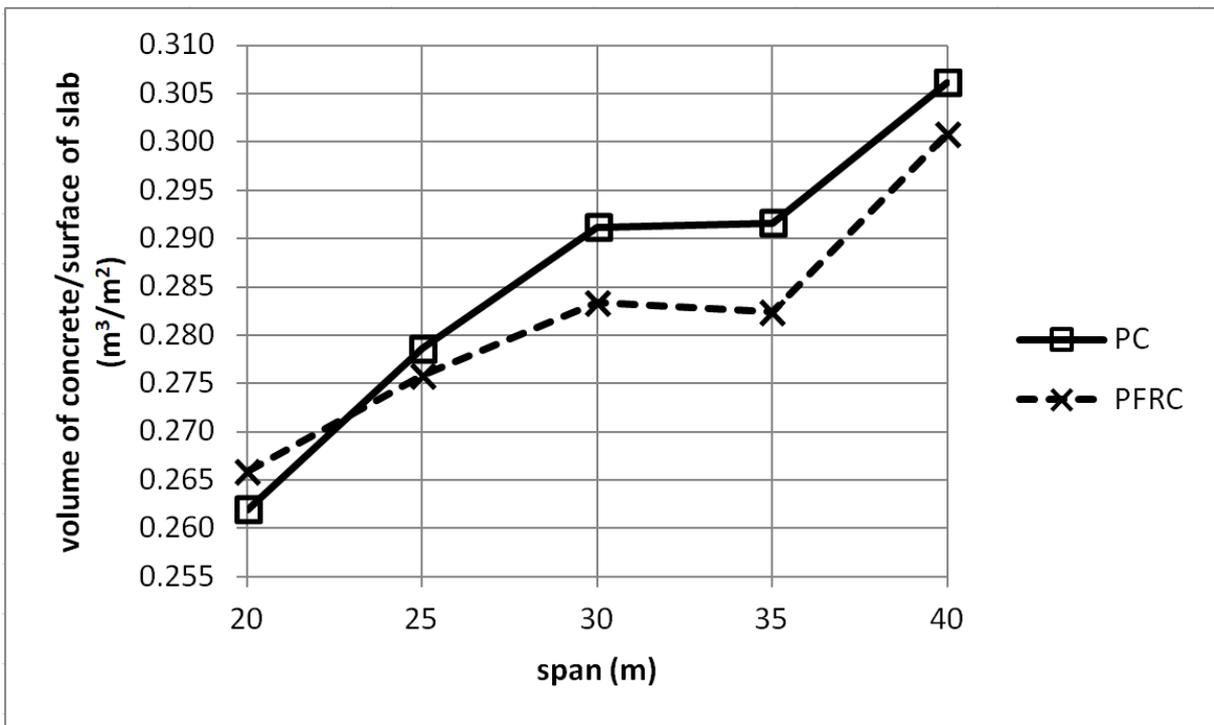
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Fig. 9. Thickness of the bottom flange of the beam in relation to the span lengths and use of

