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From the simplest equations of Hydrodynamics to science and engineering modeling skills

Desarrollo de habilidades de modelación desde las ecuaciones más simples de la Hidrodinámica

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Abstract

The development of modeling skills is a very important issue in Science teaching nowadays. The present work illustrates how, from the simplest equations of hydrodynamics, it is possible to contribute to this end. Bernoulli and continuity equations are included in Physics syllabi of secondary and university levels, and can be seen as a linking between general and professional education. By means of the proposed project, students are taken through general stages which are usually present in any engineering project or research work based on modeling and simulation. such as the formulation of the problem, the statement of the Physics model, a computational simulation and the comparison between theory and experiments. This kind of project allows for the development of modeling skills and also to some other typical skills of the scientist's and engineer's profiles nowadays, such as fitting and graphing analysis. It is common to see that secondary and first year university courses do not contribute much to the formation of modeling skills, instead they rather contribute to particular skills from the perspective of the different subjects. On the other hand, students are usually more motivated for the modeling of real world situations than for idealized ones.

El desarrollo de habilidades relacionadas con la modelación es un aspecto esencial en la enseñanza de las ciencias hoy en día. El presente trabajo ilustra una propuesta de cómo desarrollar habilidades de modelación físico-matemáticas desde las ecuaciones más simples de la hidrodinámica, es decir, la ecuación de Bernoulli y la ecuación de continuidad. Estas ecuaciones representan la conservación de la energía y de la masa, respectivamente, y están presentes comúnmente en los programas de Física para la Enseñanza Secundaria y Universidad. A través del proyecto propuesto, el estudiante transita a través de etapas generales usualmente presentes en los proyectos de innovación ingenieril o de investigación, es decir, el surgimiento de la idea inicial, el planteamiento del modelo físico, la exploración computacional del mismo, y la comparación con medidas experimentales. El proyecto presentado hace uso directo de habilidades tales como la realización de ajustes y análisis gráficos, típicas en los perfiles de ingenieros e investigadores en la actualidad. Por otro lado, los estudiantes presentan más motivación por aquellas situaciones más cercanas a la realidad que por las muy idealizadas.

Keywords: modeling skills, Physics teaching.
Palabras clave: habilidades de modelación, enseñanza de la Física.

1. Introduction

Science and engineering modeling skills are highly demanded in modern society. There is a great variety of scientific and technological challenges that should be faced what makes the development of this type of skills a crucial issue in science teaching nowadays.

There is a number of works which indicate the lack of these skills in secondary and university students. Wedelin et al. (2009) have investigated on how engineering students approach and deal with mathematical modeling problems. Their findings indicate that students had almost no previous experience in problem solving and are unaware about the different alternatives to modeling a given situation. The metacognitive domain was found to highly impact on the course results. Important factors positively influencing the courses were the nature of the problems, the supervision and the follow-up lectures. In line with this, Nair et al. (2009) pointed that university graduates do not fulfill the expectancies of the employers. Commonly, they lack of communication, decision-making, problem-solving and leadership skills, well as of the ability to work in multidisciplinary and multicultural environments which are fundamental professional attributes (Radcliffe, 2005; Wellington et al., 2002; Patil, 2005). All these issues should be addressed at earlier stages of the educational system, such as the secondary level.

Justi and Gilbert (2002) carried out a study to enquire into the knowledge of models and modeling held by a total sample of 39 Brazilian science teachers: 10 working in 'fundamental' (ages 6-14 years); 10 at the 'medium' (ages 15-17 years); 10 undergraduate pre-service 'medium'-level teachers and 9 university teachers of chemistry. Results showed that teachers are aware of the value of models in the development of science but not of their value in learning about science. Teachers were uncertain of the relationship that could exist in the classroom between various types of models. On the student's side, results indicated that modeling was recognized in theory but not widely considered in practice.

The relationship between modeling and argumentation in a context of Chemistry has been addressed by Cardoso-Mendosa and Justi (2013). Modeling has been proven to be an argumentative process. Authors identified argumentative situations when students performed all of the modeling stages. They also show that representations are important resources for argumentation.

From the theoretical point of view, Justi and Gilbert (2002) proposed the Model of Modeling Diagram (MDD) (also cited by Cardoso-Mendosa and Justi, 2013). This model was structured in four stages. The first corresponds to the production of a mental model. This stage was subsequently divided into other four sub-stages. The first one refers to the purposes of the model. The second sub-stage relates to acquire knowledge required for the model, that is, the object of the model and everything helpful for its development. The other two sub-stages include the individual's creativity and thinking skills which combine with the occurrence of the previous sub-stages in the mental realization of the model. The stage 2 of MMD refers to the dynamic relationship between mental and expressed models. The model is expressed by means of any of the modes of representation, i.e. material, visual, and verbal. The stage 3 corresponds to the model tests by any of both thought and empirical experiments. Finally, the stage 4 corresponds to the evaluation of the model, that is, to find out what was or not able to explain regarding the initial purposes.

Scientific models are on the grounds of the design of any new technology. Once the intuitive idea comes up, the engineer should be able to get it under way following the methodologies of modeling, simulation and design. There is a long road between the initial idea and the final technological innovation, and the engineer should be able to walk it along. Physics concepts pave the road of technical engineering concepts. In this respect, the construction of Physics models to characterize real world situations result very motivating for the students and allow them to approach global schemes of work very similar to those characterizing the scientist's and engineer's profiles in the job market.

It is usual to see that from the first year of technical engineering studies no appreciable contribution is made to innovation skills which are rather postponed for higher academic years where the skills developed by the single subjects are integrated. With some pedagogical effort, the traditional classes of Physics can contribute enormously to the development of modeling skills. Physics concepts can be taught in such a way that their application to solve practical problems be professionally illustrated.

In the present article, a Physics course project is introduced and carried out. It corresponds to the topic of Hydrodynamics within the field of Fluid Mechanics (Resnick et al., 1999; Fishbare et. al., 1996; and Alonso and Finn, 1992) which is present in Secondary and first university year of science and technical engineering studies. This is a topic for which there are a few student projects and laboratory experiments available. This project contributes towards this need and involves very simple Mathematics. Only two simple equations are related, namely, Bernoulli's continuity equations.

The project consists basically of the characterization of a water container placed at a higher place from where the water comes down by gravity. This situation may correspond to water tanks placed on roofs of houses, mainly, of those typical from the countryside or farmhouses. This is also the case of many houses in developing countries. In this respect, there are several questions that may come up, for example: "Taking into account a daily consumption of water, how long will we have water available in the container?"; "in case of contamination of water, how long does it take to evacuate the container?"; and "what is the volume of water available at a given moment?" It is very interesting to illustrate how, by using the two simplest equations of Hydrodynamics, present in Secondary and first university year, the questions above can be addressed. While developing the project, students should go through a modeling stage and an experimental stage (where theory is contrasted). These steps are always present somewhat in the road towards a technological innovation.

The outline of the paper is the following. In section 2, the Motivation for the Project is presented along with the steps followed in its development which are introduced in two subsections: (2.1) The Physical Model , and (2.2) Computational Exploration and Experiments. In section 3, some conclusions are drawn. The reader will notice that there is a first part where Secondary level math is used and a second, where very simple derivatives and integrals are introduced, which make the project also appropriate for first-year university year courses.

2. Development of the project

Every one of us has at least one time noticed that sometimes the water slower at the shower or any other tap in small countryside or farm houses where the water reservoir usually consists of a tank placed on the roof. It is very easy to notice that this happens because the water container on the roof is fuller when the water at the tap flows faster or emptier when it flows slower, respectively. This situation suggests directly that there is a relation between the velocity of the water at the shower and the volume of water in the container. By knowing the velocity of the water at the shower, and with the help of the proper physics model, the water level at the container can be known. Subsequently, if the geometry is known, the volume of the container can be calculated as well. This statement is itself a hypothesis and, as it is supported by some direct observations, it can be considered a scientific hypothesis. This is the starting point of our proposal.