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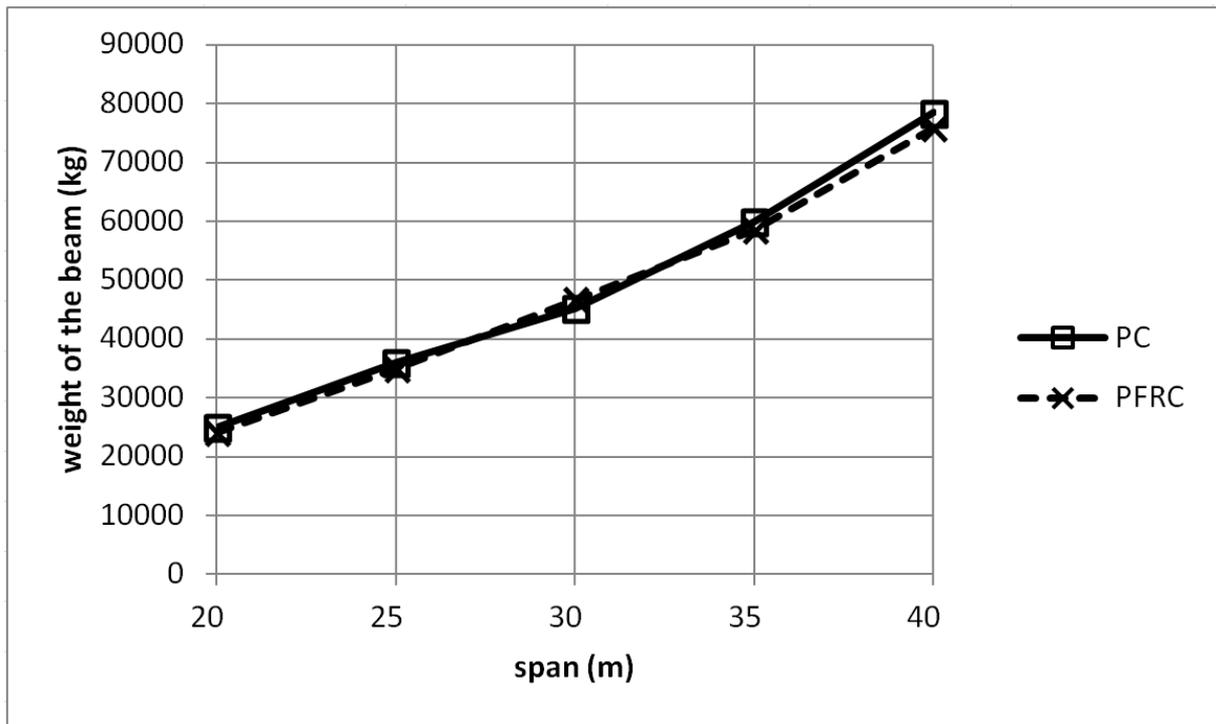
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Fig. 10. Mean volume of concrete-to-surface of slab ratio in relation to the span lengths and use

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of fibers

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527 **Fig. 11.** Mean weight of the beam in relation to the span lengths and use of fibers

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530

531 **Table 1.** Unit cost values.

Input parameter	Unit	Value
Cost of beam steel (B-500-S)	US\$/kg	3.65
Cost of slab steel (B-500-S)	US\$/kg	1.95
Cost of active steel (Y1860-S7)	US\$/kg	4.71
Cost of beam formwork	US\$/m	104.48
Cost of slab formwork	US\$/m ²	41.60
Cost of slab concrete HA-25	US\$/m ³	91.00
Cost of slab concrete HA-30	US\$/m ³	97.50
Cost of slab concrete HA-35	US\$/m ³	104.00
Cost of slab concrete HA-40	US\$/m ³	110.50
Cost of beam concrete HP-35	US\$/m ³	170.05
Cost of beam concrete HP-40	US\$/m ³	185.56
Cost of beam concrete HP-45	US\$/m ³	197.73
Cost of beam concrete HP-50	US\$/m ³	212.67
Cost of beam steel fiber	US\$/kg	1.43

532

533 **Table 2.** Steel reinforcement, cost correction coefficients.

Diameter (mm)	Beam		Slab	
	Material	Labor	Material	Labor
D6	1.250	1.400	1.250	1.400
D8	1.170	1.250	1.170	1.250
D10	1.075	1.100	1.075	1.100
D12	1.000	1.000	1.000	1.000
D16	0.980	0.900	0.980	0.900
D20	0.980	0.900	0.980	0.900
D25	-	-	1.000	0.800
D32	-	-	1.000	0.800

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537 **Table 3.** Beam transport costs (distance up to 50 km/one way).

Maximum beam weigh (kN)	Transport cost (US\$)
550	1356
660	1773
800	2295
1000	2538
2000	3929
4000	5320

538

539 **Table 4.** Beam placing costs.

Maximum beam length (m)	Placing cost (US\$)
20	4034
25	4173
30	7094
35	7233
40	8624

540

541 **Table 5.** Input parameters for analysis.