2.1. The physical model

Bernoulli's equation (Resnick et al., 1999; Fishbare et. al., 1996; and Alonso and Finn, 1992) states that along a streamline, the left-hand side of the following expression equals constant:

$$P + \frac{1}{2}\rho v^2 + \rho g h = H, \tag{1}$$

where P is the pressure, v the velocity of the flow, h the level of elevation with respect to an adopted reference, g the Earth's gravity, ρ the density of the fluid, and H the total energy head. This equation represents the law of Conservation of Energy. This model is valid for incompressible, inviscid and laminar fluids. Even though, we will show that it can be applied to real fluids, whenever their viscosity, turbulence and compressibility are little manifested. This aspect should be emphasized in the case of engineering students since the right knowledge of the applicability of the model is a very important issue which can be contrasted by the direct comparison with experiments. The length of the pipes are considered small for the sake of simplicity and to keep the head losses low.

Another equation which is basic in Hydrodynamics is the continuity equation (Resnick et al., 1999; Fishbare et. al., 1996; and Alonso and Finn, 1992), which states that along a streamline, the product of the velocity (v) by the area of the cross section (S) is a constant, which is called flow rate, Q. This equation is an expression of the conservation of the mass of the fluid along different cross sections,

$$vS = Q. \tag{2}$$

Let us apply equations 1 and 2 to determine the velocity of the flow at an open tap inside a house, considering the water coming from a container placed on the roof, at a given level of elevation, h from above the tap. The container is considered as an open container, that is, under the influence of the atmospheric pressure. A schematic representation of the situation is shown in Figure 4. The model of a single container and a tap is still general when all taps of the house are closed except one. From now on, for simplicity and given that it is still valid for practical cases, the shape of the container will be considered cylindrical. The cross section of the pipes and tap are circular.

The Bernoulli's equation adopts the following form for this case:

$$v_1^2 + 2g(h + h_2) = v_2^2, \tag{3}$$

where v_1 is the velocity of the water level of the container, v_2 the velocity of the water at the tap, P_{at} the atmospheric pressure, and ρ the density of the water. The remaining quantities are indicated in Figure 4.

Let S_1 be the area of the cross section of the container and S_2 the cross section of the tap. The continuity equation can be rewritten as:

$$v_1 S_1 = v_2 S_2. \tag{4}$$

Combining equations 3 and 4, we get:

$$h(v_2) = \frac{1 - (S_2/S_1)^2}{2g} v_2^2 - h_2. \tag{5}$$

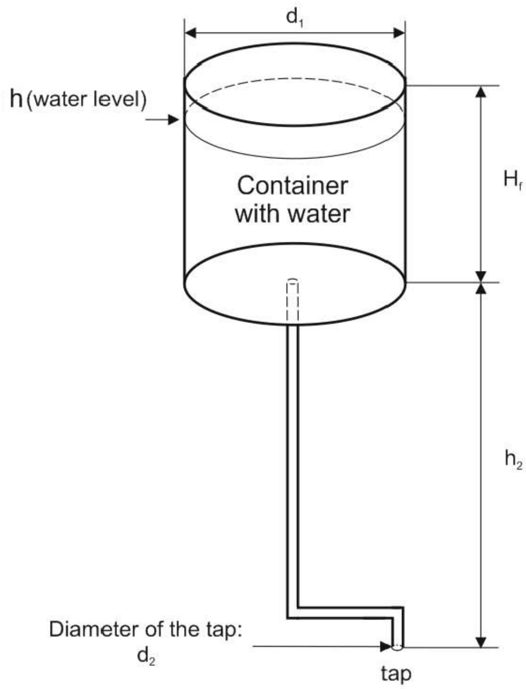

Figure 1: Schematic representation of the container and the tap. Variables used in the equations are shown.

Considering the diameters of the circular cross sections of the tap, d_2, and the container, d_1, the equation above can be rewritten in the following form:

$$h(v_2) = \frac{1 - (d_2/d_1)^4}{2g} v_2^2 - h_2. \tag{6}$$

Taking into account the cylindrical form of the container, its volume for a given velocity of the water flow at the tap is expressed as:

$$V(v_2) = \frac{\pi d_1^2}{4} \left(\frac{1 - (d_2/d_1)^4}{2g} v_2^2 - h_2 \right). \tag{7}$$

By using the equation above, the volume of the container can be calculated if the velocity of the fluid at the tap is known.

Simple method for the calculation of the velocity of the water at the tap

In the following we will calculate the velocity of the water at the tap, v_2. Let the small container be a cylindrical body of area of the base, S_c, diameter d_c, height h_c, and volume V_c. According to the continuity equation, the volume of water that fills up the small container over a time period (t_f) equals the velocity (v_2) multiplied by the area of the cross section (S_2), that is:

$$\frac{V_c}{t_f} = v_2 S_2 \tag{8}$$

Writing v_2 as a function of the diameters (d_2 and d_c) and the height h_c, we get:

$$v_2 = \frac{d_c^2 h_c}{d_2^2} \frac{1}{t_f}. \tag{9}$$

Equation 9 shows a simple method to calculate the velocity of the water at the tap by using a small container and measuring the necessary time to fill it up. Until this point, only secondary school math operations have been involved. In the following, very simple integrals from the first university math course will be introduced what makes the model appropriate for the introductory Physics courses at the university level.

Taking v_2 apart in equation 6, it is possible to calculate the time elapsed to evacuate the container, t_e, which is initially filled up to a given height, h. Let us denote the variable representing the variation of h by h_1. By considering the continuity equation 4 and the definition of instant velocity, we obtain:

$$v_2 S_2 = \left(\frac{2g(h_1 + h_2)}{1 - (d_2/d_1)^4} \right)^{1/2} S_2 = v_1 S_1 = -\frac{dh_1}{dt} S_1. \tag{10}$$

Separating variables in equation 10, we obtain:

$$\frac{S_2}{S_1} \int_0^{t_e} dt = -\int_{h_2+h}^{h_2} \left(\frac{2g(h_1 + h_2)}{1 - (d_2/d_1)^4} \right)^{1/2} S_2 \, dh_1, \tag{11}$$

where t_e is the time elapsed to evacuate the container initially with water level h. By integrating at both sides of the equation, we get:

$$t_e(h) = -(4h_2)^{1/2} \left(\frac{1 - (d_2/d_1)^4}{g} \right)^{1/2} \left(1 - \left(\frac{2h_2 + h}{2h_2} \right)^{1/2} \right). \tag{12}$$

Equation 12 allows for the calculation of the elapsed time (t_e) to evacuate a cylindrical tank initially filled up to the water level h. The evacuation time can be estimated roughly by knowing the amount of water that is used daily on average, V_{ave}. The evacuation time (in days) or the number of days with water available can be expressed as $V_{tot}(h)/V_{ave}$, where $V_{tot}(h)$ is the volume for a given water level h.

By using the equation 7, and by determining the velocity of the water at the tap, v_2, the volume of the container at a given moment can be calculated. For this purpose, if we consider a uniform filling (at constant velocity) of a small container, it is possible to calculate the velocity of the water at the tap, v_2. If the volume of the small container is very small in comparison to the volume of the big container, the velocity of the water at the tap will not vary appreciably.

Combining equations 6 and 9, the height of the water level at the container is obtained as a function of the filling time of the small container, t_f.

$$h(t_f) = \left(\frac{\left(1 - (d_2/d_1)^4\right)(d_c/d_2)^4 h_c^2}{2g} \right) \frac{1}{t_f^2} - h_2. \tag{13}$$

Combining equations (7) y (9), the volume of the container is obtained as a function of the filling time, t_f.

$$V(t_f) = \frac{\pi d_1^2}{4}\left(\frac{\left(1-(d_2/d_1)^4\right)(d_c/d_2)^4 h_c^2}{2g}\frac{1}{t_f^2} - h_2\right). \tag{14}$$

In this case, the term $(d_2/d_1)^4$ in equations 12, 13 and 14 can be neglected since d_1 is much larger than d_2. The filling time, t_f, can be measured, for example, with a chronometer app for smartphones which is a very familiar device to the students.