Input parameter	Unit	Symbol	Value
PC precast bridge width	m	W	12.00
Inclination, top flange tablet	-	n_{s3}	3
Top flange division	-	s_3	3
Inclination, bottom flange tablet	-	n_{i3}	3
Bottom flange division	-	i_4	4
Web inclination	degree	I_a	80
Beam slenderness	Span/ h_1	-	>17
Bearing center to beam face distance	m	E_{nt}	0.47
Concrete bridge barrier width	m	a_{bar}	2x0.50
Thickness of wearing surface	m	t_{ws}	0.09
Concrete bridge barrier loads	kN/m	Q_m	2x5.0
Transport distance (one way)	km	T_d	50
Active prestressing steel crops	%		25
Passive reinforcing steel (B-500-S)	N/mm ²	f_{yk}	500
Active prestressing steel (Y1860-S7)	N/mm ²	f_{pk}	1700
Strand diameter	inches	Φ_s	0.6
Beam surface reinforcement	mm	Φ_r	8
Strand sheaths	Levels 2 and 3		
Stirrups, vertical slenderness	200 (length/diameter)		

542 **Table 6.** MA-VDNS cost results from nine runs for 20-25-30-35-40 m spans.

Span (m)	PC			PFRC		
	Mean cost (US\$)	Minimum cost (US\$)	Deviation (%)	Mean cost (US\$)	Minimum cost (US\$)	Deviation (%)
20	76,779	73,052	5.1	74,706	72,877	2.5
25	92,547	86,505	7.0	93,604	91,850	1.9
30	111,848	108,308	3.3	110,813	107,054	3.5
35	134,199	128,627	4.3	132,135	121,733	8.5
40	153,829	140,759	9.3	158,534	139,255	13.8

543 **Table 7.** MA-VDNS best solutions for 20-25-30-35-40 m spans.
544

Span (m)		h_1 (m)	e_4 (m)	b_1 (m)	b_3 (m)	e_1 (m)	e_2 (m)	e_3 (m)	$f_{c,beam}$ (MPa)	$f_{c,slab}$ (MPa)	p_1 (n)	p_2 (n)	p_3 (n)	p_4 (n)	S_v (m)	f_{r3k} (MPa)
20	a	1.13	0.18	1.28	0.23	0.20	0.10	0.15	45	30	22	17	0	2	5.67	-
	b	1.07	0.18	1.45	0.25	0.15	0.11	0.10	40	35	26	8	0	4	5.65	4.0
25	a	1.35	0.19	1.26	0.23	0.15	0.10	0.15	35	25	22	22	0	2	5.39	-
	b	1.31	0.20	1.08	0.32	0.15	0.10	0.10	45	35	18	18	0	2	5.60	5.5
30	a	1.61	0.19	1.07	0.24	0.15	0.10	0.17	35	35	18	18	0	4	5.67	-
	b	1.65	0.18	1.22	0.30	0.19	0.10	0.15	35	30	21	21	0	2	5.73	6.0
35	a	1.83	0.17	1.33	0.23	0.16	0.10	0.16	45	30	23	23	0	2	5.46	-
	b	1.78	0.17	1.35	0.23	0.15	0.10	0.15	40	30	24	24	0	2	5.61	5.0
40	a	2.07	0.18	1.29	0.23	0.18	0.10	0.15	35	40	22	22	0	2	5.64	-
	b	2.11	0.17	1.25	0.25	0.15	0.10	0.11	35	25	22	22	0	2	5.25	3.0

545 (a) PC

546 (b) PFRC

547 **Table 8.** MA-VDNS basic measurements for 20-25-30-35-40 m spans.

Span (m)		Beam reinforcement (kg)	Slab reinforcement (kg)	Total reinforcement (kg/m ²)	Beam concrete (m ³ /m ²)	Slab concrete (m ³ /m ²)
20	a	2,794	8,137	43.38	0.079	0.183
	b	1,666	8,938	42.08	0.076	0.189
25	a	3,645	8,740	39.69	0.092	0.186
	b	2,107	11,830	44.67	0.090	0.186
30	a	5,399	10,089	41.63	0.097	0.194
	b	3,673	11,756	41.47	0.100	0.183
35	a	6,895	11,562	42.73	0.111	0.181
	b	4,377	15,318	45.59	0.108	0.174
40	a	7,968	10,343	37.22	0.128	0.179
	b	5,778	17,598	47.51	0.123	0.178

548 (a) PC

549 (b) PFRC