2.2. Computational exploration and experiments

In order to illustrate a real example, let us apply the theory presented in section 2.1 to the real case of the water pipes of the countryside house of one of the authors. In Table 1 and 2, the geometrical features of the container, the tap and the small container used to measure the velocity of the water at the tap, are registered. It can be seen that everyday measurement instruments such as the ruler and the metric tape are used.

Quantity	**Value**	**Instrument**	**Precision of the instrument used in the measurement**
Diameter of the cylindrical water tank	$d_1 = 1.53$ m	Metric tape	$\Delta d_1 = 0.001$ m
Height of the cylindrical water tank	$H_f = 1.27$ m	Metric tape	$\Delta d_1 = 0.001$ m
Diameter of the tap	$d_2 = 0.008$ m	Ruler	$\Delta d_2 = 0.001$ m
Level of elevation above the bottom of container	$h_2 = 1.670$ m	Metric tape	$\Delta h_2 = 0.001$ m
Total volume of the container = 2.33 m^3			

Table 1: Direct measurements of d_1, d_2 and h_2 and the corresponding precision of the instruments used.

Quantity	**Value**	**Instrument**	**Precision of the instrument used in the measurement**
Diameter of the small container (d_c)	$d_c = 0.28$ m	Ruler	$\Delta d_c = 0.001$ m
Height of the small container (h_c)	$h_c = 0.09$ m	Ruler	$\Delta h_c = 0.001$ m
Filling time (t_f)		Chronometer	$\Delta t_f = 0.01$ s

Table 2: Direct measurements of d_c, h_c and the corresponding precision of the instruments. The precision of the instrument used to measure the filling time t_f is also included.

The volume of the small container that has been used to measure the velocity of the water at the tap is 2.37 % of the total volume of the container. This means that the velocity at the tap will not vary considerably during the measuring interval. A Fortran (Chapman, 2003) code has been used to perform the simulations, but this is not the only option available. A simple Microsoft Excel sheet or any other calculus spreadsheet can be used for this purpose.

In Figure 2, the theoretical curve of $h(t_f)$ (equation 13) is shown (solid line) in comparison to the experimental points (open triangles). It can be observed that the longer it takes to fill up the small container the smaller the level of elevation of the water in the big container. There is almost a linear dependency between both variables.

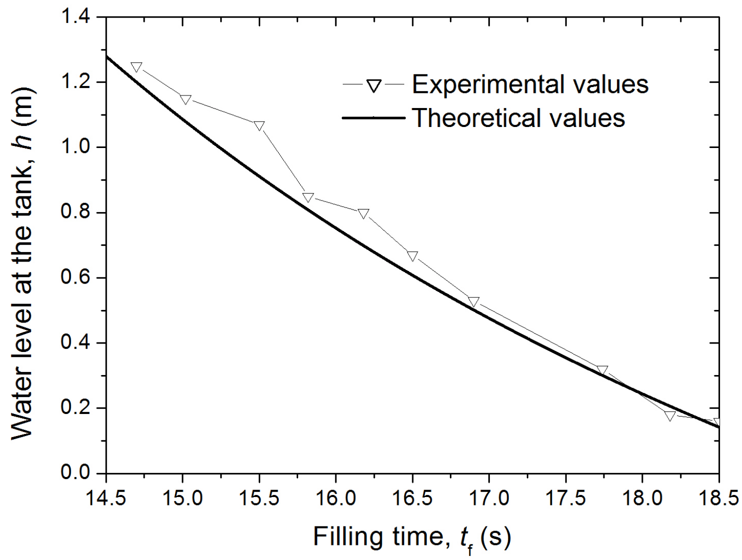

Figure 2: Theoretical values of $h(t_f)$ (solid line) in comparison to the experimental points (open triangles).

In Figure 3, the volume $V(t_f)$ of the container is plotted versus the filling time, t_f. As the volume of the container is proportional to its height, the features of this figure are similar to the previous one.

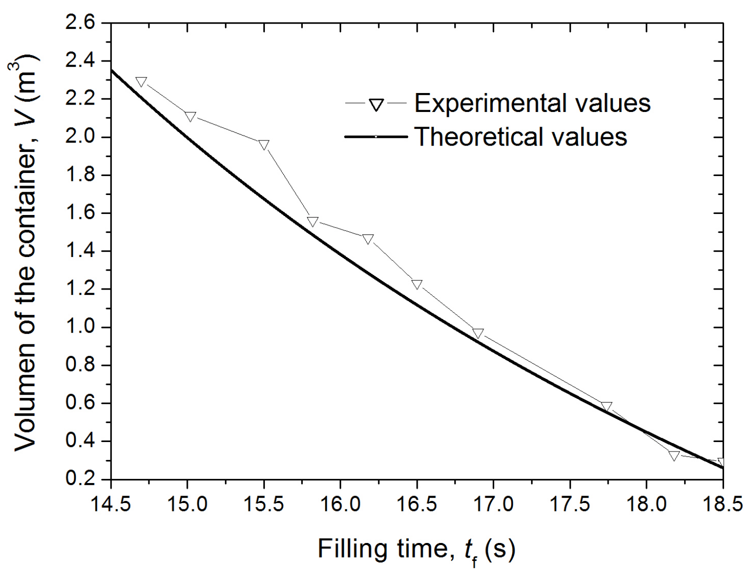

Figure 3: The volume $V(t_f)$ of the container is plotted versus the filling time t_f.

In Figure 4, the water level at the tank, h is represented as a function of the total time necessary to empty it, t_e. It takes 4.6 hours to evacuate a volume of 2.33 m^3.

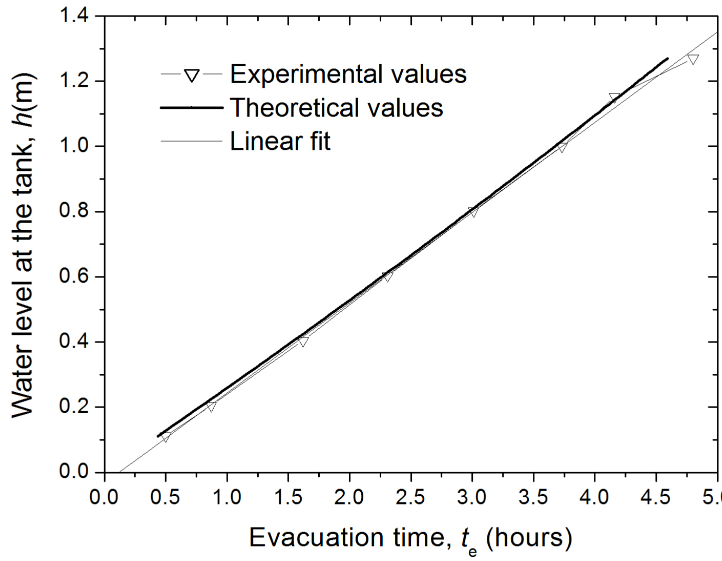

Figure 4: Time elapsed to evacuate the container as a function of its water level h.

As the curve of Figure 4 shows a nearly linear trend, a linear fit has been applied. The resulting linear model can be useful for practical purposes. This way of proceeding represents a common engineering scheme. The parameters of the fit have been included in Table 3.

$h(t_e) = A + Bt_e$		
		Error
A	−0.033	0.013
B	0.277	0.004
R	SD	N
0.99922	0.018	8

Table 3: Results for the linear fit to $h(t_e)$ points. The Least Squares method has been applied. In the table, R is the linear correlation coefficient and SD, the standard deviation.

Let us calculate as follows the velocities of the water at the tap and of the water level at the container, respectively, as a function of the water level, h. Let us write $v_2(h)$ apart in equation 6,

$$v_2(h) = \left(\frac{2g(h + h_2)}{1 - (d_2/d_1)^4} \right)^{1/2}. \tag{15}$$

Using the continuity equation, $v_1 S_1 = v_2 S_2$ and the equation above, we can write:

$$v_1(h) = \frac{S_2}{S_1} \left(\frac{2g(h + h_2)}{1 - (d_2/d_1)^4} \right)^{1/2}. \tag{16}$$

Equations 15 and 16 represent the velocities at the tap and of the water at the container as a function of the water level, h, respectively. In Figure 5, the curve of $v_2(h)$ is plotted.

The velocity of the water level in the container ranks from 0.1616 mm/s (for $h = 0.11$ m) to 0.2075 mm/s (for $h = 1.27$ m). In fact, we can calculate the evacuation time by following a linear proportion, for example, by measuring the time elapsed for a given decrease of the water level at the tank. On the other hand, the velocity of the water flow at the tap ranks from 6.99 m/s (for $h = 0.11$ m) to 7.59 m/s (for $h = 1.27$ m). It does not vary much. It can be seen that both velocities, the one of the water in the container and the one of the water flow at the tap have a nearly linear behavior with the water level at the tank. This fact can be appreciated in the linear fit in Figure 5.

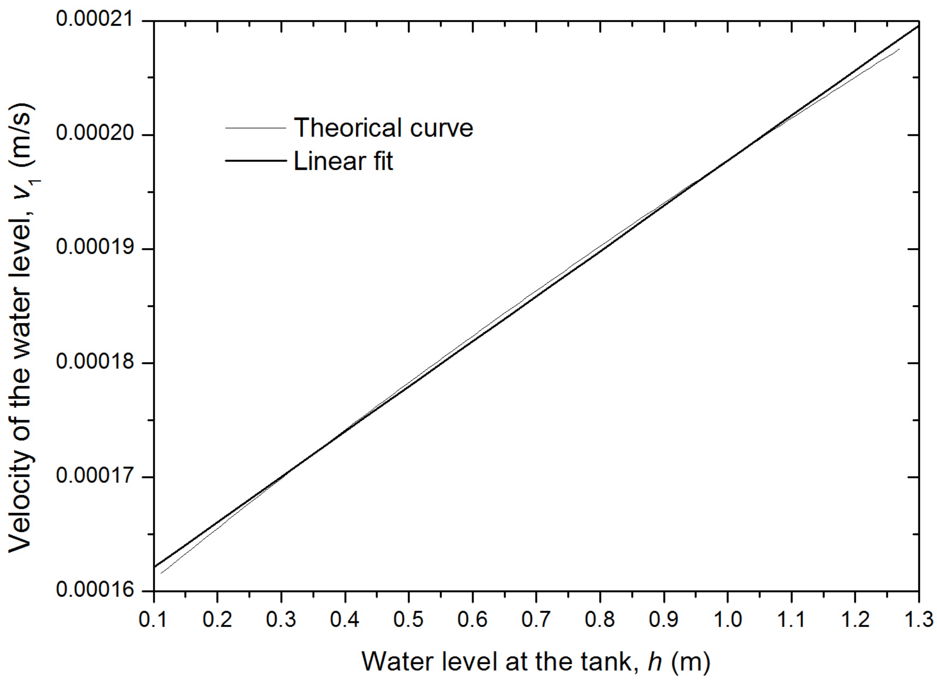

Figure 5: Theoretical curve of the velocity of the water flow at the tap versus the water level at the tank.

At this point of the article, the student has found a professional answer for the initial questions. The whole way from the intuitive idea to the professional solution has been walked. It can be seen also how very basic and simple Physics' laws and concepts helped explaining an everyday situation. This fact results meaningful and motivating for the students.

3. Conclusions

A proposal of student project for developing science and engineering modeling skills from secondary and introductory university Physics courses has been presented. The students, applying a very basic knowledge of Physics, can solve a small engineering problem. A practical situation which involves a typical water container placed on a roof of a farmhouse has been fully explored. The starting point is an intuitive idea that may arise from the direct observation of a water tap. By using a simple Physics model, students can provide answers to practical questions. They not only get to a simple and good model based on Physics equations, but can also explore all kind of conditions for the practical situation by playing with the variables involved, such determining as the evacuation time for a given height or the velocity of the water level in the container.

This kind of situation is typical in the professional scenario of engineers and scientists. Results reflect a reasonable match between theory and experiments which is a good indicator of the prediction capability of the physics model. By using a simple setup consisting of a water pipe and a container, students can experience the validy, with a professional focus, of simple Physics concepts and laws of Hydrodynamics.

Acknowledgements

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